Topological Aspects of Synthetic Polymers Through *NM* Polynomials

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Abstract. In this study, we delve into the versatile application of the Neighborhood M-Polynomial (NM) in predicting a wide array of material characteristics. Our research investigates the capability of the neighborhood M-polynomial to discern neighborhood degree sum-based topological indices when analyzing synthetic polymers. These indices serve as pivotal tools, enabling us to accurately predict the diverse physical, chemical, and biological properties inherent in the materials under scrutiny.

Keywords: Vulcanized rubber network, poly-methyl methacrylate network, NM-polynomial, topological index

INTRODUCTION

In recent decades, there has been a significant surge in the study of graphs and their various invariants. These invariants can take the form of matrices, polynomials, numeric values, or sequences assigned to a given graph. One noteworthy category among these invariants is the class of topological indices, which assigns a numerical value to a graph. The value of a topological index for a molecular structure is intricately linked to its shape, size, symmetry, bond patterns, and the types of atoms it contains [1]. Consequently, topological indices play a vital role in quantitatively characterizing molecular structures [2, 3, 4].

Numerous researchers have delved into different aspects and applications of topological indices, exploring their implications in understanding the physical, chemical, and biological properties of various materials. For more detailed information on recent advancements in the realm of topological indices, readers are encouraged to refer to the references [5, 6, 7, 8].

It's important to note that polymers, essential components of various materials, come in two primary types: synthetic and natural. Synthetic polymers, such as nylon, polyethylene, polyester, Teflon, and epoxy, are engineered by scientists and engineers and are derived from petroleum oil. On the other hand, natural polymers, like silk, wool, DNA, cellulose, and proteins, occur naturally and can be extracted from biological sources.

In the context of topological indices, the neighborhood M-polynomial plays a pivotal role [9, 10, 11]. It proves to be effective in recovering topological indices based on neighborhood degree sums, which, in turn, provide valuable insights into the diverse physical, chemical, and biological characteristics of the materials under investigation [12].

Recognizing the profound importance of the neighborhood M-polynomial, this paper embarks on a journey to unveil the computed closed forms of NM-polynomials for both vulcanized rubber networks and poly-methyl methacrylate networks. These NM-polynomials, in turn, serve as a valuable source for deriving various topological indices grounded in neighborhood degree analysis. For a comprehensive understanding of our findings and in-depth information, we invite readers to explore the references [13, 14].

VULCANIZED RUBBER NETWORK (VR_m^n)

Vulcanite, also known as vulcanized rubber, is a remarkable material created through a chemical transformation process. This transformation involves the addition reaction of polyisoprene, which is the primary component of natural rubber, with sulfur under the influence of steam pressure. The crucial factor in this process is the quantity of sulfur, as it significantly influences the hardness of vulcanite by fostering the formation of cross-links between the polyisoprene chains. These cross-links give rise to a rigid, dense, and exceptionally durable solid material.

I. H. Agustin (ed.), *Proceedings of the 2nd International Conference on Neural Networks and Machine Learning 2023 (ICNNML 2023)*, Advances in Intelligent Systems Research 183,
https://doi.org/10.2991/978-94-6463-445-7_6

TABLE 1. Formulae of some neighbourhood degree-based topological indices

TI	Degree based index	Derivation from f(x,y)= NM(G;x,y)
$M_1(\Psi)$ [15]	$\sum_{xy\in E(\Psi)} [d_x+d_y]$	$(D_x+D_y)(NM(\Psi;x,y))\mid_{x=y=1}$
$M_2(\Psi)$ [15]	$\sum_{xy\in E(\Psi)} [d_x.d_y]$	$(D_x.D_y)(NM(\Psi;x,y))\mid_{x=y=1}$
$F(\Psi)$ [16]	$\sum_{xy\in E(\Psi)} [d_x^2+d_y^2]$	$(D_x^2.D_y^2)(NM(\Psi;x,y))\mid_{x=y=1}$
$mM_2(\Psi)$ [17]	$\sum_{xy\in E(\Psi)} \left[\frac{1}{d_x.d_y}\right]$	$(S_x.S_y)(NM(\Psi;x,y))\mid_{x=y=1}$
$R_K(\Psi)$ [18]	$\sum_{xy\in E(\Psi)} [d_x.d_y]^k$	$(D_x^\alpha.D_y^\alpha)(NM(\Psi;x,y))\mid_{x=y=1}$
$ReZG_3(\Psi)$ [19]	$\sum_{xy\in E(\Psi)} d_xd_y[d_x+d_y]$	$(D_xD_y)(D_x+D_y)(NM(\Psi;x,y)\mid_{x=y=1}$
$SDD(\Psi)$ [20]	$\sum_{xy\in E(\Psi)} \left[\frac{d_x^2+d_y^2}{d_x.d_y}\right]$	$(D_xS_y+S_xD_y)(NM(\Psi;x,y))\mid_{x=y=1}.$
$H(\Psi)$ [21]	$\sum_{xy\in E(\Psi)} \left[\frac{2}{d_x+d_y}\right]$	$2S_xJ(NM(\Psi;x,y))\mid_{x=y=1}.$
$ReZG_2(\Psi)$ [19]	$\sum_{xy\in E(\Psi)} \left[\frac{d_x.d_y}{d_x+d_y}\right]$	$S_xJD_xD_y(NM(\Psi;x,y)\mid_{x=y=1}.$
$A(\Psi)$ [22]	$\sum_{xy\in E(\Psi)} \left[\frac{d_x.d_y}{d_x+d_y-2}\right]^3$	$S_x^3Q_{-2}JD_x^3D_y^3(NM(\Psi;x,y)\mid_{x=y=1}.$

What sets vulcanized rubber apart is its remarkable array of physicochemical properties, far superior to those of natural rubber. This transformation, credited to Charles Goodyear's pioneering work in 1839, marked a pivotal moment in the world of polymers. Vulcanite swiftly earned recognition as an ideal material for crafting denture bases, maintaining its status for nearly three decades. Its popularity stemmed from its ability to offer an accurate and comfortable fit, combined with an affordable cost, making it a celebrated choice in the field of dental prosthetics [23].

Structure of VR_m^n

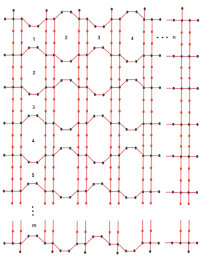

FIGURE 1. Hydrogen depleted molecular graph of VR_m^n.

Results of VR_m^n

Theorem 1. *Let VR_m^n be the vulcanized rubber network then NM-polynomial is*

$$\begin{aligned}NM((VR_m^n,x,y) = {} & 2x^3y^7+4x^4y^7+(2m-2)x^4y^8+(4n+2)x^5y^6\\ & +2x^5y^7+2nx^5y^8+(3mn+2m+n-2)x^6y^6\\ & +2x^6y^7+4nx^6y^8+(4m-4)x^6y^9+(6mn-6n)\\ & x^6y^{10}+4x^7y^8+(2n-2)x^8y^8+(2m-2)x^9y^{10}\\ & +(mn-m-n+1)x^{10}y^{10}.\end{aligned}$$

Proof. In the realm of vulcanized rubber networks, denoted as VR_m^n, an intriguing interplay of edges and vertices defines its intricate structure. This network boasts a total of $10mn+9m+6n+5$ edges and $8mn+8m+6n+6$ vertices, shaping a complex web of connections. What adds depth to its complexity is the partitioning of edges based on the neighboring degrees within VR_m^n.

Specifically, when we delve into this partitioning based on the sum of neighbors' degrees, fascinating patterns emerge. For instance, there are 2 edges denoted as $E_{(3,7)}$, 4 edges represented by $E_{(4,7)}$, and a quantity of $2m-2$ edges designated as $E_{(4,8)}$. Additionally, the network exhibits 4 edges for $E_{(5,6)}$ and 2 edges each for $E_{(5,7)}$ and $E_{(5,8)}$. Moving further, we encounter $3mn+2m$ edges for $E_{(6,6)}$, 2 edges for $E_{(6,7)}$, and 4n edges for $E_{(6,8)}$. Moreover, $4m-4$ edges emerge in the context of $E_{(6,9)}$, while $6mn-6$ edges define the intriguing dynamics of $E_{(6,10)}$. The interplay continues with 4 edges in the category of $E_{(7,8)}$, $2n-2$ edges for $E_{(8,8)}$, $2m-2$ edges for $E_{(9,10)}$ and a distinct count of $mn-m$ edges encapsulated within $E_{(10,10)}$.

This detailed partitioning illuminates the complex structures and interconnections within the vulcanized rubber network, unveiling a fascinating tapestry of interconnected elements, eagerly awaiting exploration and comprehension.

$$\begin{aligned}Let, NM(VR_m^n),x,y) = {} & \sum_{i\leq j} m_{ij}(VR_m^n))x^iy^j\\ = {} & \mid E_{(3,7)}\mid x^3y^7+\mid E_{(4,7)}\mid x^4y^7+\mid E_{(4,8)}\mid x^4y^8+\mid E_{(5,6)}\mid\\ & x^5y^6+\mid E_{(5,7)}\mid x^5y^7+\mid E_{(5,8)}\mid x^5y^8+\mid E_{(6,6)}\mid x^6y^6+\\ & \mid E_{(6,7)}\mid x^6y^7+\mid E_{(6,8)}\mid x^6y^8+\mid E_{(6,9)}\mid x^6y^9+\mid E_{(6,10)}\mid\\ & x^6y^{10}+\mid E_{(7,8)}\mid x^7y^8+\mid E_{(8,8)}\mid x^8y^8+\mid E_{(9,10)}\mid x^9y^{10}\\ & +\mid E_{(10,10)}\mid x^{10}y^{10}.\\ NM((VR_m^n,x,y) = {} & 2x^3y^7+4x^4y^7+(2m-2)x^4y^8+(4n+2)x^5y^6+2x^5y^7\\ & +2nx^5y^8+(3mn+2m+n-2)x^6y^6+2x^6y^7+4nx^6y^8\\ & +(4m-4)x^6y^9+(6mn-6n)x^6y^{10}+4x^7y^8+(2n-2)\\ & x^8y^8+(2m-2)x^9y^{10}+(mn-m-n+1)x^{10}y^{10}.\end{aligned}$$

□

Theorem 2. *Let VR_m^n be the vulcanized rubber network then*

$$\begin{aligned}1.M_1^*(VR_m^n) &= 126m+54n+152mn+38.\\ 2.M_2^*(VR_m^n) &= 432m+96n+568mn+32.\\ 3.F_N^*(VR_m^n) &= 934m+134n+1232mn+78.\\ 4.M_2^{nm}(VR_m^n) &= (0.204)m+(0.190)n+(0.193)mn+(0.245).\\ 5.NR_{(\frac{1}{2})}(VR_m^n) &= (61.681)m+(27.795)n+(74.475)mn+(18.331).\\ 6.\ ND_3(VR_m^n) &= (6292)m+(8872)n+(9056)mn+(15392).\\ 7.\ ND_5(VR_m^n) &= (19.682)m+(10.687)n+(21.6)mn+(11.522).\\ 8.\ NH(VR_m^n) &= (1.31)m+(1.172)n+(1.35)mn+(0.854).\\ 9.\ NI(VR_m^n) &= (30.207)m+(6.209)n+(36.5)mn+(9.924).\\ 10.\ S(VR_m^n) &= (570.824)m+(94.228)n+(783.712)mn-(294.36).\end{aligned}$$

Proof.

$$\begin{aligned}
Let,\ NM((VR_m^n,x,y) &= 2x^3y^7+4x^4y^7+(2m-2)x^4y^8+(4n+2)\\
&\quad x^5y^6+2x^5y^7+2nx^5y^8+(3mn+2m+n\\
&\quad -2)x^6y^6+2x^6y^7+4nx^6y^8+(4m-4)x^6\\
&\quad y^9+(6mn-6n)x^6y^{10}4x^7y^8+(2n-2)x^8\\
&\quad y^8+(2m-2)x^9y^{10}+(mn-m-n+1)\\
&\quad x^{10}y^{10}.\\
1.D_xNM((VR_m^n),x,y) &= 6x^3y^7+16x^4y^7+4(2m-2)x^4y^8+5(4n\\
&\quad +2)x^5y^6+10x^5y^7+10nx^5y^8+6(3mn+\\
&\quad 2m+n-2)x^6y^6+12x^6y^7+24nx^6y^8+6\\
&\quad (4m-4)x^6y^9+6(6mn-6n)x^6y^{10}+28x^7\\
&\quad y^8+8(2n-2)x^8y^8+9(2m-2)x^9y^{10}+10\\
&\quad (mn-m-n+1)x^{10}y^{10}.\\
2.D_yNM((VR_m^n),x,y) &= 14x^3y^7+28x^4y^7+8(2m-2)x^4y^8+6(4n\\
&\quad +2)x^5y^6+14x^5y^7+16nx^5y^8+6(3mn+\\
&\quad 2m+n-2)x^6y^6+14x^6y^7+32nx^6y^8+9\\
&\quad (4m-4)x^6y^9+10(6mn-6n)x^6y^{10}+32\\
&\quad x^7y^8+8(2n-2)x^8y^8+10(2m-2)x^9y^{10}\\
&\quad +10(mn-m-n+1)x^{10}y^{10}.\\
3.D_xD_yNM((VR_m^n),x,y) &= 42x^3y^7+112x^4y^7+32(2m-2)x^4y^8+30\\
&\quad (4n+2)x^5y^6+70x^5y^7+80nx^5y^8+36(3\\
&\quad mn+2m+n-2)x^6y^6+84x^6y^7+192n\\
&\quad x^6y^8+54(4m-4)x^6y^9+60(6mn-6n)x^6\\
&\quad y^{10}+224x^7y^8+64(2n-8)+90(2m-2)\\
&\quad x^9y^{10}+100(mn-m-n+1)x^{10}y^{10}.\\
4.S_yNM((VR_m^n),x,y) &= \frac{2}{7}x^3y^7+\frac{4}{7}x^4y^7+\frac{(m-1)}{4}x^4y^8+\frac{(2n+1)}{3}\\
&\quad x^5y^6+\frac{2}{7}x^5y^7+\frac{n}{8}x^5y^8+\frac{(3mn+2m+n-2)}{6}\\
&\quad x^6y^6+\frac{2}{7}x^6y^7+\frac{n}{2}x^6y^8+\frac{(4m-4)}{9}x^6y^9\\
&\quad +\frac{(6mn-6n)}{10}x^6y^{10}+\frac{1}{2}x^7y^8+\frac{(2n-2)}{8}\\
&\quad x^8y^8+\frac{(2m-2)}{10}x^9y^{10}+\frac{(mn-m-n+1)}{10}\\
&\quad x^{10}y^{10}.\\
5.S_xS_yNM((VR_m^n),x,y) &= \frac{2}{21}x^3y^7+\frac{1}{7}x^4y^7+\frac{(m-1)}{16}x^4y^8+\frac{(2n+1)}{15}\\
&\quad x^5y^6+\frac{2}{35}x^5y^7+\frac{n}{40}x^5y^8+\frac{(3mn+2m+n-2)}{36}\\
&\quad x^6y^6+\frac{2}{42}x^6y^7+\frac{n}{12}x^6y^8+\frac{(4m-4)}{54}x^6y^9+\\
&\quad \frac{(6mn-6n)}{60}x^6y^{10}+\frac{4}{56}x^7y^8+\frac{(2n-2)}{64}x^8y^8\\
&\quad +\frac{(2m-2)}{90}x^9y^{10}+\frac{(mn-m-n+1)}{100}x^{10}y^{10}.
\end{aligned}$$

$$6.D_x^{\alpha}D_y^{\alpha}NM((VR_m^n),x,y) = 2(3^{\alpha}7^{\alpha})x^3y^7+4(4^{\alpha}7^{\alpha})x^4y^7+(2m-2)(4^{\alpha}8^{\alpha}) x^4y^8+(4n+2)(5^{\alpha}6^{\alpha})x^5y^6+2(5^{\alpha}7^{\alpha})x^5y^7+2n(5^{\alpha}8^{\alpha})x^5y^8+(3mn+2m+n-2)(6^{\alpha}6^{\alpha})x^6y^6 +2(6^{\alpha}7^{\alpha})x^6y^7+4n(6^{\alpha}8^{\alpha})x^6y^8+(4m-4)(6^{\alpha}9^{\alpha})x^6y^9+(6mn-6n)(6^{\alpha}10^{\alpha})x^6y^{10}+4(7^{\alpha}8^{\alpha}) x^7y^8+(2n-2)(8^{\alpha}8^{\alpha})x^8y^8+(2m-2)(9^{\alpha}10^{\alpha}) x^9y^{10}+(mn-m-n+1)(10^{\alpha}10^{\alpha})x^{10}y^{10}.$$

$$7.S_x^{\alpha}S_y^{\alpha}NM((VR_m^n),x,y) = \frac{2}{21^{\alpha}}x^3y^7+\frac{4}{28^{\alpha}}x^4y^7+\frac{(2m-2)}{32^{\alpha}}x^4y^8+\frac{(4n+2)}{30^{\alpha}} x^5y^6+\frac{2}{35^{\alpha}}x^5y^7+\frac{2n}{40^{\alpha}}x^5y^8+\frac{(3mn+2m+n-2)}{6^{\alpha}} x^6y^6+\frac{2}{42^{\alpha}}x^6y^7+\frac{4n}{48^{\alpha}}x^6y^8+\frac{(4m-4)}{54^{\alpha}}x^6y^9+ \frac{(6mn-6n)}{60^{\alpha}}x^6y^{10}+\frac{4}{56^{\alpha}}x^7y^8+\frac{(2n-2)}{64^{\alpha}}x^8y^8 +\frac{(2m-2)}{90^{\alpha}}x^9y^{10}+\frac{(mn-m-n+1)}{100^{\alpha}}x^{10}y^{10}.$$

$$8.D_x.S_yNM((VR_m^n),x,y) = \frac{6}{7}x^3y^7+\frac{16}{7}x^4y^7+\frac{(4m-4)}{4}x^4y^8+\frac{(5(2n+1))}{3} x^5y^6+\frac{10}{7}x^5y^7+\frac{5n}{8}x^5y^8+\frac{6(3mn+2m+n-2)}{6} x^6y^6+\frac{12}{7}x^6y^7+\frac{6n}{2}x^6y^8+\frac{6(4m-4)}{9}x^6y^9+ \frac{6(6mn-6n)}{10}x^6y^{10}+\frac{7}{2}x^7y^8+\frac{8(2n-2)}{8}x^8y^8 +\frac{9(2m-2)}{10}x^9y^{10}+\frac{10(mn-m-n+1)}{10}x^{10}y^{10}.$$

$$9.S_x.D_yNM((VR_m^n),x,y) = \frac{14}{3}x^3y^7+7x^4y^7+2(2m-2)x^4y^8+\frac{6(4n+2)}{5} x^5y^6+\frac{14}{5}x^5y^7+\frac{16n}{5}x^5y^8+(3mn+2m+n-2) x^6y^6+\frac{7}{3}x^6y^7+\frac{16n}{3}x^6y^8+\frac{9(4m-4)}{6}x^6y^9+ \frac{10(6mn-6n)}{6}x^6y^{10}+\frac{32}{7}x^7y^8+(2n-2)x^8y^8 +\frac{10(2m-2)}{9}x^9y^{10}+(mn-m-n+1)x^{10}y^{10}.$$

$$10.JNM((VR_m^n),x,y) = 2x^{10}+4x^{11}+(2m-2)x^{12}+(4n+2)x^{11}+2x^{12} +2nx^{13}+(3mn+2m+n-2)x^{12}+2x^{13}+4nx^{14} +(4m-4)x^{15}+(6mn-6n)x^{16}+4x^{15}+(2n-2) x^{16}+(2m-2)x^{19}+(mn-m-n+1)x^{20}.$$

$$11.S_xJNM((VR_m^n),x,y) = \frac{1}{5}x^{10}+\frac{4}{11}x^{11}+\frac{(m-1)}{6}x^{12}+\frac{(4n+2)}{11}x^{11} +\frac{1}{6}x^{12}+\frac{2n}{13}x^{13}+\frac{(3mn+2m+n-2)}{12}x^{12} +\frac{2}{13}x^{13}+\frac{2n}{7}x^{14}+\frac{(4m-4)}{15}x^{15}+\frac{3(mn-n)}{8} x^{16}+\frac{4}{15}x^{15}+\frac{(n-2)}{8}x^{16}+\frac{2(m-1)}{19}x^{19} +\frac{(mn-m-n+1)}{20}x^{20}$$

$$\begin{aligned}
12.JD_xD_yNM((VR_m^n),x,y) &= 42x^{10}+112x^{11}+32(2m-2)x^{12}+30(4n+2)\\
&x^{11}+70x^{12}+80nx^{13}+36(3mn+2m+n-8)\\
&x^{12}+84x^{13}+192nx^{14}+54(4m-4)x^{15}+60(6\\
&mn-6n)x^{16}+224x^{15}+64(2n-2)x^{16}+90(2m\\
&-2)x^{19}+100(mn-m-n+1)x^{20}.\\
13.S_xJD_xD_yNM((VR_m^n),x,y) &= \frac{21}{5}x^{10}+\frac{112}{11}x^{11}+\frac{8(2m-2)}{3}x^{12}+\frac{30(4n+2)}{11}\\
&x^{11}+\frac{70}{12}x^{12}+\frac{80}{13}x^{13}+\frac{36(3mn+2m+n-2)}{12}\\
&x^{12}+\frac{84}{13}x^{13}+\frac{96n}{7}x^{14}+\frac{54(4m-4)}{15}x^{15}+\\
&\frac{15(6mn-6n)}{4}x^{16}+\frac{112}{7}x^{15}+\frac{64(2n-2)}{16}x^{16}+\\
&\frac{90(2m-2)}{19}x^{19}+\frac{100(mn-m-n+1)}{20}x^{20}.\\
14.S_x^3Q_{-2}JD_x^3D_y^3NM((VR_m^n),x,y) &= (36.175)x^8+(120.44)x^9+(32.768)(2m-2)x^{10}\\
&+(37.03)(4n+2)x^9+(85.75)x^{10}+(96.16)nx^{11}\\
&+(46.65)(3mn+2m+n-8)x^{10}+(111.32)x^{11}\\
&+256nx^{12}+(71.672)(4m-4)x^{13}+(78.717)\\
&(6mn-6n)x^{14}+(319.73)x^{13}+(95.53)(2n-2)\\
&x^{14}+(148.38)(2m-2)x^{17}+(171.46)(mn-m\\
&-n+1)x^{18}.\\
15.(D_x^2+D_y^2)NM((VR_m^n),x,y) &= 2(3^2+7^2)x^3y^7+4(4^2+7^2)x^4y^7+(2m-2)(4^2\\
&+8^2)x^4y^8+(4n+2)(5^2+6^2)x^5y^6+2(5^2+7^2)\\
&x^5y^7+2n(5^2+8^2)x^5y^8+(3mn+2m+n-2)\\
&(6^2+6^2)x^6y^6+2(6^2+7^2)x^6y^7+4n(6^2+8^2)x^6\\
&y^8+(4m-4)(6^2+9^2)x^6y^9+(6mn-6n)(6^2+\\
&10^2)x^6y^{10}+4(7^2+8^2)x^7y^8+(2n-2)(8^2+8^2)\\
&x^8y^8+(2m-2)(9^2+10^2)x^9y^{10}+10(mn\\
&-m-n+1)(10^2+10^2)x^{10}y^{10}.
\end{aligned}$$

□

By substituting these cardinalities into the definitions of topological indices, we attain the desired outcomes.

3D structure of VR_m^n

The 3D-plot of NM-polynomials of VR_m^n with the help of Maple software. A three-dimensional plot can depict intricate connections among three variables. For example, it could visually demonstrate the interplay between temperature, time, and cross-link density in the vulcanization process. Utilizing such a plot facilitates the optimization of vulcanization conditions, leading to the attainment of specific material properties as desired.

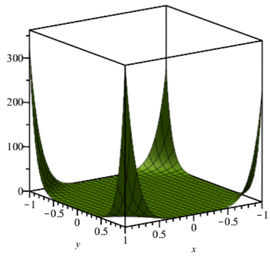

FIGURE 2. 3D plot of NM-polynomial of VR_m^n.

POLY-METHYL METHACRYLATE NETWORK($PMMA_n$)

Poly-methyl methacrylate ($PMMA$), a versatile synthetic resin recognized by its trade name acrylic glass, has gained widespread popularity as an exceptional glass substitute. This versatile material finds applications in various sectors, including the manufacturing of instrument panels, aircraft canopies, skylights, and cutting-edge medical technologies. Interestingly, its history dates back to 1937 when Walter Wright introduced $PMMA$ as the first-ever replacement for vulcanite, pioneering a revolution in the field of dental prosthodontics. Since then, $PMMA$ has evolved to become the primary choice for fabricating denture bases and is now the most widely utilized material in the dental industry, offering both functional and aesthetic benefits to countless patients.

Structure of $PMMA_n$

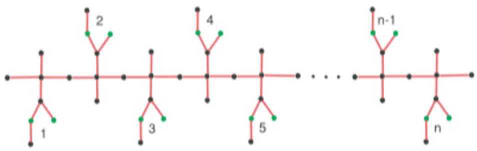

FIGURE 3. Hydrogen depleted molecular graph of $PMMA_n$.

Results of $PMMA_n$

Theorem 3. *Let $PMMA_n$ be the poly-methyl methacrylate network then NM-polynomial is*

$$NM((PMMA_n,x,y) = nx^2y^4 + x^2y^5 + nx^3y^7 + nx^4y^7 + (n+1)x^4y^{10} \\ + x^5y^{10} + nx^7y^{10} + (2n-2)x^8y^{10}$$

Proof. Let $PMMA_n$ be the poly-methyl methacrylate network shown in Figure with n monomers, then total number of vertices and edges are $7n+2$ and $7n+1$, respectively. Now, if we partitioning edges based on neighbors degree sum of $PMMA_n$ we get, $E_{(2,4)} = n$, $E_{(2,5)} = 1$, $E_{(3,7)} = n$, $E_{(4,7)} = n$, $E_{(4,10)} = n+1$, $E_{(5,10)} = 1$, $E_{(7,10)} = n$, $E_{(8,10)} = 2n-2$.

$$
\begin{aligned}
Let, NM(PMMA_n), x, y) &= \sum_{i \le j} m_{ij}(PMMA_n) x^i y^j \\
&= \sum_{2 \le 4} m_{24}(PMMA_n) x^2 y^4 + \sum_{2 \le 5} m_{25}(PMMA_n)) x^2 y^5 \\
&+ \sum_{3 \le 7} m_{37}(PMMA_n) x^3 y^7 + \sum_{4 \le 7} m_{47}(PMMA_n) x^4 y^7 \\
&+ \sum_{4 \le 10} m_{410}(PMMA_n)) x^4 y^{10} + \sum_{5 \le 10} m_{510}(PMMA_n) x^5 y^{10} \\
&+ \sum_{7 \le 10} m_{710}(PMMA_n)) x^7 y^{10} + \sum_{8 \le 10} m_{810}(PMMA_n) x^8 y^{10}. \\
&= \sum_{uv \in E_{(1,2)}} m_{14}(PMMA_n) x^1 y^2 + \sum_{uv \in E_{(2,2)}} m_{22}(PMMA_n) x^2 y^2 \\
&+ \sum_{uv \in E_{(2,3)}} m_{23}(PMMA_n) x^2 y^3. \\
&= | E_{(2,4)} | x^2 y^4 + | E_{(2,5)} | x^2 y^5 + | E_{(3,7)} | x^3 y^7 + | E_{(4,7)} | \\
&\quad x^4 y^7 + | E_{(4,10)} | x^4 y^{10} + | E_{(5,10)} | x^5 y^{10} + | E_{(7,10)} | x^7 y^{10} \\
&\quad + | E_{(8,10)} | x^8 y^{10} \\
NM(PMMA_n), x, y) &= n x^2 y^4 + x^2 y^5 + n x^3 y^7 + n x^4 y^7 + (n+1) x^4 y^{10} + x^5 y^{10} \\
&\quad + n x^7 y^{10} + (2n-2) x^8 y^{10}.
\end{aligned}
$$

□

Now we compute some degree-based topological indices of $PMMA_n$ from this NM-polynomial.

Theorem 4. *Let $PMMA_n$ be the poly-methyl methacrylate network then*

1. $M_1^*(PMMA_n) = 94n + 20.$
2. $M_2^*(PMMA_n) = 327n - 60.$
3. $F_N^*(PMMA_n) = 736n - 58.$
4. $M_2^{nm}(PMMA_n) = (0.272)n + (0.12).$
5. $NR_{(\frac{1}{2})}(PMMA_n) = (45.282)n - (1.330).$
6. $ND_3(PMMA_n) = 5196n - 1500.$
7. $ND_5(PMMA_n) = (16.711)n + (4.2).$
8. $NH(NM(PMMA_n) = (1.086)n + (0.339).$
9. $NI(PMMA_n) = (21.842)n - (1.269).$
10. $S(PMMA_n) = (444.84)n - (148.08).$

Proof.

$$
\begin{aligned}
let,\ \ NM((PMMA_n),x,y) &= nx^2y^4+x^2y^5+nx^3y^7+nx^4y^7+(n+1) \\
&\quad x^4y^{10}+x^5y^{10}+nx^7y^{10}+(2n-2)x^8y^{10}. \\
1.D_xNM((PMMA_n),x,y) &= 2nx^2y^4+2x^2y^5+3nx^3y^7+4nx^4y^7+4 \\
&\quad (n+1)x^4y^{10}+5x^5y^{10}+7nx^7y^{10}+8(2n \\
&\quad -2)x^8y^{10}. \\
2.D_yNM((PMMA_n),x,y) &= 4nx^2y^4+5x^2y^5+7nx^3y^7+7nx^4y^7+10 \\
&\quad (n+1)x^4y^{10}+10x^5y^{10}+10nx^7y^{10}+10 \\
&\quad (2n-2)x^8y^{10}. \\
3.D_xD_yNM((PMMA_n),x,y) &= 8nx^2y^4+10x^2y^5+21nx^3y^7+28nx^4y^7 \\
&\quad +40(n+1)x^4y^{10}+50x^5y^{10}+70nx^7y^{10} \\
&\quad +80(2n-2)x^8y^{10}. \\
4.S_yNM((PMMA_n),x,y) &= \frac{n}{4}x^2y^4+\frac{1}{5}x^2y^5+\frac{n}{7}x^3y^7+\frac{n}{7}x^4y^7+ \\
&\quad \frac{(n+1)}{10}x^4y^{10}+\frac{1}{10}x^5y^{10}+\frac{n}{10}x^7y^{10}+ \\
&\quad \frac{(2n-2)}{10}x^8y^{10}. \\
5.S_xS_yNM((PMMA_n),x,y) &= \frac{n}{8}x^2y^4+\frac{1}{10}x^2y^5+\frac{n}{21}x^3y^7+\frac{n}{28}x^4y^7+ \\
&\quad \frac{(n+1)}{40}x^4y^{10}+\frac{1}{50}x^5y^{10}+\frac{n}{70}x^7y^{10}+ \\
&\quad \frac{(2n-2)}{80}x^8y^{10}. \\
6.D_x^{\alpha}D_y^{\alpha}NM((PMMA_n),x,y) &= n(8^{\alpha})x^2y^4+(10^{\alpha})x^2y^5+n(21^{\alpha})x^3y^7+ \\
&\quad n(28^{\alpha})x^4y^7+(n+1)(40^{\alpha})x^4y^{10}+(50^{\alpha}) \\
&\quad x^5y^{10}+n(70^{\alpha})x^7y^{10}+(2n-2)(80^{\alpha})x^8y^{10}. \\
7.S_x^{\alpha}S_y^{\alpha}NM((PMMA_n),x,y) &= \frac{n}{(8^{\alpha})}x^2y^4+\frac{1}{(10^{\alpha})}x^2y^5+\frac{n}{(21^{\alpha})}x^3y^7+ \\
&\quad \frac{n}{(28^{\alpha})}x^4y^7+\frac{(n+1)}{(40^{\alpha})}x^4y^{10}+\frac{1}{(50^{\alpha})}x^5y^{10} \\
&\quad +\frac{n}{(70^{\alpha})}x^7y^{10}+\frac{(2n-2)}{(80^{\alpha})}x^8y^{10}. \\
8.D_xS_yNM((PMMA_n),x,y) &= \frac{n}{2}x^2y^4+\frac{2}{5}x^2y^5+\frac{3n}{7}x^3y^7+\frac{4n}{7}x^4y^7+ \\
&\quad \frac{4(n+1)}{10}x^4y^{10}+\frac{5}{10}x^5y^{10}+\frac{7n}{10}x^7y^{10}+ \\
&\quad \frac{8(2n-2)}{10}x^8y^{10}.
\end{aligned}
$$

$$
\begin{aligned}
9.S_xD_yNM((PMMA_n),x,y) &= \frac{4n}{2}x^2y^4+\frac{5}{2}x^2y^5+\frac{7n}{3}x^3y^7+\frac{7n}{4}x^4y^7+\\
&\frac{10(n+1)}{4}x^4y^{10}+\frac{10}{5}x^5y^{10}+\frac{10n}{7}x^7y^{10}+\\
&\frac{10(2n-2)}{8}x^8y^{10}.\\
10.S_xJNM((PMMA_n),x,y) &= \frac{n}{6}x^6+\frac{1}{7}x^7+\frac{n}{10}x^{10}+\frac{n}{11}x^{11}+\\
&\frac{(n+1)}{14}x^{14}+\frac{1}{15}x^{15}+\frac{n}{17}x^{17}+\\
&\frac{(2n-2)}{18}x^{18}.\\
11.JD_xD_yNM((PMMA_n),x,y) &= 8nx^6+10x^7+21nx^{10}+28nx^{11}+\\
&40(n+1)x^{14}+50x^{15}+70nx^{17}+\\
&80(2n-2)x^{18}.\\
12.S_xJD_xD_yNM((PMMA_n),x,y) &= \frac{4n}{3}x^6+\frac{10}{7}x^7+\frac{21n}{10}x^{10}+\frac{28n}{11}x^{11}+\\
&\frac{20(n+1)}{7}x^{14}+\frac{10}{3}x^{15}+\frac{70n}{17}x^{17}+\\
&\frac{40(2n-2)}{9}x^{18}.\\
13.S_x^3Q_{-2}JD_x^3D_y^3NM((PMMA_n),x,y) &= 8nx^4+8x^5+(18.087)nx^8+(30.11)n\\
&x^9+(n+1)(37.03)x^{12}+(56.89)x^{13}+\\
&(101.62)nx^{15}+(n-1)(250)x^{16}.\\
14.(D_x^2+D_y^2)NM((PMMA_n),x,y) &= 20nx^2y^4+29x^2y^5+58nx^3y^7+65nx^4y^7\\
&+116(n+1)x^4y^{10}+125x^5y^{10}+149nx^7\\
&y^{10}+328(n-1)x^8y^{10}.
\end{aligned}
$$

□

3D Plot of $PMMA_n$

The 3D-plot of NM-polynomials of $PMMA_n$ with the help of Maple software. In a three-dimensional context, PMMA's polymer chain extends into space, forming a intricate and three-dimensional framework. The inclusion of methyl groups $(-3CH3)$ along the primary polymer backbone significantly influences the overall spatial configuration. This three-dimensional arrangement is distinguished by the rotational freedom permitted around single bonds, resulting in a polymer that is both flexible and amorphous. The presence of methyl groups introduces potential steric effects, exerting an impact on the spatial organization and characteristics of the polymer. The physical properties of PMMA are intricately tied to its three-dimensional structure. Notably, the polymer exhibits transparency, rigidity, and possesses a high glass transition temperature.

CONCLUSION

A remarkable method for computing topological indices based on neighborhood degree sums is through the utilization of the neighborhood M-polynomial. In this study, we have delved into the topological characteristics of specific synthetic polymers. Our approach involved initially deriving the general form of the neighborhood M-polynomial for these structures. Subsequently, we harnessed these polynomials to recover several neighborhood degree sum-based indices. To facilitate a clearer understanding of our findings, we have also provided graphical representations of the results.

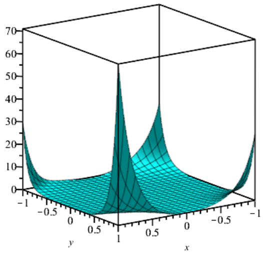

FIGURE 4. 3D plot of NM-polynomial of $PMMA_n$.

ACKNOWLEDGMENTS

The author expresses genuine gratitude to the Department of Mathematics at V. S. K. University in Ballari, India and to PUI-PT Combinatorics and Graph at ICONSSMAL Universitas Jember in Indonesia for their valuable assistance in finalizing this paper. Appreciation is also extended to the anonymous referee for their invaluable comments and constructive suggestions, which markedly enhanced the paper's readability.

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