Public Goods in Information Cascades: Is There Still a Free Rider Problem?

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Abstract. The phase "public goods" has been with the theories of economics for quite a long time and there are multiple ways to analyze and interpret why these goods will cause the free-rider problem. This paper suggests a variant of the basic information cascade game by introducing an idea of mutual benefit such that for each player, he receives benefit partially from himself making a correct choice and the last person making the correct choice. It is possible to design a information cascade game such that the final payoff for each player is both non-rival and non-excludable to satisfy the definition of a public good. By observing the results, we can prove that the free-rider problem still exists through a different approach. Moreover, it is possible to adjust and refine the model into such a way that even when the collective gain for each person is much larger than the private sacrifice for the person to do so, the free-rider problem still exist. In this paper, we Observe the key features of a public good and observe whether it is possible for us to obtain similar features in an information cascade game, propose a variant of the basic information cascade game, explore the threshold value for some important parameters, extend the model from 4 person to infinite and test whether it is still viable. Prove that it is possible to explain the free-rider problem in the context of information cascades, thus showing that the free-rider problem can be explained by information cascades.

Keywords: Public Goods; Information Cascade; Free-rider

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1 Introduction

Since the influencing work of Samuleson (1954), the phase public good have been brought into the world of economics, and been developed as one of the basic theories. It is defined as being non-excludable such that it is not possible to exclude one from benefiting from it and non-rival such that the consumption of public good by one individual will not reduce the availability of others.[3] The main problem of public goods as mentioned in M. Olson (1965) is the free rider problem, as many scholars have proven later in multiple ways, has caused the lack of provision in public goods. [4]After exploring the theories embedded in the fundamentals of public goods, real life agents have designed multiple approaches to address the problem, such as taxes, subsidies, patents etc. These approaches have been proven to be effective but costly in occasions. Therefore, it is incentivized to try a different approach to explain the free-rider problem and perhaps spark new ways to counter the problem.

The appearance of information cascades on the other hand is much later, since the influential work of Bikhchandani, Hirshleifer, and Welch (1992).[1] The most basic structure of information cascades has been established and has been a widely studied topic in fields of game theory and computer science. Later work has been focusing on implications in real life researches such as the paper by LR Anderson and CA Hult(1997) which lays foundation of information cascades in laboratory. [5]Many others simply uses it as a phenomenon in the study of herding effects such as the work of W Galuba and K Aberer and D Chakraborty.(2010)[6] Although the model itself have been mentioned in 1992, little advance has been made on the model itself. However, the interesting laboratory paper mentioned above by LR Anderson and CA Hult(1997) does conclude by mentioning cascades can be fragile and can be broken by the introduction of public signals.[5]

Interestingly, the method of public signal has also been addressed as a common solution to the free-rider problem in the context of public goods. Reputation for example can be seen as a typical type of public signal. And as mentioned by Andreoni, J., & Petrie, R. (2004) the provision of public good does increase if contributions were made publicly compared to confidentiality. [8]With that being said, it may also be possible to address the free-rider problem as a mutant of the information cascade game.

This paper approach endeavors to address the past free-rider problem using information cascade theory, despite the extensive body of research incorporating elements such as variable posterior beliefs and differential signal accuracy, little of which tries to explore the implication of information cascades when mutual benefit exists. This paper aims to provide an innovative perspective on information cascade models by introducing the concept of mutual benefit. By this way, we explore the implications of public goods and whether it still exhibits the free-rider problem under this scenario.

The principal motive behind this research stems from the desire to blend the untouched aspects of information cascade models into explaining one of the classic theories of economics. By doing so, this paper seeks to enhance our comprehension of public goods and perhaps offer a fresh outlook on the dynamics of crowd behavior.

There are three primary dimensions of this concept. Firstly, to investigate whether cascades form under these conditions. Subsequently, to ascertain the critical ratio between individual and collective benefits that triggers the cascade. Lastly, to study how these introduced variables influence individual choices and the ultimate outcomes and whether they are still applicable in a large group of players. In the following section, this paper starts by illustrating the basic model, the simple mutant of the original information cascade game.

2 The Basic Model

At the heart of this study, like all information cascade models, imagine a sequence of 'n' rational and utility-maximizing individuals, denoted as i=1 to n. Each player acts in sequence, with each subsequent player (i=3, i=4, etc.) making their move after the immediate predecessor (i=2, i=3, etc.).

In this setup, a new behavior exists, designated by nature, with a 50% probability of being beneficial (denoted as A) and 50% probability of being detrimental (denoted as R). Each player receives a private signal $X_i \in \{H, L\}$, $(1 >= H >= 0.5 >= L >= 0)$. The high signal (H) implies a higher probability that the behavior is correct, while the low signal (L) indicates a greater likelihood that the behavior is erroneous. The accuracy of these private signals is not guaranteed, such that

$$P(H|A) = P(L|R) = p, \tag{1}$$

and

$$P(L|A) = P(H|R) = (1-p). \tag{2}$$

This accuracy of signal p must be strictly larger than 0.5, this is for simplicity since p is an information known to all, a value p smaller than 0.5 will only result in individuals playing opposite to the signal they have received.

Upon receiving their private signal, each player decides to either accept or reject the behavior. This choice is observable to subsequent players. Expectedly, a cascade ensues when the players' Bayesian updated beliefs reaches a point when the private signal of a player is not enough to change its actions.

With a keen interest to try and break the inevitable cascade, it is necessary to introduce the collective benefits and create our unique model for our purpose. Our strategy to incorporate mutual benefit involves introducing a parameter, α, into the equation. Under this scenario, a player receives an α proportion of the benefit when they make a correct choice, and the remaining (1-α) proportion when the final player makes a correct choice. Assuming all other variables remain constant, the expected utility for an individual can be expressed using the following equation.

$$u_i = \alpha \times \left(Pr_i(Xi = a|A) + Pr_i(Xi = r|R) \right) + (1 - \alpha) \times \left(Pr_n(Xn = a|A) + Pr_n(Xn = r|R) \right) \tag{3}$$

For the simplicity of our computations, this paper maintains a decision rule wherein an individual, when confronted with two equally persuasive updated beliefs, conforms their choice with their private signal. This rule prompts our players to initially calculate their Bayesian updated belief regarding the correct behavior. Subsequently, they are expected to envisage the potential moves of succeeding individuals, thus determining the most optimal response.

3 Multi-Player Models

In order to systematically analyze the problem, it is reasonable to firstly focus on a smaller game with limited players. By optimizing the expected payoffs of each player in the game, it is possible to establish a sequence of optimal strategies and associated alpha values that underpin their choices.

3.1 Player Model

The first player will adhere to their private signal. Similarly, in accordance with our specified decision rule, the second player follows their private signal.

The complexity emerges on the third player. Suppose the first two players both choose 'A', and the third player receives the 'R' private signal. The updated belief of the third player can be computed as follows:

$$\Pr_3(A|observation=aa, Xi=r) = \Pr_3(R|observation=rr, Xi=A) = \frac{\frac{1}{2}p^2(1-p)}{\frac{1}{2}p^2(1-p) + \frac{1}{2}p(1-p)^2}$$

$$= p$$

(4)

Should the third player decide to conform with the cascade, they can be confident that the fourth player will also play 'A', regardless of their private signal. Given that the updated belief is 'p', the payoff for the third player would be 'p', independent of alpha.

However, if the third player opts to follow their private signal, their immediate payoff would be α(1-p), and their subsequent payoff, contingent on the final player's action, would be (1-α)(p^2+(1-p)p). The total payoff would then amount to α(1-p)+(1-α)p, which is considerably less than 'p' since 1-p < p. Therefore, a cascade forms in our game with mutual benefits.

Yet, an intriguing question arises - is the formation of the cascade an inevitable outcome as this game is extended to a scenario with a larger number of players?

3.2 Player Model

Under a similar setting as previous, arriving to the same part when two previous players have the same result and the third obtains a conflicting one. Under classic information cascades, it is easy to arrive at a point to compute the Bayesian updated belief of individual three same as Equation(1).

Without loss of generosity, assume that player three has observation=aa, Xi=R. Which allows to start deriving the utilities from two different actions:

$$u_3(a) = \alpha \times p + (1-\alpha) \times \left(Pr_5\left(X_5=a|A\right) + Pr_n\left(X_5=r|R\right)\right) \tag{5}$$

and

$$u_3(r) = \alpha \times (1-p) + (1-\alpha) \times \left(Pr_5\left(X_5 = a|A\right) + Pr_5\left(X_5 = r|R\right)\right) \tag{6}$$

For the third individual choosing to accept the behavior, the decision for player four and player five is simple. For player 4, it is perfectly clear that whatever it chooses could not have affect player 5, since the previous three has the same result and it is optimal for player 5 to follow through the cascade. It is optimal for player 4 and player 5 following through with the cascade. Thus, the utility of player three in this occasion is as following:

$$u_3(a) = \alpha \times p + (1-\alpha) \times p = p \tag{7}$$

However, if it decides to play reject and follow its own private signal, the equation is changed to:

$$u_3(r) = \alpha \times (1-p) + (1-\alpha) \times \left(Pr_5\left(X_5 = a|A\right) + Pr_5\left(X_5 = r|R\right)\right) \tag{8}$$

In which the third individual would start to simulate the actions of later individuals, and would start with the fourth individual, assuming it has observation= aar. Similarly, computing the Bayesian updated belief then its decisions for individual 4 both occasions, X4=a and X4=r. Assuming the first occasion, it has observation = aar, X4=A:

$$\Pr_4(A|observation = aar, Xi = A) = \frac{\frac{1}{2}p^3(1-p)}{\frac{1}{2}p^3(1-p) + \frac{1}{2}p(1-p)^3} = \frac{p^2}{p^2 + (1-p)^2} = \frac{p^2}{2p^2 - 2p + 1} \tag{9}$$

And observation = aar, X4=R:

$$\Pr_4(A|observation = aar, Xi = R) = \frac{\frac{1}{2}p^2(1-p)^2}{\frac{1}{2}p^2(1-p)^2 + \frac{1}{2}p^2(1-p)^2} = \frac{1}{2} \tag{10}$$

Thus, the optimal choice for individual 4 if individual 4 has gotten private signal a suggesting it to accept the behavior is comparing between:

$$u_4(a|X4=a) = \frac{p^2}{p^2 + (1-p)^2} \tag{11}$$

and

$$u_4(r) = \alpha \times \left(1 - \frac{p^2}{p^2+(1-p)^2}\right) + (1-\alpha) \times \left(p\left(1 - \frac{p^2}{p^2+(1-p)^2}\right) + (1-p)\left(\frac{p^2}{p^2+(1-p)^2}\right)\right)$$
$$= \alpha \times \left(1 - \frac{p^2}{p^2+(1-p)^2}\right) + (1-\alpha)\frac{-2p^3+3p^2-2p+1}{p^2+(1-p)^2}$$

(12)

Since for all values of p>0.5, Equation (8) is larger than equation (9), individual 4 will choose to accept the behavior if its private signal suggests it to do so.

The optimal choice for individual 4 if individual 4 has gotten private signal r is quite simple. Since it has updated belief 1/2as Equation (7) has suggested will follow the private signal due to our assumption and it will choose to reject the behavior.

Back to Individual three, it is obvious now, upon rejecting the behavior when observing aa and Private signal X3=R. It choosing to reject will mean that on probability p, individual 4 will choose to accept and the final decision at individual 5 will also be to accept. At (1-p), individual 4 will get reject as its private signal and individual 5 will play whatever its private signal suggests. It is possible to compute the utility of individual 3 choosing to reject given as below:

$$u_3(r) = \alpha \times (1-p) + (1-\alpha) \times \left((1-p)p^2 + p^2 + p^2(1-p)\right) = \alpha(1-p) + (1-\alpha)\left(3p^2 - 2p^3\right)$$

(13)

To prevent a cascade from forming, it must satisfy $u_3(r) > u_3(a)$, thus:

$$\alpha(1-p) + (1-\alpha)\left(3p^2 - 2p^3\right) > p \tag{14}$$

With some simplification, the final equation for alpha is:

$$\alpha < \frac{3p^2 - 2p^3 - p}{3p^2 - 2p^3 - (1-p)} \tag{15}$$

in which there is a different threshold value for alpha for different values of p.

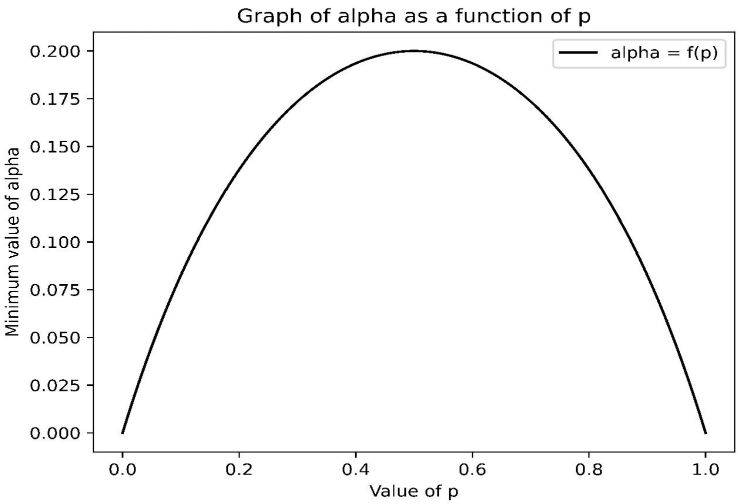

Fig. 1. The curve for threshold values of Equation (12), the threshold values of alpha

Figure 1 suggests that there is an threshold alpha for every p such that any alpha larger than the threshold would lead to a result no different to the original information cascade game. And in the entire 5-player game, for alpha greater than 0.20, the result is still the same as the original information cascade game.

It now seems that alpha does to some extent matter, and at alphas less than the threshold value, it is in fact able to prevent to appearance of a cascade. With the clearly conflicting results between a 5 player game and a 4 player one, it is necessary to proceed the research into a more generalized version, to try and observe what would happen if there is a larger number of individuals.

3.3 The Generalized Model

To understand this issue from a generalized perspective, it is important to adopt a different approach. Imagine if we are using the approach as before, the first person is considering the results for both of its actions, in which in their own simulation have to consider the results of its following players. This process is non-stopping to an extent that the first player has to come up with all the possible results of the game. Making the problem NP-complete. Thus it becomes exceedingly complex to use the same method as before, since considering all possible choices of each subsequent player is rather difficult especially computationally.

Instead, let's consider what each player is primarily focused on when deciding. It is quite obvious how each player make decisions, they simply compare their expected payoff from either decision. In which case, we can decompose the expected payoff for both choices into the following components: the updated belief, denoted Prn, which is determined by the actions of preceding players. Define qAn, the probability that the final player chooses 'A' given that the current player chooses 'A' and 'A' is correct. Similarly, define qRRn and qRAn which respectively denote the probabilities that the final player chooses 'R' given the current player chooses 'R' and 'R' is correct, and that the final player chooses 'A' given the current player chooses 'R' and 'A' is correct. There is a sequence of an infinite number of players, with the player of interest being the nth in the sequence, followed by an infinite number of players after them.

Given this setup, a player's decision is a comparison between the following expressions:

$$\alpha \times Pr_n + (1-\alpha)(q^A\text{n} \times Pr_n) \tag{16}$$

and

$$\alpha \times (1-Pr_n) + (1-\alpha)(q^{RR}\text{n} \times (1-Pr_n) + q^{RA}(Pr_n)) \tag{17}$$

For computational simplicity, it is enough for us to only consider a case that where a cascade is most likely to form in the sequence of players. In other words, if it is possible to prevent even this player from joining the cascade, it is quite safe to assume the rest won't either. In this case, as the player is on the verge of a possible cascade, q^An equals 1 since it is certain that the final player will continue the cascade if the current player chooses 'A'. For q^{RR}n, the maximum it can be is 'p', signifying that immediately after the next player receives a low signal and chooses 'R', the final player also chooses 'R'. For q^{RA}n, given an infinite number of players after the current one, the highest it can reach is when the next player gets a high signal and the final player in the infinite horizon chooses 'A'. In these circumstances, q^{RA}n and q^{RR}n equal 'p'. Substitute this into the former equations, the results show that there are no positive value of alpha for $0.5<p<1$ assuming that there is no cascade.

However, why is it possible to have an alpha that prevents a cascade in a 5-player game while not in a game with infinite number of player? The detailed reasoning might be difficult and will require some sort of calculations. But it may be helpful to take a step back considering the original version of the public goods and free-rider problem. Assume a reasonably small group of individuals, lets say 4 individuals, upon contributing a total of 1 units of some good, they will each receive 1.1 units in return, or else they will receive nothing in return. It is quite clear in this case that nothing for all result will not be a Nash equilibrium, since we are quite sure that if individuals are caught up in that situation, they are willing to contribute the unit to receive a larger payoff. However if there is an arbitrarily large group of individual, assuming that the return per person in this setting stays the same if there is a provision of public good, and the total cost is enlarged by scale, eventually there will be a Nash equilibrium such that all individual chooses not to contribute is sustainable: for example a 5 player version of the above gives 4.4/5 units while the player has to pay 1 unit to obtain it.

Therefore, rather counterintuitively, when there is an infinite number of players, The results will be quite different to those obtain by a much smaller group of individuals.

4 Conclusion

This paper has proven quite obviously that, it is possible to represent public goods and the problem it encounters quite adequately using information cascade model as a foundation while making some modifications to it. By thinking conveying information as a contribution to the collective gain, and the loss in expected payoff in current stage as the sacrifice, we can obtain an outcome similar to the free-rider problem. Even if the collective gain approaches infinity(a sequence of infinite players that will all gain from the collective gain), individuals will definitely not follow the cascade.

This paper has also proven that under a large enough sequence of players, a mutual benefit of close to one will not be able to alter the players into conveying information and will eventually lead to a cascade. The results have proven that the free rider problem of public goods still exist in this new scenario but perhaps it will be possible for us to find new ways to address the problem. However, this paper is quite limited to the theoretical part to try and simulate the free-rider problem. It has not been tested in experiments thus there may be unnoticed flaws. Further researches can be focused on experiment the theoretical method to make sure real life situations will not occur to be off by too much. Also, it will appear to be valuable to start thinking about the ways in which we can avoid the cascade thus not be trapped into the free rider problem.

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