Analysis for influencing factors of slope stability and multivariate polynomial regression

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Abstract. The stability of slope may be affected by many factors. The influence quantity and influence law of each factor are not clear. Unified and comprehensive multi-factor regression formula does not come out so far. So the strength reduction method is used to obtain 33 sets of effective sample parameter results by finite element analysis to solve these problems. The multivariate linear and multivariate polynomial fitting formulas of various influencing factors are constructed, and the fitting accuracy is compared. A set of parameters outside the sample space are selected for finite element calculation to verify the accuracy and applicability of the fitting formula. The results show that the multivariate linear and multivariate polynomial fitting formulas are reliable, and the goodness of fit coefficients are 0.922 and 0.9908 respectively. Compared with the multiple linear fitting process, the root mean square error, mean absolute deviation and mean absolute percentage error of the latter are greatly reduced. In the finite element calculation verification results, the errors of the two are within 3%, which meets the engineering needs, but the multivariate polynomial fitting formula calculation is more consistent with the finite element calculation results.

Keywords: strength reduction method; finite element; factor of safety; multivariate polynomial fitting; comparative analysis.

1 Introduction

Slope safety has always been the focus of attention in the field of engineering and construction[1]. And the stability calculation is the core of the slope engineering[2]. The complexity of geology, the difference of soil, the creep of soil, the drainage consolidation rate, the aging deformation of slope, bad weather and boundary conditions lead to the uncertainty and occasional occurrence of slope landslide prediction. The current research on slope mainly involves the calculation of slope finite element stability[4-5,7], slope risk analysis[3,6], sensitivity analysis of the influencing factors[2], regression analysis[4]. The theoretical methods include strength reduction method, gravity proportional loading method, limit equilibrium method, etc. Most of these methods require too much calculation and rely on computer and professional software[8-10].

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It is a new attempt to establish or fit an accurate empirical formula to intuitively understand the various factors affecting the slope safety and their possible degree of influence. Then it's possible to calculate the slope safety by a relatively simple formula. The single mapping relationship between each factor and the safety coefficient was analyzed in previous study without multivariate function fitting. The influence of multiple factors cannot be considered at one time in such fitting results so as to affect the calculative accuracy of the slope safety coefficient. In this study, strength reduction method and finite element calculation were adopted to obtain 33 groups of effective sample parameters results. The influence rules and ranking of each factor were summarized from the results. The fitting methods of unary linear, unary polynomial, multivariate linear and multivariate polynomial are used for comprehensive analysis. The multivariate linear and multivariate polynomial fitting formulas taking various influencing factors into account are obtained. The fitting accuracy is compared and a set of parameters outside the sample space are selected for finite element calculation to verify the accuracy and applicability of the fitting formula.

2 Strength reduction method

The basic idea of strength reduction method is to divide the strength parameters (internal friction angle and cohesion) of rock and soil materials by one strength subtraction coefficient to obtain the reduced strength parameter and then finite element calculation is carried out. In the calculation process, the reduction coefficient is continuously increased and the critical strength parameters are searched. When the group of strength parameters just makes the calculation not converge or makes the slope form completed sliding surface, the strength reduction coefficient at this time is the slope stability safety factor. The specific principle is shown in Formulas (1) to (2).

$$c_r = \frac{c}{F_r} \tag{1}$$

$$\varphi_r = \arctan\left(\frac{\tan\varphi}{F_r}\right) \tag{2}$$

In the equation: c_r is the cohesive force played by the soil, c is the cohesive force what can be provided by the soil, φ_r is the internal friction angle played by the soil, φ is the internal friction angle what can be provided by the soil, F_r is the strength reduction coefficient.

3 Finite element calculation and regression analysis

3.1 Finite-element calculation

The internal friction angle of one homogeneous slope is 25°, cohesion is 12 kPa, and weight density is 20 kN/m^3. The slope is 10m high and its angle is 45°. The Poisson's ratio of soil is 0.25 and elastic modulus is 50MPa. The calculation was performed by finite element software using the Mohr Coulomb yield criterion and plane strain unit CPE4 for analysis. The grid was divided using a quadrilateral swept form. Typical mesh division is shown in Figure 1 and equivalent plastic strain is shown in Figure 2.

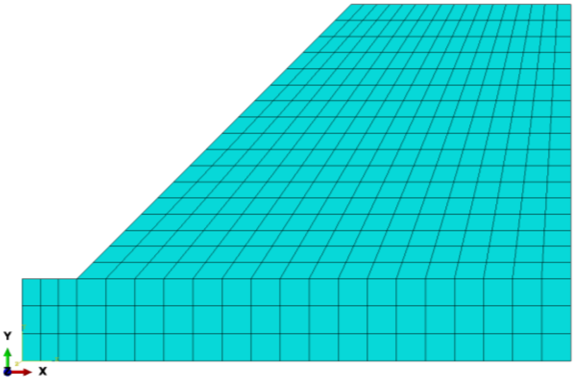

Fig. 1. Slope grid division

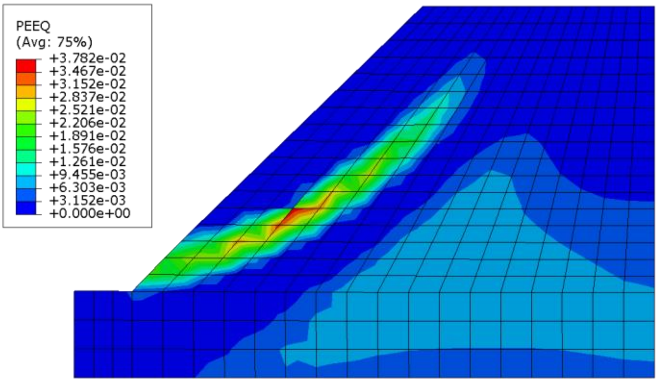

Fig. 2. Result of slope equivalent plastic strain calculation

Considering the test orthogonality and single variable controlling, the parameters were selected. The detailed parameters and calculation results are shown in Table 1. According to the parameters and results of Table 1, the influence curve of each influencing factor on the safety factor is drawn and the polynomial formula is fitted. The curve of parameter percentage change and safety factor is drawn and the influence of each factor on safety factor is analyzed.

Table 1. Analysis of the side slope parameters

No.	Internal friction angle	Cohesion	Weight density	Slope high	Slope angle	Pois-son's ratio	Safety factor
1	15	12	20	10	45	0.25	0.894
2	20	12	20	10	45	0.25	1.047
3	25	12	20	10	45	0.25	1.109
4	30	12	20	10	45	0.25	1.255
5	35	12	20	10	45	0.25	1.429
6	25	7.2	20	10	45	0.25	0.927
7	25	9.6	20	10	45	0.25	1.008
8	25	12	20	10	45	0.25	1.109
9	25	14.4	20	10	45	0.25	1.196
10	25	16.8	20	10	45	0.25	1.392
11	25	12	12	10	45	0.25	1.495
12	25	12	16	10	45	0.25	1.312
13	25	12	20	10	45	0.25	1.109
14	25	12	24	10	45	0.25	1.026
15	25	12	28	10	45	0.25	0.974
16	25	12	20	6	45	0.25	1.508
17	25	12	20	8	45	0.25	1.220
18	25	12	20	10	45	0.25	1.109
19	25	12	20	12	45	0.25	1.020
20	25	12	20	14	45	0.25	0.966
21	25	12	20	10	27	0.25	1.704
22	25	12	20	10	36	0.25	1.400
23	25	12	20	10	45	0.25	1.107
24	25	12	20	10	54	0.25	0.958
25	25	12	20	10	63	0.25	0.837
26	25	12	20	10	45	0.15	1.114
27	25	12	20	10	45	0.2	1.111
28	25	12	20	10	45	0.25	1.109
29	25	12	20	10	45	0.3	1.105
30	25	12	20	10	45	0.35	1.109
31	25	12	20	10	45	0.4	1.122
32	25	12	20	10	45	0.45	1.136
33	25	12	20	10	45	0.495	1.219

Note: The unit of internal friction angle is °, the unit of cohesion is kPa, the unit of weight density is kN/m^3, the unit of slope height is m, and the unit of slope angle is °.

3.2 Unary regression analysis

In the calculation, it's found that the elastic model has little effect on the safety factor, so it is not analyzed as an influencing factor. The Poisson's ratio is significant linear before 0.3 and the first half of the curve is selected for linear regression. According to the analysis of single-factor linear regression (Table 2), the influence of the safety coefficient is ranked as slope angle> weight density > slope height> internal friction angle> cohesion> Poisson's ratio. The slope angle has the greatest influence on the safety factor of the slope while the Poisson 's ratio has the least influence.

Table 2. Results of the linear fitting of the slope parameters

Ratio of change	Internal friction angle	Cohe-sion	Weight density	Slope high	Slope angle	Safety factor
-0.4	0.894	0.927	1.495	1.508	1.704	1.114
-0.2	1.047	1.008	1.312	1.220	1.400	1.111
0	1.109	1.109	1.109	1.109	1.107	1.109
0.2	1.255	1.196	1.026	1.020	0.958	1.105
0.4	1.429	1.392	0.974	0.966	0.837	1.109
Slope	0.1277	0.1119	-0.1326	-0.1285	-0.2177	-0.0015
Intercept	0.7636	0.7907	1.581	1.55	1.854	1.1142
R^2	0.980	0.9652	0.9386	0.8938	0.9608	0.5362

According to the influence curves of each influencing factor and the safety factor, the safety factor gradually increases with the increase of internal friction angle and cohesion. The safety factor decreases with the increase of weight density, slope height and angle. The safety factor decreases with the increase of Poisson's ratio slightly. The changes of all factors more or less reflect have some nonlinearity while the Poisson's ratio shows the linear relationship before 0.3 and the nonlinear relationship after 0.3. It can achieve good results that the internal friction angle, cohesion, weight density, slope height, slope angle are fitting with binomial and the goodness of fit coefficient is 0.9868,0.991,0.9946,0.996,0.9869,9982 respectively while the Poisson's ratio is fitting with trinomial and the goodness of fit coefficient is 0.9548.

3.3 Multiple regression analysis

Unary regression has its own shortage that it cannot reflect the effect of multiple factors acting at the same time. Therefore, on the basis of unary regression analysis, multiple regression analysis is conducted and the fitting formula is obtained. The parameter analysis of a specific slope is carried out and then the empirical formula is regressed, which can facilitate the estimation and prediction of the safety of similar slopes. In this paper, the multiple linear regression method is used firstly. The regression results are shown in Formula (3) and the goodness of fit coefficient reaches 0.922. Using the idea of substitution, according to the single factor fitting in the sensitivity analysis results, it is reasonable to judge the existence of cubic and quadratic terms in the established equation. At the same time, it is judged that each factor is relatively independent, so the mixed quadratic and cubic terms are not considered. The results of cubic and quadratic terms were calculated in advance and set as a new one-dimensional term and the coefficient regression of 13 terms was completed. The results of formula (4) were obtained and the goodness of fit coefficient reached 0.9908. For the fitting process, Formulas (5) to (7) are generally used to evaluate the fitting effect. The smaller the root mean square error, the average absolute deviation and the average absolute percentage error are, the better fitting effect is.

$$F_s = 0.0255\varphi + 0.0466c - 0.0332\gamma - 0.0643H - 0.0242\theta + 0.042\upsilon + 2.3407 \tag{3}$$

$$F_s = 0.0005\varphi^2 - 0.0009\varphi + 0.0017c^2 + 0.0054c + 0.002\gamma^2 - 0.1119\gamma + 0.0073H^2 \\ -0.2101H + 0.0005\theta^2 - 0.0691\theta + 12.2044\upsilon^3 - 9.8868\upsilon^2 + 2.5\upsilon + 5.2186 \tag{4}$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(y_t - y_t^{'})^2}{n}} \tag{5}$$

$$MAD = \frac{\sum_{t=1}^{n}\left|y_t - y_t^{'}\right|}{n} \tag{6}$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{y_t - y_t^{'}}{y_t}\right| \tag{7}$$

In the formula: φ is the internal friction angle provided by the soil and the unit is °; c is the cohesive force provided by the soil and the unit is kPa; γ is the soil weight and the unit is kN/m^3; H is slope high and the unit is m; θ is slope angle and the unit is °; υ is Poisson's ratio. $RMSE$ is error of root mean square; MAD is mean absolute deviation; $MAPE$ is mean absolute percentage error; n is sampling numbers; y_t is measured value; $y_t^{'}$ is predicted value; $\overline{y_t}$ is the average of the measured values.

From Table 3, It can be seen that the results of multiple linear fitting is good and the average values of residual and relative residual were 0 and 0.09% respectively. However, the residual and relative residual of individual calculation results still varied greatly, ranging from-0.071 to 0.117 and-14.00% to 5.94% respectively. When multivariate polynomial fitting is used the residual and relative residuals of the calculation results decrease significantly, ranging from-0.052 to 0.047 and-4.51 % to 4.28 % respectively. The average values of residual and relative residuals are only 0 and 0.02 % respectively.

Table 3. Result of regression analysis

No.	multivariate linear			multivariate polynominal		
	Fitting values	Residual error	Relative residuals	Fitting values	Residual error	Relative residuals
1	0.899	-0.005	0.59%	0.912	-0.018	1.99%
2	1.027	0.020	-1.91%	1.000	0.047	-4.51%
3	1.155	-0.046	4.16%	1.114	-0.006	0.51%
4	1.282	-0.028	2.20%	1.255	0.000	0.03%
5	1.410	0.018	-1.29%	1.423	0.006	-0.42%
6	0.931	-0.004	0.48%	0.930	-0.004	0.38%
7	1.043	-0.035	3.43%	1.012	-0.004	0.40%
8	1.155	-0.046	4.16%	1.114	-0.006	0.51%
9	1.267	-0.071	5.94%	1.236	-0.040	3.38%
10	1.378	0.014	-0.99%	1.378	0.015	-1.05%
11	1.420	0.075	-5.00%	1.506	-0.011	0.72%
12	1.287	0.024	-1.84%	1.278	0.033	-2.53%
13	1.155	-0.046	4.16%	1.114	-0.006	0.51%
14	1.022	0.004	-0.38%	1.013	0.013	-1.25%
15	0.889	0.085	-8.71%	0.975	-0.001	0.07%
16	1.412	0.096	-6.39%	1.488	0.020	-1.34%
17	1.283	-0.063	5.20%	1.272	-0.052	4.28%
18	1.155	-0.046	4.16%	1.114	-0.006	0.51%
19	1.026	-0.006	0.63%	1.015	0.005	-0.47%
20	0.898	0.068	-7.03%	0.974	-0.008	0.86%
21	1.590	0.113	-6.66%	1.711	-0.008	0.46%
22	1.372	0.028	-1.98%	1.372	0.028	-1.99%
23	1.155	-0.047	4.29%	1.114	-0.007	0.63%
24	0.937	0.021	-2.15%	0.937	0.021	-2.15%
25	0.719	0.117	-14.00%	0.841	-0.004	0.51%
26	1.151	-0.036	3.27%	1.110	0.004	-0.34%
27	1.153	-0.042	3.75%	1.119	-0.008	0.69%
28	1.155	-0.046	4.16%	1.114	-0.006	0.51%
29	1.157	-0.052	4.67%	1.106	-0.001	0.09%
30	1.159	-0.049	4.46%	1.104	0.006	-0.52%
31	1.161	-0.039	3.49%	1.116	0.006	-0.55%
32	1.163	-0.027	2.41%	1.152	-0.016	1.40%
33	1.165	0.054	-4.41%	1.212	0.007	-0.58%
μ	1.156	0.000	0.09%	1.156	0.000	0.02%
ξ	1.590	0.117	5.94%	1.711	0.047	4.28%
ζ	0.719	-0.071	-14.00%	0.841	-0.052	-4.51%

Note: μ is mean value, ξ is maximum value, ζ is minimum value.

From table 4, it can be seen that multivariate polynomial fitting is better than multivariate linear fitting, and the root mean square error decreases from 0.053 to 0.018, which decreased by 65.69%. The mean absolute deviation decreased from 0.045 to 0.013, which decreased by 71.49%. The average absolute percentage error decreased from 0.039 to 0.011, which decreased by 71.85%.

Table 4. Comparison of fitting effect characterization parameters

Multivariate linear			Multivariate polynominal		
RMSE	*MAD*	*MAPE*	*RMSE*	*MAD*	*MAPE*
0.053	0.045	0.039	0.018	0.013	0.011

4 Validation

To verify the validity of the fitting results, the selected parameters are as follows: internal friction angle is 18°, cohesion is 10 kPa, the weight density is 22 kN/m^3, the slope height is 9 m, the slope angle is 50°, Poisson's ratio is 0.23, while the boundary and elastic modulus keep unchanged using finite element software. And the slope safety factor is 0.7788. The result obtained using the multivariate linear fitting formula is 0.7590, the residual error is -0.0198 and the relative residuals is -2.54%. The result calculated by multivariate polynomial fitting formula is 0.7825, the residual error is 0.0037 and the relative residual is 0.48%. It can be seen that the calculation of multivariate linear and multivariate polynomial fitting formula is reliable and the result errors are both less than 3%, which meet the needs of the project, while the calculation of multivariate polynomial fitting formula is more consistent with the finite element results.

5 Conclusion

In this paper, a method for comprehensive analysis of various factors affecting slope stability is proposed. Multiple linear and polynomial regressions are performed on multiple factors including internal friction angle, cohesion, weight density, slope height, slope angle and Poisson 's ratio, and the fitting effects are compared. Finally through the finite element analysis by a set of parameters outside the sample space, to verify the accuracy and validity of the fitting formula, the conclusion is as follows:

(1) The strength reduction method is used to analyze the stability of the slope. The results show that the safety factor increases with the increase of internal friction angle and cohesion. With the increase of weight density, slope height and slope angle, the safety factor decreases gradually. With the increase of Poisson 's ratio, the safety factor tends to increase, but the increase is small.

(2) The linear fitting results of the univariate function show that the order of influence on the safety factor is slope angle > weight density > slope height > internal friction angle > cohesion > Poisson 's ratio. The slope angle has the greatest influence on the safety factor of the slope while the Poisson 's ratio has the least influence and the goodness of fit coefficient is far less than 1, only 0.5362.

(3) The fitting effect of univariate polynomial is better than that of linear fitting and the goodness of fit coefficient is closer to 1. The fitting results can provide a basis for the construction of multivariate polynomial fitting formula.

(4) The results of multivariate polynomial fitting are better than those of multivariate linear fitting. The goodness of fit coefficients of the two are 0.9908 and 0.922 respectively. The goodness of fit coefficient of multivariate polynomial is closer to 1. The root mean square error of multivariate polynomial fitting decreased from 0.053 to 0.018, which decreased by 65.69 %. The mean absolute deviation decreased from 0.045 to 0.013, which decreased by 71.49 %. The average absolute percentage error decreased from 0.039 to 0.011, which decreased by 71.85 %.

(5) It is verified that the multivariate linear and multivariate polynomial fitting formulas are reliable and the error of the results is within 3 %, which meets the engineering needs. However, the multivariate polynomial fitting formula is more consistent with the finite element calculation results, and the error is only 0.48 %.

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