Optimizing Knapsack Allocation: The Preemptive Multiple Bounded Knapsack Problem

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Abstract. The Multiple Bounded Knapsack Problem (MBKP) involves the task of allocating a set of items, each with bounded availability, into different knapsacks with the goal of maximizing the overall profit from the selected items while ensuring that the capacity of each knapsack is not exceeded. Knapsacks in the MBKP can be prioritized based on their importance. Priority refers to the sequence or level of importance in a system. Preemptive priority is an approach where certain objectives are given higher priority than others, allowing for faster handling or higher service for higher-priority objectives. This enables designers or systems to focus on objectives assumed most important. The MBKP with prioritized knapsacks is referred to as the Preemptive Multiple Bounded Knapsack Problem (PMBKP). It involves a process in solving the problem. The PMBKP algorithm begins by establishing a canonical form of the problem. It initiates the solving process starting with the first priority. The result obtained in the first process is then substituted into the second process with constraints on the second-priority knapsack, and this process continues until solving the knapsack in the last priority order. The solutions from the first to last priority are consolidated to form the solution for the problem Solving the PMBKP will optimize the knapsacks based on priority.

Keywords: multiple bounded knapsack problem, preemptive multiple bounded knapsack problem, priority, preemptive, bounded knapsack

1 Introduction

The Multiple Knapsack Problem (MKP) entails distributing a portion of n items among m distinct knapsacks, aiming to maximize the combined profit of the chosen items without surpassing each knapsack's capacity [1]. Every subset should match the capacity of the knapsack to which it is designated. MKP is an optimization challenge focused on allocating items to achieve the highest profit, while also ensuring that each subset can fit into knapsacks with ample capacity [2].

The Multiple Bounded Knapsack Problem (MBKP) deals with a variety of item types and multiple knapsacks, which is also referred to as the Multiple Bounded Knapsack Problem with Divisible Item Sizes (MBKP-DS) [3]. Each item type comes with

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its own size, value, and upper limit. These item sizes can be divided, and each knapsack has its own specific capacity [4].

Definition 1. [3] (Multiple Bounded Knapsack Problem (MBKP)) Given a collection of n distinct item types and a group of knapsacks denoted as $M = \{1, \ldots, m\}$. Each item of type j has a size $w_j \in Z^+$, a value $v_j \in Z^+$ and an upper bound $b_j \in Z^+$. Each knapsack i has a capacity $W_i \in Z^+$. The formulation of the MBKP can be written as follows [3]: Maximize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} v_j x_{ij}$

Subject to

$$\sum_{j=1}^{n} w_j x_{ij} \leq W_i\,; i = 1,2, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j; j = 1,2, \ldots, n$$

$$x_{ij} \in \mathbb{N}_0$$

Lemma 1. The solution to the Multiple Bounded Knapsack Problem (MBKP) exists in which each x_{ij}, the weight w_j is placed into the knapsack using most a capacity W_i for each knapsack $i = 1,2, \ldots, m$.

Proof. Since the optimal solution of MBKP exist [3], then solution of MBKP exist.

The MKP holds significant managerial implications and is acknowledged as highly challenging to solve in practical scenarios, particularly for instances of realistic scale [5]. Despite progress in solving the MKP, challenges persist due to its complexity and uncertainty, with several effective methods such as "branch and bound" by Martello and Toth in 1981, and the MULKNAP method developed by Pisinger, while recent algorithms like those proposed by Fukunaga and Korf in 2007 also demonstrate effectiveness in specific cases [6]. The solution of the Multiple Knapsack Problem (MKP) with prioritization of objects is applied to a system that manages energy consumption on the consumer's side. The results of this research indicate that the solution of the MKP is more effective when prioritization is applied [7].

Priority is the order or level of importance assigned to tasks, jobs, or objects within a system. It is used to determine the sequence of handling or servicing based on their level of importance [8]. Preemptive is a concept in which an entity or action has the right to take precedence or priority over another entity or action in receiving service or attention [9]. Preemptive priority is an approach in design problem-solving where certain objectives are assumed more important than others. In many cases, some objectives may need to be fulfilled before other goals can be considered [10]. The preemptive priority has been implemented in addressing various issues such as queueing theory [11], goal programming [12], and scheduling [13]. Its application has demonstrated streamlined problem-solving [11, 12] and effective outcomes [13].

In this paper, a new algorithm is presented to solve the Multiple Bounded Knapsack Problem by assigning priority orders to each knapsack. This algorithm is called Preemptive Multiple Bounded Knapsack Problem. Section 2 describes the PMBKP.

Section 3 elaborates on the PMBKP algorithm. In Section 4, an example of the PMBKP algorithm is provided, and the final section summarizes the conclusions drawn from the presented results.

2 Preemptive Multiple Bounded Knapsack Problem

The Preemptive Multiple Bounded Knapsack Problem (PMBKP) involves allocating quantities of distinct item types to a group of knapsacks, each with its own priority and capacity constraints. The goal is to optimize the allocation, considering item sizes, values, and upper bounds, while respecting knapsack priorities. Preemptive Multiple Bounded Knapsack Problem (PMBKP) can be stated as follows.

Definition 2. (Preemptive Multiple Bounded Knapsack Problem (PMBKP)) Given a collection of n distinct item types and a group of knapsacks denoted as $M = \{1, \ldots, m\}$. Each item of type j has a size $w_j \in Z^+$, a value $v_j \in Z^+$ and an upper bound $b_j \in Z^+$. Each knapsack i has a capacity $W_i \in Z^+$. Let there be m priorities, where p_i represents the priority of the i-th knapsack with $i = 1, \ldots, m$. The problem is to allocate the quantity x_{ij} of items of type j, for $j = 1,2, \ldots, n$, to be assigned to each knapsack i while considering its priority. The canonical form for PMBKP: Maximize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} v_j x_{ij}$$

Subject to

$$p_i\colon \sum_{j=1}^{n} w_j x_{ij} \le W_i \quad (i = 1,2, \ldots, m)$$

$$\sum_{i=1}^{m} x_{ij} \le b_j; j = 1,2, \ldots, n$$

$$x_{ij} \in \mathbb{N}_0$$

The PMBKP begins by establishing a canonical form of the problem. It initiates the solving process starting with the highest priority. The approach involves solving the problem for the highest priority first. The results obtained for this priority are then used as inputs for solving the problem at the next priority level. This process is iterated sequentially through priorities, with each set of results being used as a basis for solving the subsequent priority level. This process continues until the $(m-1)$-th priority is addressed. Subsequently, the results from the $(m-1)$-th priority are integrated into the solution process for the m-th priority. Finally, all solutions from the first to the m-th priority are aggregated to provide the comprehensive solution for the problem.

3 An Algorithm for PMBKP

In this section algorithm for PMBKP are presented. Let k represents the different priority orders for each knapsack, such that $p_k > p_{k+1}; k = 1,2,3,\dots,m-1$. The algorithm for PMBKP can be seen in **Fig. 1**.

Algorithm for PMBKP
Input: An instance of PMBKP
Output: A solution maximizing the value of the assigned items

1. Sort the knapsacks based on their priorities, starting from the first priority to the last.
   While $k \coloneqq 1: m$
   begin
2. Determine x_{kj} as the assignment of item x_j into the k-th priority knapsack by solving the knapsack problem with constraints of maximizing capacity W_k and the availability constraint of item b_j.
3. Calculate the remaining items b_j after determining the placement of item x_{kj} into the k-th priority knapsack.
4. Set $k = k + 1$.
   End

Fig. 1. An Algorithm for PMBKP

Based on an algorithm for PMBKP, the solution to PMBKP exists in which assigning each weight w_j represented by x_{ij} to a knapsack with a maximum capacity W_i, where $i = 1,2,\dots,m$, each having its priority. It can be concluded that PMKP has a solution if the MBKM problem without priority has a solution. However, the solutions in PMBKP and MBKP are not necessarily the same.

4 An Example of PMBKP

There are 3 knapsacks with maximum capacities of 10 kg, 20 kg, and 15 kg, respectively. There are 4 items with weights, values, and availability as follows:
Item 1: Weight = 2 kg, Value = 10, Availability = 4
Item 2: Weight = 3 kg, Value = 7, Availability = 6
Item 3: Weight = 5 kg, Value = 12, Availability = 5
Item 4: Weight = 7 kg, Value = 8, Availability = 9
Determine the formulation of item combinations that should be put into the knapsacks to maximize the total value without exceeding the maximum capacities while fulfilling the first, second, and third knapsacks.
The PMBKP algorithm used in the problem above can be seen in **Fig.2**.

Input: The canonical form for PMBKP above can be written as follows:
maximize $Z = 10(x_{11} + x_{21} + x_{31}) + 7(x_{12} + x_{22} + x_{23}) + 12(x_{13} + x_{23} + x_{33}) + 8(x_{14} + x_{24} + x_{34})$
Subject to:

$p_1: 2x_{11} + 3x_{12} + 5x_{13} + 7x_{14} \leq 10$
$p_2: 2x_{21} + 3x_{22} + 5x_{23} + 7x_{24} \leq 20$
$p_3: 2x_{31} + 3x_{32} + 5x_{33} + 7x_{34} \leq 15$
$x_{11} + x_{21} + x_{31} \leq 4$
$x_{12} + x_{22} + x_{32} \leq 6$
$x_{13} + x_{23} + x_{33} \leq 5$
$x_{14} + x_{24} + x_{34} \leq 9$
$x_{ij} \in \mathbb{N}_0;\ j = 1,2,3,4;\ i = 1,2,3$
where x_{ij} is item j in knapsack i

Output: The maximum value of the assigned items

1. The knapsack sequence starts with the first knapsack, followed by the second and third knapsacks.

While $k := 1:3$
$k = 1$

2. The first priority knapsack is determined as follows: maximize
$Z = 10x_{11} + 7x_{12} + 12x_{13} + 8x_{14}$
Subject to:
$2x_{11} + 3x_{12} + 5x_{13} + 7x_{14} \leq 10$
$x_{11} + x_{21} + x_{31} \leq 4$
$x_{12} + x_{22} + x_{32} \leq 6$
$x_{13} + x_{23} + x_{33} \leq 5$
$x_{14} + x_{24} + x_{34} \leq 9$
$x_{1j} \in \mathbb{N}_0;\ j = 1,2,3,4$
The solution for the first-priority knapsack is $x_{11} = 4$ and $x_{12} = x_{13} = x_{14} = 0$, meaning that item 1 is placed into knapsack 1 with a quantity of 4.

3. The remaining quantities for each item are $x_1 = 0, x_2 = 6, x_3 = 5$ and $x_4 = 9$

Continue for $k = 2$

4. The process then continues by solving the second-priority knapsack as follows: maximize
$Z = 10x_{21} + 7x_{22} + 12x_{23} + 8x_{24}$
Subject to:
$2x_{21} + 3x_{22} + 5x_{23} + 7x_{24} \leq 20$
$x_{21} + x_{31} \leq 0$
$x_{22} + x_{32} \leq 6$
$x_{23} + x_{33} \leq 5$
$x_{24} + x_{34} \leq 9$
$x_{2j} \in \mathbb{N}_0;\ j = 1,2,3,4$
The solution for the second-priority knapsack is $x_{23} = 4$ and $x_{21} = x_{23} = x_{24} = 0$, meaning that item 3 is placed into knapsack 2 with a quantity of 4.

5. The remaining quantities for each item are $x_1 = 0, x_2 = 6, x_3 = 1$ and $x_4 = 9$

Continue for $k = 2$

6. The process then continues by solving the last-priority knapsack with the following formulation. Maximize
$Z = 10x_{31} + 7x_{23} + 12x_{33} + 8x_{34}$
Subject to:

$2x_{31} + 3x_{32} + 5x_{33} + 7x_{34} \leq 15$
$x_{31} \leq 0$
$x_{32} \leq 6$
$x_{33} \leq 1$
$x_{34} \leq 9$
$x_{3j} \in \mathbb{N}_0;\ j = 1,2,3,4$
The solution for the last-priority knapsack is $x_{32} = 5$ and $x_{31} = x_{32} = x_{34} = 0$, meaning that item 2 is placed into knapsack 3 with a quantity of 5.

End

Fig. 2. The application of the PMBKP algorithm to solve the example

The maximum value obtained from all priorities is 123 by placing item 1 into knapsack 1 with a quantity of 4, item 2 into knapsack 3 with a quantity of 5, and item 3 into knapsack 2 with a quantity of 4.

5 Conclusion

The Preemptive Multiple Bounded Knapsack Problem (PMBKP) involves optimizing the allocation of various item types to a set of knapsacks, each with distinct priorities and capacity constraints. The problem is approached by establishing a canonical form and solving it iteratively, starting with the highest priority. The solutions obtained at each priority level serve as inputs for solving the subsequent priority level. This process continues sequentially until the $(m-1)$-th priority, and the results are then integrated into the solution process for the m-th priority. In the end, solutions from the first to the m-th priority are combined to provide a comprehensive solution for the entire problem.

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