



Revisiting the Application of Virtual Simulation Experiment in Methods of Mathematical Physics

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Abstract. In order to stimulate students' learning enthusiasm and enhance their learning experience, the article focuses on "student-centered and personalized development", using "case simulation" as a starting point to explore the implementation of "interactive and visual" teaching reform. Abstract knowledge is visualized, and simulation experiments are used to assist teaching, increase interaction links, and stimulate real learning. The simulation experiments produced are shared through the Internet to supplement information technology teaching resources. This measure enriches the teaching content, enriches teaching methods, and closely links teaching with practical problems, making the course more challenging and innovative, and achieving good teaching effect.

Keywords: component; Virtual simulation; Informatization; Methods of Mathematical Physics.

1 Introduction

Methods of Mathematical Physics are fundamental course in the field of physics and important tools for solving complex mathematical and physical problems or specific interdisciplinary problems [1-5]. Jiang Ping et al. studied the application of simulation experiments in courses from the perspectives of 3D visualization methods, increasing course challenges, and course innovation [6]; Zhu Jun et al. studied the practice of ideological and political education in curriculum courses using "multiple solutions to one problem" as an example, and believed that the focus should be on cultivating students' scientific thinking awareness, dialectical thinking ability, and innovative ability[7]; Tian Xiuyun et al. used MATLAB visualization methods to make traditional classroom teaching more visual and strengthen students' understanding of teaching content [8]; Zhao Xiaoyun et al. studied the organization of course teaching content from the perspective of improving learning efficiency [9]; Zhu Yongsheng's teaching reform was carried out from the aspects of neglecting the combination of theory and physical phenomena in the curriculum, being disconnected from practical examples in daily life, and being disconnected from the analysis of specific physical backgrounds[10]; Liu Yi

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corrected the misconception among students that mathematics and physics methods course are mathematics classe and unrelated to physics by demonstrating the sound of music on site and analyzing its Fourier spectral lines in class [11].

Through model building, approximation processing, boundary setting, discussion and analysis, simulation and competition, this paper helps students understand that the essence of mathematical physics equations comes from specific problems, and they need to return to physical knowledge, and then use physical knowledge to explain specific problems. In short, it is a process of "problem-knowledge-problem".

2 Curriculum reform content

2.1 Curriculum reform objective

In view of the problems such as strong comprehension, disconnection between theory and application, and large content of knowledge and information, the author has explored and implemented the "interactive and visual" teaching reform. Centering on the overall goal of "student-centered and personalized development", in science education, interactive teaching is used to encourage real learning, simulation experiments are used to help teaching, teaching methods are improved, and information teaching resources are built. In classroom teaching, students are guided to apply the familiar idea of "combination of number and shape" to the field of complex variable functions, and simulation interaction is carried out with the help of computer software "LabVIEW or matlab", so that abstract knowledge and concepts can be displayed to students intuitively and graphically to enhance learning experience.

2.2 Curriculum reform ideas and design

In the process of classroom instruction, students are guided to apply the familiar concept of "combination of number and form" to the realm of complex variable functions. Through simulation and interaction, abstract content is visualized, thereby enhancing teaching effectiveness and enabling students to experience the intimate connection between this knowledge domain and materialism as well as methodology. They are encouraged to consciously analyze and address problems from a Marxist perspective. For instance, this includes solving elementary functions and typical mathematical physics equations in the complex field through simulation. The course employs interactive and visual teaching methods such as group discussions and concentrated sharing. To ensure teaching quality while continuously improving students' problem-solving abilities, assessments for this course incorporate student-led discussions on results achieved, shared case studies, analysis of applied curriculum theories, as well as solutions for discipline-based competition projects.

3 Typical case

3.1 Simulation, "Numeral-form association" of complex function theory

The combination of number and form is an important idea in mathematics teaching. "Number" and "form" are two aspects of the intrinsic attributes of things. Just as Mr. Hua Luogeng said, "when the number lacks form, it is less intuitive, and when the form is few, it is difficult to understand." In the study of mathematical knowledge, the abstract mathematical language, complex mathematical formulas and intuitive images are combined, that is, the combination of abstract thinking and image thinking, the purpose of which coincides with the establishment of ideal models in physics, that is, to grasp the essence of complex problems, make them simple, easy to do, can be solved, make abstract problems concrete, image, visible. Many concepts and formulas in complex variable function theory are very abstract. Through computer simulation, "combination of number and form" can keep pace with The Times and adapt to the teaching needs of the new era.

Elementary Function Case

Taking the power function in elementary functions as an example:

$$Y = X^2 \quad (1)$$

$$\omega = z^2 \quad (2)$$

Among them, equation (1) is a real variable function, equation (2) is a complex variable function, where "z" is a complex number and the expression is "z=x+iy". "x" and "y" are the real and imaginary parts of the complex number "z", respectively, and "i" is an imaginary unit. When "i" does not appear in the expression of "z", "z" is simplified to a real number, and the graphs of the two functions are made, as shown in Figure 1 and Figure 2:

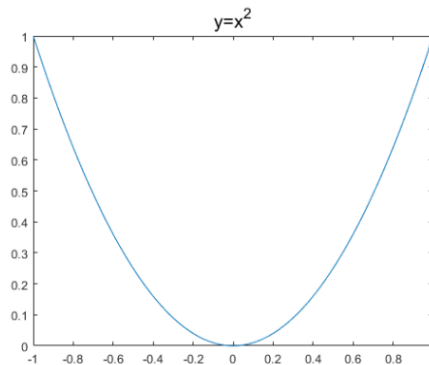


Fig. 1. Graph of the real variable function $Y = X^2$

By using computer simulation methods, the mathematical graphics of radical complex functions can be represented and compared with knowledge related to real functions, making it easy to understand new knowledge, as shown in Figures 1 and 2. You can visually see the symmetry of the " $Y = X^2$ " image about the Y-axis from the graph. For complex functions $\omega = z^2$, when the imaginary part $y=0$, the complex number z is equal to the real number x , the complex function is simplified into a real function, and the graph will also be the same as the real function. Therefore, it can be seen that real numbers are a special case where the imaginary part of complex numbers is equal to zero. In teaching practice, changing the perspective of the function $\omega = z^2$ graph, for example, selecting the perspective as "view (0,0)", as shown in Figure 3, can intuitively express this relationship and help students understand the relationship between power functions in different number systems. In Figure 4, the value of ω is related to both the real and imaginary parts of the argument z , i.e. x and y , while the value of Y in the real variable function is only related to the independent variable X . From the figure 4, it can be seen that the graph with $Y = X^2$ after rotation is the same when the imaginary part of $\omega = z^2$ is zero, which is consistent with the theoretical analysis results.

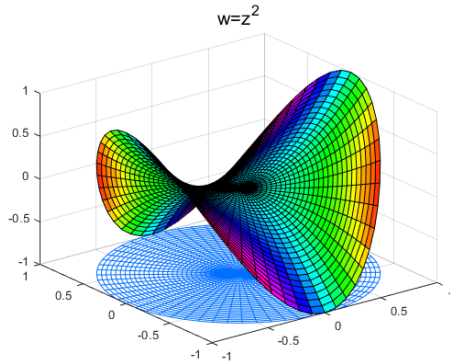


Fig. 2. Diagram of the complex function $\omega = z^2$

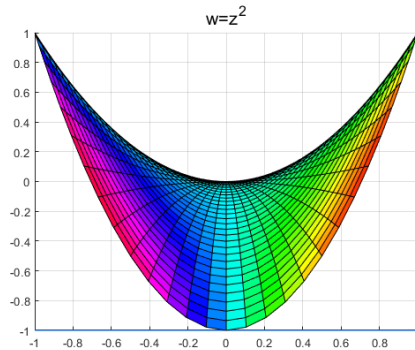


Fig. 3. Complex function $\omega = z^2$ view (0,0) graph

In summary, both are power functions. In the real field of view, the image is a parabola, while in the complex field, the image is a surface as shown in Figure 2. In response to this phenomenon, it can be understood from two aspects: on the one hand, when the range of numbers expands from the real range to the complex range, the function graph changes from a two-dimensional curve to a three-dimensional surface, indicating that after the number system is expanded, the power function image becomes more complex and the information expressed is richer; On the other hand, there are still similarities between the function images before and after the expansion of the number system. By changing the viewing angle, the images tend to be the same. When the imaginary part of the complex function is zero, the images are the same as the real function. Based on the above knowledge, students can learn to look at things and analyze problems from the perspective of connection, and it is easier to see the whole picture of the same thing from multiple angles. For the images of power functions, the similarities and differences between real and complex numbers are found out by comparison, and the dialectical relationship (such as the relationship between special and general) is understood.

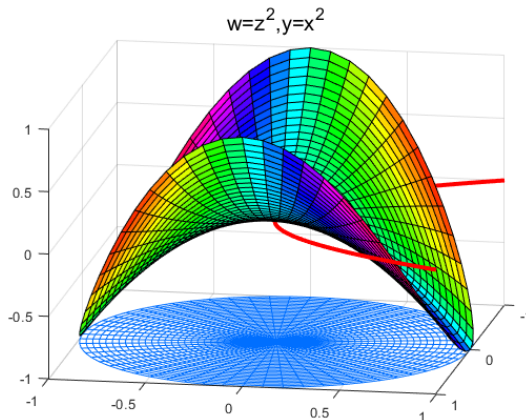


Fig. 4. Function graph of view (100,20)

Fourier transform Case

The Fourier transform, as an important mathematical tool for converting time domain to frequency domain, has a wide range of applications in signal processing, electronics, communication, and other fields. Therefore, when explaining the Fourier transform, it should be combined with specific physical meanings, supplemented by virtual simulation experiments, in a dynamic and interactive form, to make abstraction concrete and help students understand the Fourier transform. Encourage students to develop the habit of using Fourier transform to analyze and process physical quantities. The Fourier transform formula is [11]:

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x)e^{-i\omega x} dx \quad (3)$$

In the formula (3), $f(x)$ represents the signal to be analyzed, and $F(\omega)$ represents the amplitude value of the ω signal in the frequency of the signal to be analyzed. The meaning of multiplying $F(x)$ by $e^{-i\omega x}$ is to find the signal with the frequency of ω in the signal $f(x)$ to be analyzed. If there is a signal with frequency ω in the signal to be analyzed, the value of formula (3) is not zero; otherwise, the value of the integral is zero. Next, we will explain this phenomenon from two aspects: mathematical operations combined with computer simulation.

For the integral $I = \int_{-\infty}^{+\infty} f(x)e^{-i\omega x}dx$, let $(x) = \sin \omega_0 x$, then,

$$I = \int_{-\infty}^{+\infty} \sin \omega_0 x e^{-i\omega x} dx = \int_{-\infty}^{+\infty} \sin \omega_0 x (\cos \omega x - i \sin \omega x) dx = \int_{-\infty}^{+\infty} \sin \omega_0 x \cos \omega x dx - \int_{-\infty}^{+\infty} i \sin \omega_0 x \sin \omega x dx \quad (4)$$

The integral interval of formula (4) is infinite, may wish to truncate the integral interval into an integer period interval, and write formula (4) into formula (5):

$$I = \int_{-NT}^{NT} \sin \omega_0 x \cos \omega x dx - \int_{-NT}^{NT} i \sin \omega_0 x \sin \omega x dx \quad (5)$$

Where T represents the period, N takes an integer. According to the orthogonality of trigonometric functions, the first term on the right side of equation (4) is zero, when $\omega_0 \neq \omega$, the second term on the right side of equation (4) is also zero, so $I=0$; Only when $\omega_0 = \omega$, the integral is not equal to zero, so,

$$I = - \int_{-NT}^{NT} i \sin \omega x \sin \omega x dx = -\frac{i}{2} \int_{-NT}^{NT} (1 - \cos 2x) dx = -iNT \quad (6)$$

Based on the above calculation, it can be concluded that when $\omega_0 \neq \omega$, then $F(\omega) = 0$ which means that there is no frequency in the analyzed signal ω Signal, when $\omega_0 = \omega$, then $F(\omega) \neq 0$, which indicates the presence of a frequency of ω The signal has an amplitude value of $F(\omega)$. Take $\omega_0 = 3$ and $f(x) = \sin 3x$ as an example: consider the complex function $u = \sin 3x \times e^{-i\omega x}$, and use LabVIEW to draw the graph of u on the complex plane when $\omega=1.5, 6$ and 3 , as shown in Figures 5, 6, and 7. From Figures 5 and 6, it can be seen that when $\omega \neq 3$, all the points in the figure are evenly distributed in the positive and negative regions, so the algebraic sum of their values is equal to zero. According to the meaning of Fourier transform, it can be inferred that there is no signal with an angular frequency equal to 3 in the analyzed signal; Figure 7 illustrates that when the angular frequency is equal to 3, the function graph is only distributed in the lower half plane of the complex plane, and the algebraic sum of its values is not zero. Therefore, there is a signal with an angular frequency equal to 3 in the analyzed signal. This is consistent with the theoretical analysis results of equations (5) and (6). In summary, if Fourier transform is applied to the function "sin 3x", only signals with an angular frequency equal to 3 can be found, that is, $\sin 3x$.

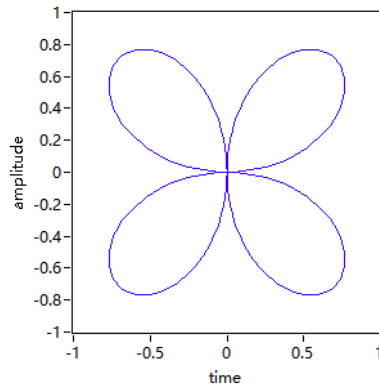


Fig. 5. curve of function $u = \sin 3x \times e^{-i\omega x} (\omega = 1.5)$

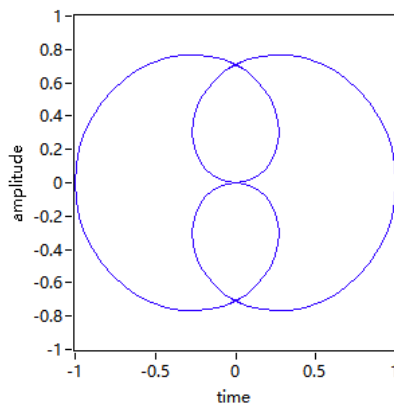


Fig. 6. curve of function $u = \sin 3x \times e^{-i\omega x} (\omega = 6)$

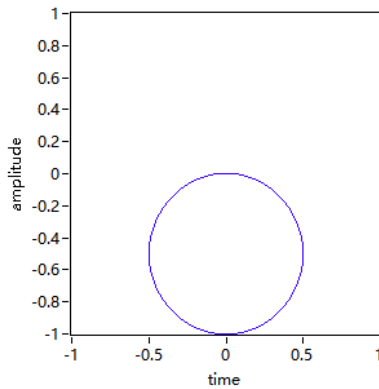


Fig. 7. curve of function $u = \sin 3x \times e^{-i\omega x} (\omega = 3)$

Let $g(x) = \sin(2\pi * 10)x + 2 \sin(2\pi * 30)x + 3\sin(2\pi * 40)x$, use Matlab to solve the Fourier transform of the mixed signal $g(x)$, as shown in Figure 8. From the graph, it can be seen that signals with frequencies of 15Hz, 20Hz, and 30Hz were identified, and their amplitudes correspond to 1, 2, and 3, respectively, which are consistent with reality.

In order to make it easier to understand Fourier integration and Fourier transform, the author designed a virtual simulation experiment of Fourier transform based on LabVIEW. Figure 9 is the screenshot of simulation animation. Among them, the red point moves according to the law of sine, its angular frequency is ω_0 , the black thick line rotates counterclockwise around the origin of the coordinates (0,0), and the angular frequency is ω . The blue line represents the trajectory generated by the movement of the above red points limited to the black thick line, and gives the value of the integral. The simulation experiment once again verified that when $\omega_0 \neq \omega$, the integral value is zero.

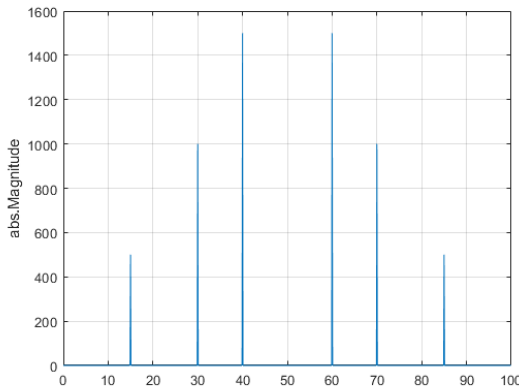


Fig. 8. Fourier transform of the function $\sin 20\pi x + 2 \sin 60\pi x + 3\sin 80\pi x$

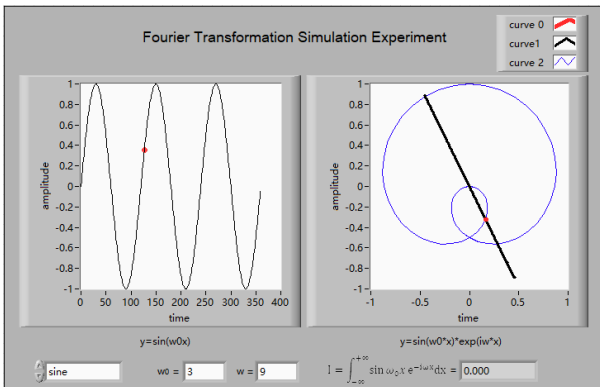


Fig. 9. Fourier Transform Simulation Experiment

3.2 Examples of Mathematical and Physical Equations

Mathematical physics equation refers to a series of partial differential equations, which are used to express the change law of the physical quantity studied in space or time. It highlights the characteristics of physics with mathematics as the language, and aims to guide students to transform specific physical problems into mathematical definite solution problems for the study of corresponding physical processes [12-13]. Specific physical problems come from natural phenomena, because natural phenomena are complex and changeable, so the problems studied by mathematical physics equations are also complex and changeable. In this part of teaching, the simulation experiment is applied to teaching, and strive to combine the boring and abstract formula deduction with the image and intuitive image to enhance the teaching effect.

One-dimensional wave equation

Problem 1: A light rope is fixed at both ends, pull the rope at position x_0 , so that it is a short distance away from the balance position, and study the vibration of the rope after releasing the hand.

The problem of fixed rope vibration at both ends is described in mathematical language as shown in formula (7):

$$\begin{cases} \mathbf{u}_{tt} - \frac{1}{4} \mathbf{u}_{xx} = \mathbf{0} & (0 \leq x \leq 1, 0 \leq t \leq 10) \\ \mathbf{u}(t, 0) = \mathbf{0}, \mathbf{u}(t, 1) = \mathbf{0} \\ \mathbf{u}(t, 0) = \begin{cases} \frac{1}{10} \frac{x}{p} & 0 \leq x \leq p \\ \frac{1}{10} \frac{1-x}{1-p} & p \leq x \leq 1 \end{cases} \\ \mathbf{u}_t(0, x) = \mathbf{0} \end{cases} \quad (7)$$

In formula (7), \mathbf{u} is the vibration problem of a rope fixed at both ends under the first kind of boundary conditions, the value of \mathbf{u} represents the displacement of the rope from the equilibrium position, t is the movement time, x is the coordinate of the particle on the rope, and p is set by the initial position. The analytical solution of the fixed solution problem is obtained by using the separation variable method as follows:

$$\mathbf{u}(x, t) = \begin{cases} \left\{ \frac{1}{5p\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cos \frac{n\pi}{2} t \sin n\pi x \right. & 0 \leq x \leq p \\ \left. \left\{ \sum_{n=1}^{\infty} \frac{1}{5(1-p)} \left[1 - \frac{1}{\pi} \frac{(-1)^{n-1}}{n} \right] \cos \frac{n\pi}{2} t \sin n\pi x \right\} \right. & p \leq x \leq 1 \end{cases} \quad (8)$$

Formula (8) is the analytical solution to the definite solution problem (7), where $\mathbf{n} = 0, 1, 2, \dots + \infty$ Only from the formula (8) It is difficult to imagine the physical image it expresses. Using LabVIEW for simulation, by setting or changing parameters for human-machine interaction, the vibration of the rope at different positions and times can be simulated, and the rapid vibration process of the rope, which is difficult to observe by the human eye, can be visually displayed.

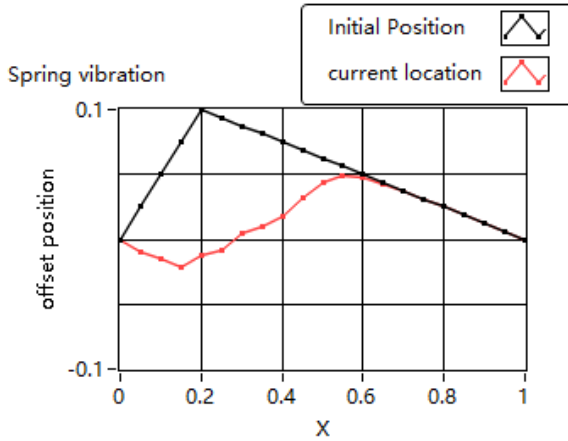


Fig. 10. position of the rope at $x_0 = 0.2\text{m}$ and $t=0.75\text{s}$

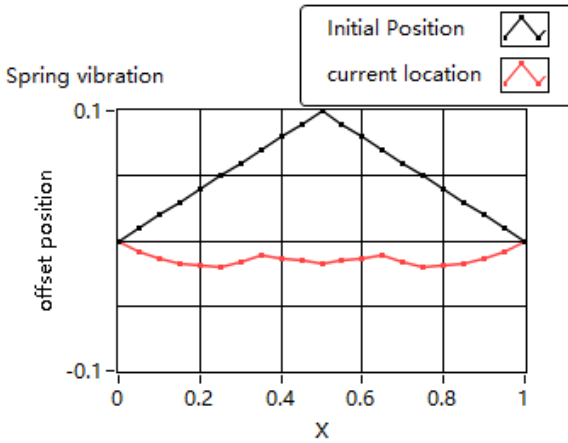


Fig. 11. position of the rope at $x_0 = 0.5\text{m}$ and $t=2.85\text{s}$

Figures 10 and 11 are screenshots of the simulation dynamic images of the displacement u changing with time t , where the rope is moved at $x=0.2\text{m}$ and 0.5m , respectively (Black line in the figure), images of the free vibration time of the rope at 0.75s and 2.85s respectively (red line in the figure), using different colors to reflect the position of the rope in Figures 12 and 13. The use of simulation experiments not only helps students understand physics problems in an intuitive way, but also guides and encourages students to dare to challenge themselves and try new technological means to assist in learning.

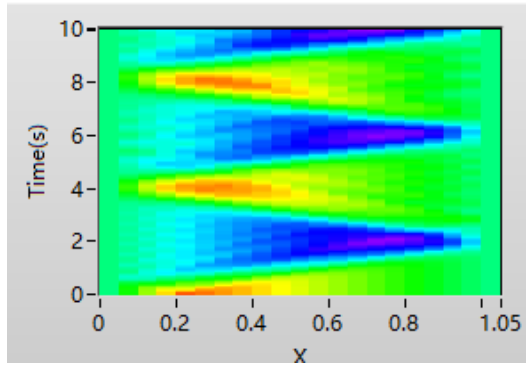


Fig. 12. $u - t$ image at $x_0 = 0.2\text{m}$

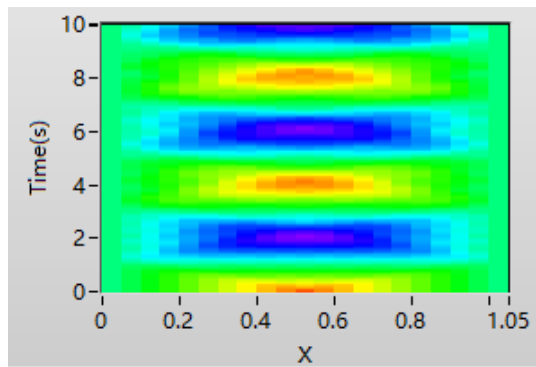


Fig. 13. $u - t$ image at $x_0 = 0.5\text{m}$

Two dimensional wave equation

Problem 2: As shown in Figure 14, one side of a rectangular heat sink b is at a higher temperature $U=100^\circ\text{C}$, and the other three sides $y=0$, $x=0$ and $x = a$ are in the cooling medium, and the lower temperature is $u_0 = 0^\circ\text{C}$. The stable temperature distribution of this rectangular cross section is studied.

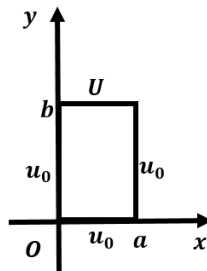


Fig. 14. Rectangular heat sink

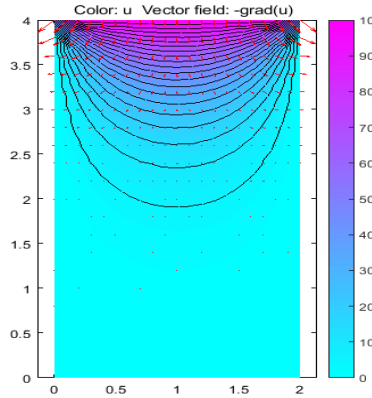


Fig. 15. Temperature distribution

The definite solution problem can be expressed as:

$$\begin{cases} u_{xx} + u_{yy} = 0 \\ u(0, y) = u(a, y) = 0 & 0 < y < b \\ u(x, 0) = 0, u(x, b) = U & 0 < x < a \end{cases} \quad (9)$$

Where u is the temperature on the cross section, x and y are the coordinates in the plane cartesian coordinate system, and a and b are the upper limits of the x and y direction boundaries respectively. The general solution of formula (9) is obtained by using the method of separating variables:

$$u(x, y) = u_0 + \frac{a(U-u_0)}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \frac{sh(2n+1)\frac{\pi y}{a}}{sh(2n+1)\frac{\pi b}{a}} sin(2n+1) \frac{\pi x}{a} \quad (10)$$

In the boundary conditions, $u_0 = 0^\circ\text{C}$, $U=100^\circ\text{C}$, $a=2$, $b=4$, MATLAB was used to simulate the stable temperature distribution, as shown in Figure 15. As can be seen from the figure, the closer to the boundary $y=4$, the greater the temperature gradient, which is consistent with the actual problem.

One-dimensional fluctuations and the heat dissipation of rectangular heat sink are common physical phenomena, but the human eye is difficult to observe its specific physical process, through physical modeling, approximate processing and other operations, after finding its general solution, and then simulation with the help of computer software, making it difficult to directly observe the physical process visible, controllable, and interactive. In the above examples, different computer software is used to meet the learning needs of different students as much as possible, and students are encouraged to actively use the technology they are good at or easy to master in the learning process according to their own preferences and strengths. When encountering problems, they should not be afraid, but dare to use their hands and brains, and be good at analyzing and solving them by comprehensive use of the knowledge they have learned. Firmly believe that the problems encountered will be solved in their own efforts and progress.

4 Conclusions

The course team takes modeling and simulation, theory examination, group discussion and other methods as course score assessment methods, encourages students to practice, applies mathematical physics modeling ideas, simulation experiments and three-dimensional visualization methods to the teaching of "Methods Of Mathematical Physics" course, supplements the information teaching resources, highlights the innovation of the course, and makes the difficulty level of the course more suitable for students to learn. It cultivates students' comprehensive ability to observe and solve problems from a perspective, enhances the learning experience and improves the teaching effect.

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