Research on Multi-Criteria Decision Making Method of Unbalanced Type-2 Fuzzy Linguistic TODIM

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Abstract. This article proposes the concept of unbalanced type-2 fuzzy linguistic term set. The membership function is studied, then TODIM model based on unbalanced type-2 fuzzy linguistic is constructed, the model takes into account human mental behavior. Considering the case that the attributes have a priority relationship and combining with the traditional TODIM model, using uncertainty instead of dominance to select and rank alternatives, the alternatives is selected and ranked, Finally, a case study is carried out.

Keywords: Unbalanced type-2 fuzzy linguistic · menbership · TODIM model

1 Introduction

With the rapid development of society, practical decision-making problems become more and more complex. It is difficult for decision-makers to accurately evaluate alternatives by using precise values in the decision-making process, people usually describe them through linguistic evaluation values. In practice, the emergence of type-1 fuzzy set theory [1] makes it possible to deal with such subjective imprecise information.

As an extension of type-1 fuzzy set, type-2 fuzzy set [2–4] are used to describe the membership of fuzzy set, the fuzzy degree of set membership value is given to enhance the set fuzziness, so it is used to deal with multi-criteria decision problems. In these multi-criteria decision problems about type-2 fuzzy set, a set of linguistic term often used by decision makers are mostly considered to be uniformly symmetric (equidistance) distributed. However, quidistant set of linguistic terms are no longer appropriate for dealing with multi-criteria decision problems, in real decision-making, due to the different cognitive levels of decision makers and the nonlinear changes of complex things. Equidistant linguistic terms do not vividly represent the degree of nonlinear variation. Decision makers are more inclined to evaluate these multi-criteria decision problems using an unbalanced linguistic terms, information on linguistic representations known in the paper as unbalanced linguistic term set. Some scholars [5, 6] have proposed a method of managing multi-criteria decision making in the context of unbanlance fuzzy linguistic. Based on this, the unbalanced type-2 fuzzy linguistic term set is proposed,

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compared with other linguistic term set, it has significant advantage in expressing fuzzy evaluation information. In this paper, the complex decision problems are studied, and the concept of non-equilibrium type-2 fuzzy linguistic term set is proposed, and the non-equilibrium type-2 fuzzy linguistic term set is used to completely represent the evaluation information of people on the decision problem.

TODIM is an extension of prospect theory and can reflect the psychological behavior of decision makers. Therefore, TODIM method has been deeply studied by many scholars. Liu [7] proposed a combinatorial weighting method based on cross entropy and entropy measure to determine target weights, extending the probabilistic linguistic TODIM method. In order to better deal with linguistic information in qualitative environment, experts and scholars have extended TODIM method to different linguistic environments [8–10].

The paper is organized as follows: Sect. 2 introduces the concept and theory knowledge of unbanlance type-2 fuzzy linguistic term set. Section 3 introduces TODIM multi-criteria decision making method of unbanlance type-2 fuzzy linguistic. Section 4 uses an example of the development of new energy vehicles is presented to illustrate the usability of the proposed method. Section 5 presents our conclusions.

2 Preliminaries

2.1 Unbalanced Type-2 Fuzzy Linguistic Term Set

Definition 2.1 Let any two unbanlance type-2 fuzzy linguistic term set:

$$S = \{s_\alpha | \alpha = -\tau, \cdots, -1, 0, 1, \cdots, \tau\} \theta = \{\rho_t | t = -\sigma, \cdots, -1, 0, 1, \cdots, \sigma\} \quad (1)$$

Then a unbanlance type-2 fuzzy linguistic term set: term set is defined as follows:

$$\mathrm{U} = \{s_{\alpha<\rho_t>}, u_{<\mu>} \varphi(\alpha, t) | \forall s_\alpha \in S, \rho_t \in \theta; u, \mu \in [0, 1]\} \quad (2)$$

where, s_α, ρ_t is the element of linguistic term set, u, μ is the corresponding semantic membership degree of s_α, ρ_t, $\varphi(\alpha, \mathrm{t})$ represents the semantics of $s_{\alpha<\rho_t>}$

2.2 Linguistic Membership Scale Functions of Unbanlance Type-2 Fuzzy Term Set

Definition 2.2 The linguistic membership function R and V can be expressed as a mapping, the form of semantic symmetry and increasing deviation is adopted.

For the first and second layer linguistic membership scale function is:

$$\mathrm{R}(s_\alpha) = \begin{cases} \frac{\varepsilon^{\frac{g-1}{2}} - \varepsilon^{-\alpha}}{2\left(\varepsilon^{\frac{g-1}{2}} - 1\right)}, \alpha < 0 \\ \frac{\varepsilon^{\frac{g-1}{2}} + \varepsilon^{\alpha} - 2}{2\left(\varepsilon^{\frac{g-1}{2}} - 1\right)}, \alpha \ge 0 \end{cases} \quad (3)$$

where, $\varepsilon(\varepsilon > 1)$ is represented the adjustment parameter, g is the granularity, and α is the subscript of the corresponding linguistic term.

$$V(\rho_t) = \begin{cases} \frac{1}{1+g^{-\sigma_1 t}}, t \geq 0 \\ \frac{1}{1+g^{-\sigma_2 t}}, t < 0 \end{cases} \tag{4}$$

where, σ_1, σ_2 are the cognitive parameters of decision makers.

For the semantic value $s_{\alpha<\rho_t>}$, $\varphi(\alpha, t)$ is the linguistic scale function.

$$\varphi(\alpha, t) = R(\alpha - 1)(1 - V(\rho) + R(\alpha + 1)V(\rho) \tag{5}$$

Definition 3.1 let $\phi = (u, \mu)$, then the uncertainty function $H(\phi)$ of the unbalanced type-2 fuzzy linguistic term set is as follows:

$$H(\phi) = \begin{cases} \pi(1-u)^{\delta}/1+\delta\,(1-\mu)^{1}/1+\delta,\ \mu \in [0, 1] \\ \pi(1-u),\ \mu = 1 \end{cases} \tag{6}$$

where, $\delta \geq 1$ is the ratio of the semantic membership of the first and second level.

3 TODIM Multi-Criteria Decision Making Method for Unbalanced Type-2 Fuzzy Linguistic

A new TODIM decision method is proposed. Let $E = \{e_1, e_2, \cdots, e_\sigma\}$ is the set of decision makers $A = \{a_1, a_2, \cdots, a_m\}$ be the set of alternatives, $C = \{c_1, c_2, \cdots, c_n\}$ is the criterion set, $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$ is the criterion weight, $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$. The specific steps are as follows:

Step 1: Each expert gives the evaluation matrix $R = [r_{ij}]_{n\times m}$, the normalization method is carried out to obtain $Y = [y_{ij}]_{n\times m}$ and computes the semantic membership matrix of the expert $Z = [z_{ij}]_{n\times m}$.
Step 2: According to the relevant weight of the priority criterion, the criterion with the highest weight is selected and calculate the relative weight ω_{jr} of each criterion.
Step 3: The uncertainty matrix $O = [o_{ij}]_{n\times m}$ of normalized matrix information.
Step 4: Calculate the individual uncertainty matrix $Q = [q_{ij}]_{n\times m}$ of alternative a_i relative to alternative a_l for all criteria.

$$\Phi_j(a_i, a_l) = \begin{cases} \frac{\omega_{jr}}{\sum_{j=1}^{n}\omega_{jr}}\left(o_{ij} - o_{lj}\right), \left(o_{ij} - o_{lj}\right) > 0 \\ 0, \left(o_{ij} - o_{lj}\right) = 0 \\ -\frac{1}{\theta}\sqrt{\frac{\sum_{j=1}^{n}\omega_{jr}\left(o_{ij}-o_{lj}\right)}{\omega_{jr}}}, \left(o_{ij} - o_{lj}\right) < 0 \end{cases} \tag{7}$$

Step 5: The global uncertainty of alternative a_l relative to all other alternatives is calculated according to the individual uncertainty matrix.

$$\zeta(a_i, a_l) = \sum\nolimits_{j=1}^{n} \Phi_j(a_i, a_l) \tag{8}$$

Step 6: The overall uncertainty of the alternatives is normalized.

$$\xi_i = \frac{\sum_{l=1}^{m}\zeta(a_i, a_l) - \min_{i\in M}\{\sum_{l=1}^{m}\zeta(a_i, a_l)\}}{\max_{i\in M}\{\sum_{l=1}^{m}\zeta(a_i, a_l)\} - \min_{i\in M}\{\sum_{l=1}^{m}\zeta(a_i, a_l)\}} \tag{9}$$

Table 1. Original decision matrix of Expert 1

	c_1	c_2	c_3	c_4
a_1	$s_{2<\rho_0>}$	$s_{1<\rho_1>}$	$s_{0<\rho_2>}$	$s_{-1<\rho_1>}$
a_2	$s_{3<\rho_1>}$	$s_{3<\rho_0>}$	$s_{-3<\rho_1>}$	$s_{2<\rho_2>}$
a_3	$s_{1<\rho_1>}$	$s_{2<\rho_{-1}>}$	$s_{3<\rho_3>}$	$s_{0<\rho_1>}$

Table 2. Original decision matrix of Expert 2

	c_1	c_2	c_3	c_4
a_1	$s_{1<\rho_1>}$	$s_{3<\rho_0>}$	$s_{-3<\rho_1>}$	$s_{-1<\rho_1>}$
a_2	$s_{2<\rho_{-1}>}$	$s_{2<\rho_0>}$	$s_{3<\rho_3>}$	$s_{0<\rho_1>}$
a_3	$s_{1<\rho_1>}$	$s_{2<\rho_2>}$	$s_{0<\rho_1>}$	$s_{3<\rho_3>}$

Table 3. Original decision matrix of Expert 3

	c_1	c_2	c_3	c_4
a_1	$s_{2<\rho_0>}$	$s_{1<\rho_1>}$	$s_{0<\rho_2>}$	$s_{-1<\rho_1>}$
a_2	$s_{3<\rho_1>}$	$s_{3<\rho_0>}$	$s_{-3<\rho_1>}$	$s_{2<\rho_2>}$
a_3	$s_{1<\rho_1>}$	$s_{2<\rho_{-1}>}$	$s_{3<\rho_3>}$	$s_{0<\rho_1>}$

Calculated according to the steps in Sect. 3
$\xi(a_1) = 0.178, \xi(a_2) = 0, \xi(a_3) = 1.$
Sort to get: $a_2 \succ a_1 \succ a_3$

4 Application Example

By comprehensively sorting out the factors that affect the development competitiveness of new energy vehicles, we need to evaluate from four aspects: driving force, pressure, state and response. There are three regions A $= \{a_1, a_2, a_3\}$ to choose, consider four criteria C $= \{c_1, c_2, c_3, c_4\}$ to evaluate alternatives, the weight information of the criterion is $\{0.2, 0.15, 0.15, 0.5\}$. Criterion priority relationship $c_1 \succ c_2 \succ c_3 \succ c_4$, three experts E $= \{e_1, e_2, e_3\}$ is shown in Tables 1, 2, and 3. Due to space limitation, Expert 1 was taken as an example.

5 Conclusion

In this paper, the concept of unbanlance type-2 fuzzy linguistic term set is proposed, and the theory of unbanlance type-2 fuzzy linguistic term set is studied. The linguistic term set can satisfy the semantic conversion needs of decision-makers in different situations and is more consistent with the actual decision-making situation. In addition, consider human psychological behavior the multi-criteria decision making method of unbalanced type-2 fuzzy linguistic TODIM proposed in this paper, takes into account the preferential relationship between the criteria when the weight information is obtained, eliminate the interference of the priority relationship between criteria on the result of decision making. Finally, a case study is carried out to verify the feasibility of the multi-criteria decision making method.

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