Advances in Economics, Business and Management Research, volume 211
Proceedings of the 2022 7th International Conference on Financial Innovation and Economic Development (ICFIED 2022)

The Optical Portfolio of the Markowitz Model and the Index Model

-Under Ten Stocks from S&P 500

Zhide Zhang[1,*]

[1] *Xian Tieyi High School, Xi'an, 710054, China*
**ZhideZhang2022@163.com*

ABSTRACT

This paper compares the effectiveness of the Markowitz model with the Single Index model. Using ten stocks of the S&P 500 index from 2000 to 2020, we first calculate the portfolio of the two models and then find the optimal portfolio to help people know better about choosing a better model under different conditions. In this paper, the portfolio of the Markowitz Model is a little better than the portfolio of the Single Index Model, which means that under the same risk condition, the portfolio of the Markowitz Model can get more returns than the portfolio of the Single Index Model.

Keywords: *Markowitz Model, Single Index Model. Portfolio*

1. INTRODUCTION

The benefits of a portfolio are obvious. Suppose investors can insist on long-term investment and prevent risks through the portfolio. In that case, they can safely tide over the short-term decline of the market, and the portfolio can also reduce risk or volatility. The investment portfolio is to buy various securities and various investment products in different industries and departments in investment portfolio activities to share risks. That is to say, the individual asset allocation of the portfolio is dispersed in different asset types, including stocks, bonds, cash (money market funds and short-term certificates of deposit are regarded as cash), real estate and gold. Each portfolio asset has different reward and risk standards, and their portfolio trends must be different. For example, the value of this type of investment product may be rising, and the value of another investment variety is declining. In this way, the losses caused to investors by the decline in value can be offset.

Generally, the investment adopts the principle of dispersing the nonsystematic risk of the stock market through the stock portfolio. However, the research conclusions are diverse due to the different sample groups, sampling methods, and empirical methods adopted by everyone in the research process. The advantage of the Markowitz model is that if your estimated value is accurate, the result of the Markowitz model is better than others [1]. For example, the Single Index model is related to many market factors, which makes it has many uncertainties. [2]. Therefore, if the original data is accurate, the Markowitz model can give better results than the Single Index model. However, if the stock estimate is large, it will be troublesome to adopt the Markowitz model. At this time, the advantages of the Single Index model are highlighted. However, the Single Index model has a great correlation with the market, which leads to an inaccurate portfolio. Therefore, it is necessary to study the comparison of the two models under different circumstances.

In this study, we intend to use the objects of ten stocks. There are three-four groups of stocks in each group belonging to one sector and an instrument representing risk-free rate, 1-month annual Fed Funds rate by comparing the two models of the Markowitz model and the Single Index Model. Form the formulation of the problem above, my research paper aims to calculate the portfolio of the Markowitz Model and the portfolio of the Single Index Model. Then compare the portfolio of the two models to gain the optimal portfolio under certain conditions. In my research paper, the portfolio of the Markowitz Model is a little better than the portfolio of the Single Index model. This is because the Single Index model is related to the market, which causes some uncertainties.

2. RESEARCH METHODOLOGY

2.1 Markowitz Model

A portfolio is the placement of funds into a set of assets that provide optimal returns with acceptable risk to investors. Markowitz model seeks to form an efficient portfolio with the highest yield under a given risk level. In addition, the Markowitz model provides an optimization process for constructing the most effective target portfolio, which has been widely used in insurance portfolio management. Markowitz model minimizes the risk borne by investors to different stocks.

Figure 1 shows a brief introduction of the efficient frontier. The efficient frontier of the Markowitz Model is a hyperbola representing portfolios with all the different combinations of assets that result in efficient portfolios. The *X*-axis represents risks, and the *Y*-axis represents return. The area inside the efficient frontier (but not directly on the frontier) represents either individual assets or all their non-optimal combinations.

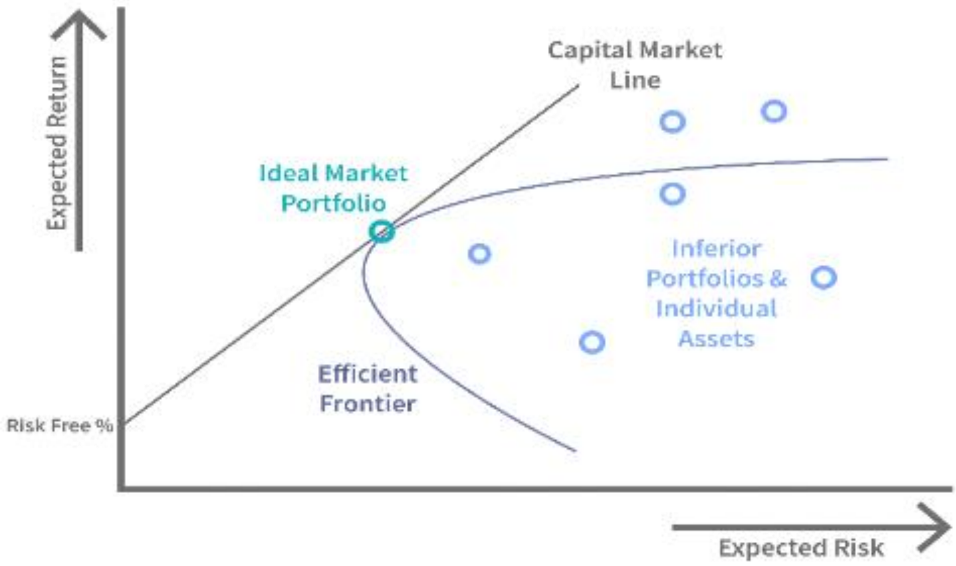

Figure 1. The efficient frontier

According to [1,3-5], we calculate the Markowitz model with the following formulations:

1. Calculate the level of return of each stock

   $R_{it} = \ln[\frac{P_t}{P_{(t-1)}}]$

   With,

   R_{it} : The rate of profit (return) stock to-i in product-t

   P_t : The closing price stock in the period-t

   $P_{(t-1)}$: The closing price of the previous stock in the period to (t-1)

2. Calculate the expected return rate of each stock return

   $E(Ri) = \frac{\sum_{t=1}^{n} R_{it}}{n}$

   With,

   E (Ri) : The level of expected return from the stock to-i

   R_{it} : The level of return of the stocks to-i in the period-t

   n : Number of observation

3. Calculate the risk (variance and standard deviation) of each stock return

   $$\sigma_i^2 = \frac{\sum_{t=1}^{n}(R_{it}-E(R_i))^2}{n-1}$$

   $$\sigma_i = \sqrt{\frac{\sum_{t=1}^{n}(R_{it}-E(R_i))^2}{n-1}}$$

   With,

   σ_i^2 : Variance of stock return to-i

   σ_i : Standard deviation of stock return to-i

   $E(R_i)$: The level of expected return from the stock to-i

   R_{it} : The level of return of the stock to-i in the period-t

   $n.$: Number of observations

4. Calculate combination between stocks

   $$C_{(r,N)} = \frac{N!}{r!\,(N-r)!}$$

   With,

   $C_{(r,N)}$: Combination of portfolio level from the number of stocks(N)

   N! : Factorial number of stocks

   r! : Factorial portfolio level is factorialized

   Determine the weight of the stock portfolio

   $$\sum_{i=1}^{N} W_i = 1$$

   With,

   W_i : The weight of the stock portfolio to-i

N : Number of observations
Calculate the expected return portfolio rate
$E(R_p)=\sum_{i=1}^{N} W_i \cdot E(R_i)$
With,
$E(R_p)$: The level of expected return of the stock portfolio
W_i : The weight of the stock portfolio to-i
ER_i : The level of expected return from the stock to-i
Calculate variances and standard deviations which are rick of stock portfolio

$$\sigma_p^2 = \sum_{i=1}^{n} W_i^2 \cdot \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \cdot \rho_{i,j}\sigma_i\sigma_j$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} W_i^2 \cdot \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \cdot \rho_{i,j}\sigma_i\sigma_j}$$

With,
σ_p^2 : Variance of stock portfolio
σ_p : Standard deviation of stock portfolio
σ_i^2 : Variance of stock return to-i
σ_i : Standard deviation of stock return to-i
σ_j : Standard deviation of stock return to-j

$\rho_{i,j}$: Correlation coefficient between stocks to-i and j
W_i : The weight of the funds invested in the stock to-i
W_j : The weight of the funds invested in the stock to-j

As for calculating $\rho_{i,j}$(correlation coefficient between stocks) can be calculated using the formula:

$$\rho_{i.j} = \frac{\frac{1}{n-1}\{\sum_{t=1}^{n}[\left(R_{it} - E(R_i)\right)\left(R_{jt} - E\left(R_j\right)\right)]\}}{\sqrt{\frac{\sum_{t=1}^{n}(R_{it} - E(R_i))^2}{n-1} \cdot \frac{\sum_{t=1}^{n}(R_{jt} - E\left(R_j\right))^2}{n-1}}}$$

With,
$\rho_{i.j}$: Correlation coefficient between stocks to-i and j
$E(R_i)$: The level of expected return from the stock to-i
$E\left(R_j\right)$: The level of expected return from the stock to-j
R_{it} : The level of return of the stock to-i in the period-t
R_{jt} : The level of return of the stock to-j in the period-t
$n.$: Number of observations

2.2 Single Index Model

The single index model assumes that the related movement between stocks is due to a single common influence or index. With the observation of stock prices, we can see that most stock prices are proportional to the stock market. This shows that one of the reasons for the possible correlation between securities returns is the common response to market changes. A useful index representing this correlation may be obtained by linking stock returns with stock returns.

According to [2, 6, 7], the formula of the Single Index Model to calculate the portfolio is shown below:

$$SP_i = \frac{E\left(R_p\right) - R_f}{\sigma_p}$$

With,

$$E\left(R_p\right) = \sum_{i=1}^{N} W_i \cdot E(R_i)$$

$$R_f = E(S) = \frac{\sum_{t=1}^{n} S_t}{n}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} W_i^2 \cdot \sigma_i^2 + 2\sum_{j=1}^{n} W_i W_j \cdot \rho_{i,j}\sigma_i\sigma_j}$$

SP_i : Sharpe portfolio index to-i
$E\left(R_p\right)$: The level of expected return of the stock portfolio
$R_f = E(S)$: The average risk-free investment interest rate return
$E(R_i)$: The level of expected return from the stock to-i

2.3 Capital Asset Line

The capital asset line (*CAL*) represents the line with an intercept point equal to the risk-free rate that is tangent to the efficient frontier of risky assets; represents the efficient frontier when a risk-free asset is available for investment. Under *CAL*, all investors will choose a position on the capital market line, in equilibrium, by borrowing or lending at the risk-free rate since this maximizes return for a given level of risk [8]. In this research paper, *CAL* is the criterion for judging the optimal portfolio of the two models.

3. RESULT AND DISCUSSION

3.1. The description of original data

The data used in this paper is the historical daily total return data for ten stocks, which belong in groups to three-four different sectors (according to Yahoo! Finance), one (S&P 500) equity index (a total of eleven risky assets) and a proxy for risk-free rate (1-month Fed Funds rate).

Table 1 represents the daily historical data of the ten stocks. From the table, we can conclude that the returns of CVX are larger than other companies' returns. The returns of IMO are the least of the ten companies. The trend of the returns of the ten companies is a positive increase in the twenty years.

Table 1. The daily historical data of the ten companies

Date	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
2001/5/11	1245.67	28.97	9.34	15.90	34.7	47.01	43.82	9.14	22.64	46.17	27.74
2001/5/31	1256.93	30.37	10.33	15.3.0	34.59	48.35	44.37	9.00	23.70	44.76	30.28
2001/6/29	1226.33	29.24	9.17	19.00	36.50	45.55	43.68	8.62	22.58	44.34	27.06
2001/7/31	1214.25	31.61	7.75	18.08	33.09	46.00	41.76	9.38	22.38	46.78	29.14
...	...	...	...	...	...	...	...	...	...	...	...
2021/5/12	6031.01	181.31	111.30	90.00	375.46	223.14	111.48	41.58	94.67	237.80	382.10

3.2 The Analysis of the Two Constraints of the Markowitz Model and the Index Model

The first constraint illustrates the area of permissible portfolios and where the efficient risky and minimal risk portfolios are located if you have no additional constraints. The second constraint is just a no-index problem, which illustrates the effect of a broad equity index as one of the risky instruments has.

3.2.1 The Characteristic of the Optimal Portfolio of the Markowitz Model

From Figure 2, we could conclude that under the same risk condition, the return under constraint 2, which w1=0, is higher than constraint 1, which means no constraint. And the gray point of the efficient risky portfolio is just the maximal sharp ratio, and the yellow point is just the minimal risk portfolio of the Markowitz Model.

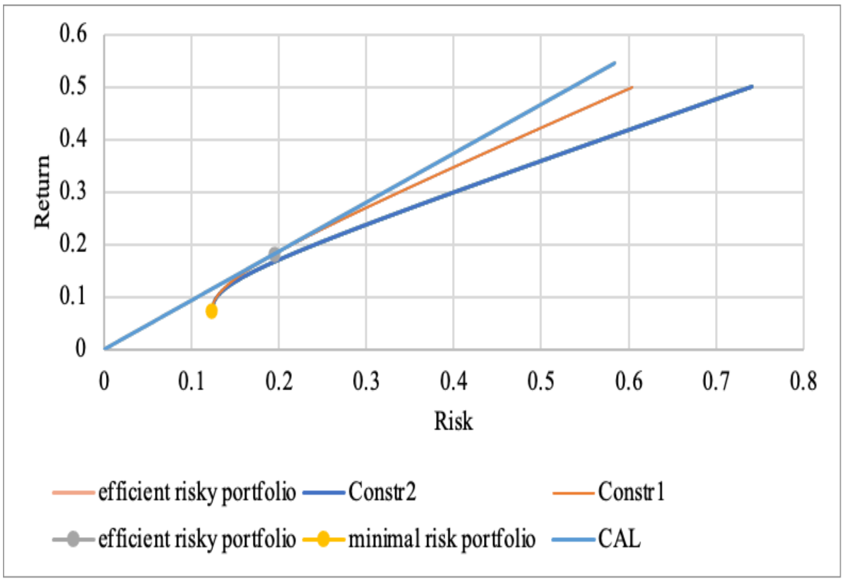

Figure 2. Constraints Analysis for Markowitz Model

3.2.2 The Characteristic of the Optimal Portfolio of the Index Model

From Figure 3, we could conclude that under the same risk condition, the return under constraint 2, which w1=0, is higher than constraint 1, which means no constraint. And the gray point of the efficient risky portfolio is just the maximal sharp ratio, and the yellow point is just the minimal risk portfolio of the Index Model. The portfolio of

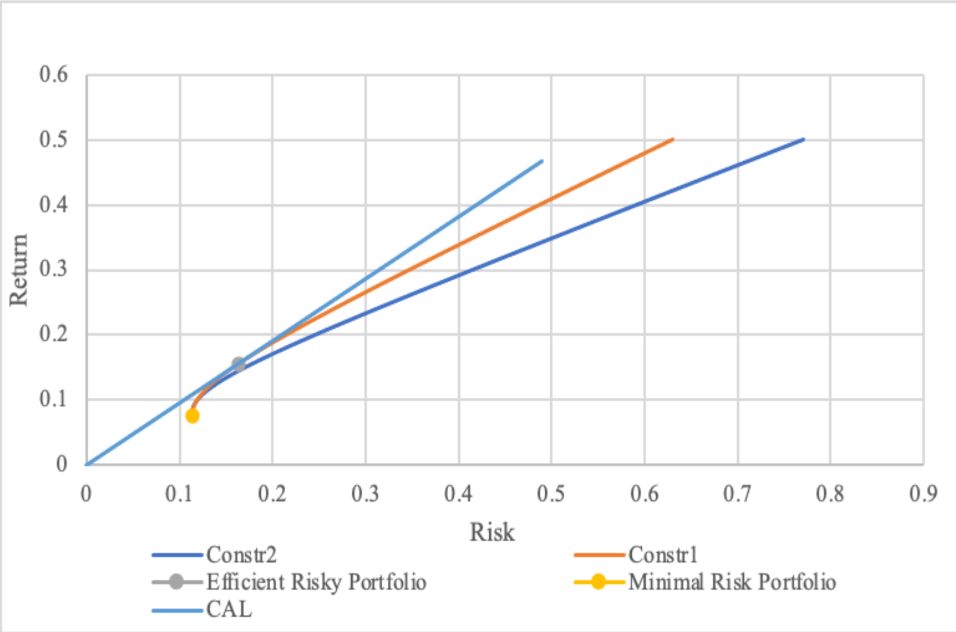

Figure 3. Constraints Analysis for Index Model

3.3 The Comparison of the Markowitz Model and the Index Model.

From Figure 4, we could conclude that with constraint 1, which indicates no constraint, the portfolio of the Markowitz Model could gain more return than the same constraint condition of the Index Model. With constraint 2, which indicates the w1=0, the portfolio of the Markowitz Model could also gain more return than the same condition of the single-index model. And under the same risk condition, the portfolio with no constraint gains more return than the one under the constraint.

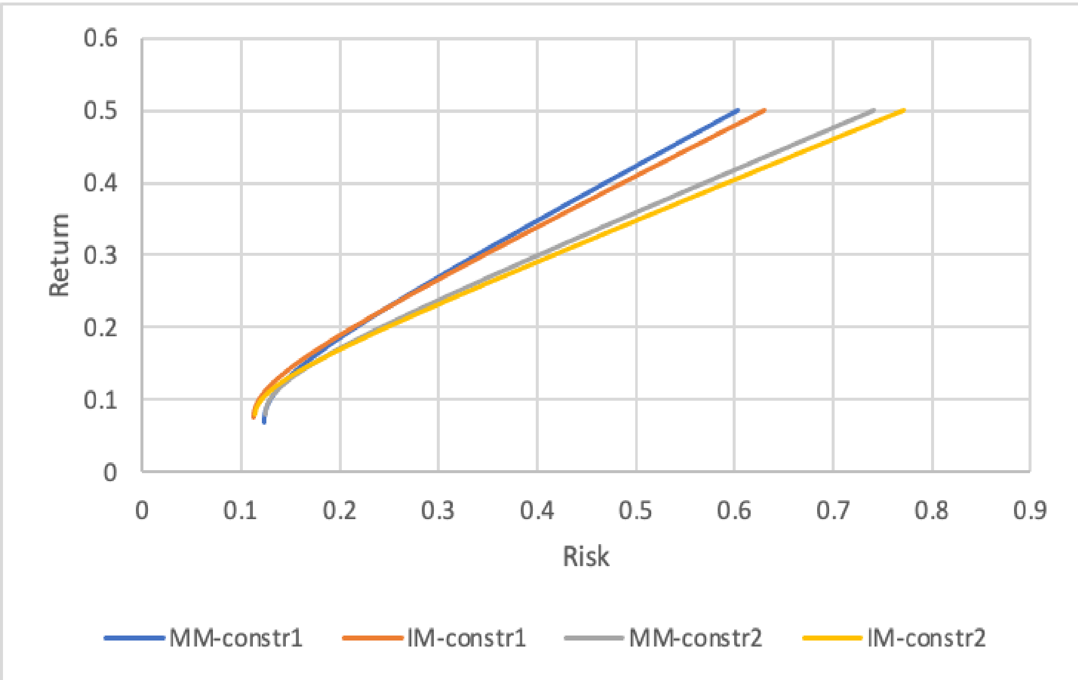

Figure 4. The comparison of the Markowitz Model and the Index Model

4. CONCLUSION

From the analysis of the Markowitz Model and the Index Model, we could find that the optimal portfolio of the Markowitz model is better than the optimal portfolio of the Index Model. Theoretically, the Index Model simplifies the estimation of the covariance matrix problem. Furthermore, it explicitly decomposes the risk into systematic and firm-specific components, which indicates that the optimal portfolio of the Index Model under the same risk condition is better than the optimal risky portfolio of the Markowitz Model. However, the

Index Model also relates to the many marketing factors, which means the results of the optimal risky portfolio of the Index Model may have many uncertainties. In this case, the research of my paper may show that the optimal portfolio of the Markowitz Model is a little better than that of the Single Index Model.

REFERENCES

[1] Markowitz H. Portfolio selection. The Journal of Finance, 1952 7 (1) 77-91.

[2] Sharpe, William F. A Simplified Model for Portfolio Analysis. Management Science Journal, 1963 9, 2.

[3] Tandelilin, Eduardus. "Analisis investasi dan manajemen portofolio." (2001).

[4] Situmorang, R. E., D. A. I. Maruddani, and R. Santoso. "Formation of the stock portfolio using Markowitz method and measurement of Value at Risk based on generalized extreme value (Case study: company's stock The IDX Top Ten Blue 2017, Period 2 January-29 December 2017)." Journal of Physics: Conference Series, 2019 Vol. 1217. No. 1. IOP Publishing.

[5] Chin L, Chendra E, Sukmana A. Analysis of portfolio optimization with lot of stocks amount constraint: case study index LQ45. InIOP Conference Series: Materials Science and Engineering, 2018 (Vol. 300, No. 1, p. 012004). IOP Publishing.

[6] Sen, Kapil, and C. D. Fattawat. "Sharpe's Single Index Model and its Application Portfolio Construction: An Empirical Study." Global Journal of Finance and Management, 2014 6.6 pp: 511-516.

[7] Andrade, Pratiba Jenifer. "Construction of Optimal Portfolio of Equity, using Sharpe's Single Index Model: A Case Study of IT Sector', International Journal of Applied Financial Management Perspectives, 2012 1(2), pp: 86-88.

[8] Blume, Marshall E., and Irwin Friend. "A new look at the capital asset pricing model." The Journal of Finance, 1973 28.1, pp: 19-33.