International Journal of Computational Intelligence Systems
Vol. **14(1)**, 2021, *pp.* 635–650
DOI: https://doi.org/10.2991/ijcis.d.210112.001; ISSN: 1875-6891; eISSN: 1875-6883
https://www.atlantis-press.com/journals/ijcis/

Research Article

Teaching Performance Evaluation Based on the Proportional Hesitant Fuzzy Linguistic Prioritized Choquet Aggregation Operator

Lei Wang[1], Lili Rong[2,3,*], Fei Teng[2], Peide Liu[2]

[1] *School of International Education, Shandong University of Finance and Economics, Jinan Shandong, 250014, China*
[2] *School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan Shandong, 250014, China*
[3] *School of Science and Technology, Shandong TV University, Jinan Shandong, 250014, China*

ARTICLE INFO

Article History

Received 11 Aug 2020
Accepted 29 Dec 2020

Keywords

Choquet integral
Proportional hesitant fuzzy linguistic term set
Multi-attribute group decision-making
Prioritized aggregation operators
Teaching performance evaluation

ABSTRACT

The quality of teaching can be improved by teaching performance evaluation from multiple experts, which is a multiple attribute group decision-making (MAGDM) problem. In this paper, a group decision-making method under proportional hesitant fuzzy linguistic environment is proposed to evaluate teaching performance. Firstly, proportional hesitant fuzzy linguistic term set (PHFLTS) is applied to express the decision makers' (DMs) preferences for teaching performance index. Secondly, the $PHFLP_rCA$ operator is developed and its properties are discussed. Then based on the $PHFLP_rCA$ operator, a MAGDM method is formulated. Thirdly, the method is applied in teaching performance evaluation of Chinese-foreign cooperative education project. Finally, this method is proved more scientific, objective and accurate by compared with other two methods.

1. INTRODUCTION

Chinese-foreign cooperative education project is the main form of transnational education in China. With the continuous expansion of its scale and the gradual upgrade of project-running, its sustainable development has become an inevitable requirement, which depends on the improvement of teaching quality. In the process of quality improving, teachers play an important role, such as curriculum designers and implementers. Teachers' qualifications, teaching methods, and teaching ability have become the factors that affect the teaching quality. In order to improve the teaching quality, it is necessary to conduct the teaching performance evaluation of Chinese-foreign cooperative education project.

The teaching performance evaluation of Chinese-foreign cooperative education project can be performed by a group of decision makers (DMs) based on multiple attributes, which may be inaccuracy, ambiguity, and uncertainty. Therefore, the teaching performance evaluation of Chinese-foreign cooperative education project can be considered as a fuzzy multiple attribute group decision-making (MAGDM) problem. It plays a vital role in determining the quality of teaching performance evaluation to apply scientific methods comprehensively and effectively. In the aspect of teaching performance evaluation, some researches on this topic have been done by using MAGDM methods [1–6]. However, there are two drawbacks in the current researches. Firstly, most of the relevant studies are hard to deal with group information. Secondly, these studies assumed that the indicators were independent of each other, without considering the priority or correlation between the indicators. However, in the process of practical evaluation, there will be mutual relationship between the indicators.

In order to overcome these two drawbacks of current researches, a better method should be proposed to solve them. In the first place, the appropriate linguistic expression is chosen for teaching performance evaluation. On the one hand, appropriate linguistic expression should enable DMs to express their preferences as clearly as possible, and reduce the subjectivity and uncertainty in the decision-making process; on the other hand, the rational expression of linguistic information is the premise and basis for solving MAGDM problems effectively [7–11]. Based on the above two aspects, this paper adopts the proportional hesitant fuzzy linguistic term sets (PHFLTSs) [12–15]. PHFLTSs are developed from traditional linguistic information, which are used to reasonably describe the subjective preference information given by DMs. Experts put forward several linguistic information representation models based on different situations, such as linguistic variables [16–18], hesitant fuzzy linguistic variables [19,20], extended hesitant fuzzy linguistic variables [21,22], linguistic distribution assessment variables [23,24], etc. By reducing the implicit constraints of the

*Corresponding author. Email: ronglili@126.com

above linguistic information representation models, a more general linguistic information representation model, i.e., proportional hesitant fuzzy linguistic elements (PHFLEs), can be proposed. PHFLEs are general forms of the above linguistic variables, which are more conducive for DMs freely expressing their subjective preferences, and more suitable for teaching performance evaluation of Chinese-foreign cooperative education project.

In the second place, after choosing the appropriate linguistic expression, another key problem is to select the information aggregation operator, that is, to find the appropriate tool for effective integration of teaching performance evaluation values. Choosing the suitable information aggregation tool is a crucial step. At present, information aggregation operators have drawn extensive attention and achieved fruitful results, especially several kinds of aggregation operators, such as Choquet integration operator [25], and prioritized integration operator [26,27]. However, the research on information aggregation operators based on PHFLEs is rare, and fails to be applied directly in teaching performance evaluation with priority and correlation.

Since Yager [26] proposed the prioritized aggregation (PrA) operator, it has been extensively improved [28–30], but the existing PrA operators cannot deal with the PHFLEs. Therefore, this paper enriches the PrA operators and proposes the proportional hesitant linguistic priority weighted average ($PHFLP_rWA_m$) operator.

Besides prioritized relation among elements, there are interdependence or interrelated features among elements, thus it is unreasonable to aggregate elements by additive measures. Sugeno [31] put forward the concept of non-additive measure (fuzzy measure), which just has monotonicity but not additive property. Choquet integral (CI) based on fuzzy measure can capture the interaction between different elements, but it hasn't been applied into the proportional hesitant linguistic information. Therefore, it is important to propose an innovative CI operator to integrate the PHFLEs and deal with the MAGDM problems in the proportional hesitant linguistic environment.

Three innovative points in this paper are:

1. The first one is to enable DMs to express their preferences more freely and accurately. The proportional hesitant linguistic information can show the advantage of a group of DMs who have their own preferences when they make decisions.
2. The second one is to make the PHFLEs be applied in the situation with priority and correlation. The phenomenon of attributes with priority and correlation in MAGDM problems is common in real life, but it has never been extended to the PHFLEs.
3. The third one is to provide a new solution to the teaching performance evaluation of Chinese-foreign cooperative education project.

The remaining sections of this paper are outlined as follows: Section 2 introduces PHFLEs and its related concepts; Section 3 proposes proportional hesitant fuzzy linguistic Choquet aggregation $\left(PHFLCA_\mu\right)$ operator, $PHFLP_rWA_m$ operator, and $PHFLP_rCA$ operator; Section 4 establishes a model based on $PHFLP_rCA$ operator method; Section 5 demonstrates a practical example about teaching performance evaluation of Chinese-foreign cooperative education project, then shows the advantages of the proposed method by comparing with other two methods. Finally, Section 6 comes to the overall conclusion.

2. PRELIMINARIES

This section introduces the concept of PHFLEs, which can be transformed into different linguistic information representation models according to the decrease of proportional constraints, and some related theories such as expectation function, deviation function, and their corresponding ranking methods are applied. In addition, the CI and the prioritized integration operators are reviewed.

2.1. PHFLEs and its Related Concepts

Definition 1. [32] Let $L = \{l_i | i = 1, 2, ..., t\}$ be a fully ordered finite discrete set, which l_i represents a linguistic term. If L meets the following conditions, L is called a linguistic term set (LTS): (1) The set L is ordered: if $i \geq \kappa$, then $l_i \geq l_\kappa$; (2) there is a negative operator: $Neg\left(l_i\right) = t - i$.

For example, when t = 5, a linguistic glossary is presented as follows:

$$L = \left\{l_1 : very\,low, l_2 : low, l_3 : \text{moderate}\; l_4 : high, l_5 : very\,high\right\}.$$

Definition 2. [33] Suppose X is a given domain $\forall x_j \in X$, and $L = \{l_i | i = 1, 2, \dots, t\}$ be a LTS, then $H_L = \left\{\left\langle x_j, h_l\left(x_j\right)\right\rangle | x_j \in X\right\}$ is called hesitant fuzzy LTS (HFLTS), $h_l\left(x_j\right) = \left\{l_{\varphi_\tau}\left(x_j\right) | l_{\varphi_\tau}\left(x_j\right) \in L, \tau = 1, 2, ..., \#\right\}$ is a set of elements of L, # is the number of linguistic terms in $h_l\left(x_j\right)$. For convenience, $h_l\left(x_j\right)$ is called hesitant fuzzy linguistic element (HFLE) and H_L is a set of HFLEs.

Definition 3. [34] Let X be a given domain and $\forall x_j \in X$, L be a given LTS and $\forall l_i \in L$, then $H_L\left(p\right) = \left\{\left\langle x_j, h_L^j\left(p\right)\right\rangle | x_j \in X\right\}$ of X is called probabilistic LTS (PLTS), where $h_L^j\left(p\right) = \left\{l^{j(\tau)}\left(p^{(\tau)}\right) | l^{j(\tau)} \in L, p^{(\tau)} \geq 0, \tau = 1, 2, ..., \#, \sum_{\tau=1}^{\#} p^{(\tau)} \leq 1\right\}$, $l^{j(\tau)}\left(p^{(\tau)}\right)$ is the τth linguistic term $l^{j(\tau)}$ and its probability distribution $p^{(\tau)}$, # is the number of different linguistic terms in $h_L^j\left(p\right)$. Linguistic terms $l^{j(\tau)}\left(\tau = 1, 2, ..., \#\right)$ are arranged in ascending order. For convenience, $h_L^j\left(p\right)$ is called the probabilistic linguistic element (PLE).

Definition 4. [24] Let X be a given domain and $\forall x_j \in X$, L be a given LTS and $\forall l_i \geq L$, then $\ddot{H}_L\left(p\right) = \left\{\left\langle x_j, \ddot{h}_L^j\left(p\right)\right\rangle | x_j \in X\right\}$ of X is called a distributed LTS, where $\ddot{h}_L^j\left(p\right) = \left\{l_i\left(p_i^j\right) | l_i \in L, p_i^j \geq 0, i = 1, 2, ..., t, \sum_{i=1}^{t} p_i^j = 1\right\}$ is called a distributed linguistic element (DLE).

Definition 5. [12] Let X be a given domain and $\forall x_j \in X$, L is a given LTS and $\forall l_i \in L$, then $\dot{\ddot{H}}_{PH}\left(\dot{p}\right) = \left\{\left\langle x_j, \dot{\ddot{h}}_{PH}^j\left(\dot{p}\right)\right\rangle | x_j \in X\right\}$ of X is called PHFLTS, where $\dot{\ddot{h}}_{PH}^j\left(\dot{p}\right) =$

$\left\{\left(l_i, \dot{p}_i^j\right) | l_i \in L, 0 \le \dot{p}_i^j \le 1, i = 1, 2, ..., t\right\}, \sum_{i=1}^{t} \dot{p}_i^j = 1, \left(l_i, \dot{p}_i^j\right)$ is LTS l_i and $\dot{p}_i^j$ denotes the degree of possibility that the alternative carries an assessment value l_i provided by a group of DM. For convenience, $\dot{\tilde{h}}_{PH}^j(\dot{p})$ is called PHFLE.

Definition 6. Let $\dot{\tilde{h}}_{PH}^j(\dot{p}) = \left\{\left(l_i, \dot{p}_i^j\right) | l_i \in L, i = 1, 2, ..., t\right\}$ be a PHFLE, then the expectation function of $\dot{\tilde{h}}_{PH}^j(\dot{p})$ is

$$E\left(\dot{\tilde{h}}_{PH}^j(\dot{p})\right) = \sum_{i=1}^{t} i \cdot \dot{p}_i^j \tag{1}$$

Definition 7. Let $\dot{\tilde{h}}_{PH}^j(\dot{p}) = \left\{\left(l_i, \dot{p}_i^j\right) | l_i \in L, i = 1, 2, ..., t\right\}$ be a PHFLE, $E\left(\dot{\tilde{h}}_{PH}^j(\dot{p})\right) = \sum_{i=1}^{t} i \cdot \dot{p}_i^j$, then the deviation function of $\dot{\tilde{h}}_{PH}^j(\dot{p})$ is

$$\sigma\left(\dot{\tilde{h}}_{PH}^j(\dot{p})\right) = \left(\sum_{i=1}^{t}\left(\dot{p}_i^j\left(i - E\left(\dot{\tilde{h}}_{PH}^j(\dot{p})\right)\right)\right)^2\right)^{1/2} \tag{2}$$

Definition 8. Let $\dot{\tilde{h}}_{PH}^1(\dot{p})$ and $\dot{\tilde{h}}_{PH}^2(\dot{p})$ be any two PHFLEs, then there are

1. if $E\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) > E\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$, then $\dot{\tilde{h}}_{PH}^1(\dot{p}) > \dot{\tilde{h}}_{PH}^2(\dot{p})$;
2. if $E\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) < E\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$, then $\dot{\tilde{h}}_{PH}^1(\dot{p}) < \dot{\tilde{h}}_{PH}^2(\dot{p})$;
3. if $E\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) = E\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$,
    1. if $\sigma\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) < \sigma\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$, then $\dot{\tilde{h}}_{PH}^1(\dot{p}) > \dot{\tilde{h}}_{PH}^2(\dot{p})$;
    2. if $\sigma\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) > \sigma\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$, then $\dot{\tilde{h}}_{PH}^1(\dot{p}) < \dot{\tilde{h}}_{PH}^2(\dot{p})$;
    3. if $\sigma\left(\dot{\tilde{h}}_{PH}^1(\dot{p})\right) = \sigma\left(\dot{\tilde{h}}_{PH}^2(\dot{p})\right)$, then $\dot{\tilde{h}}_{L}^1(\dot{p}) = \dot{\tilde{h}}_{L}^2(\dot{p})$.

2.2. CI Operator and PrA Operator

2.2.1. Fuzzy measure and CI

Definition 9. [31] Let $P(X)$ be the power set of $X = \{x_1, x_2, ..., x_n\}$, and the fuzzy measure $\mu : P(X) \to [0, 1]$ of X satisfies the following conditions:

1. $\mu(\varnothing) = 0, \mu(X) = 1$;
2. if $\forall A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \le \mu(B)$.

Fuzzy measure can be regarded as monotone set function, the fuzzy measure on X has the following characteristics:

1. Additivity: $\mu(A \cup B) = \mu(A) + \mu(B)$.
2. Sub-additivity: $\forall A, B \in P(X), \mu(A \cup B) \le \mu(A) + \mu(B)$.
3. Super-additivity: $\forall A, B \in P(X), \mu(A \cup B) \ge \mu(A) + \mu(B)$.

In MAGDM, $\mu(A)$ can be regarded as the importance of attribute subset $A \in P(X)$. The monotony of fuzzy measure means that when new attributes are added to attribute subset, the importance of attribute subset will not decrease [35]. Nonadditivity is the main feature of fuzzy measure, which can more flexibly express the relationships between decision attributes from redundancy (negative interaction) to complementarity (positive interaction) [25,36].

Since the fuzzy measure is a function defined on the power set, it is necessary to determine $2^n - 2$ parameters for calculating the fuzzy measure of n attribute, and the calculation amount for solving the fuzzy measure is large. Facing the large-scale calculation, Sugeno [31] proposed λ fuzzy measure to replace the general fuzzy measures, which simplified the computational complexity of fuzzy measures.

Definition 10. [31] Let $P(X)$ be the power set of $X = \{x_1, x_2, ..., x_n\}$, $\forall A, B \in P(X), A \cap B = \varnothing$, if the fuzzy measure g on X satisfies the following conditions:

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B),\ \lambda \in (-1, \infty)$$

then g is called λ fuzzy measure.

For $\forall A, B \in P(X)$, $A \cap B = \phi$: if $\lambda = 0$, $g(A \cup B) = g(A) + g(B)$, attribute subsets A and B are independent; if $-1 < \lambda < 0, g(A \cup B) < g(A) + g(B)$, attribute subset A and B are redundant; if $0 < \lambda < 1$, $g(A \cup B) > g(A) + g(B)$, attribute subsets A and B are complementary. In attribute-related multi-attribute decision-making, the role of attribute subset $D \in P(X)$ in decision-making process is determined not only by $g(D)$ itself, but also by other attribute subsets. If $g(D) = 0$, then attribute subset D is irrelevant. For attribute subset $H \in P(X), g(H \cup D) - g(H) > 0$ indicates that attribute subset H is important.

According to the definition of λ fuzzy measure g, $\forall x_j \in X, j, k = 1, 2, ..., n, j \ne k, x_j \cap x_k = \varnothing, \bigcup_{j=1}^{n} x_j = X$, then λ fuzzy measure is shown as follows:

$$g(X) = g\left(\bigcup_{j=1}^{n} x_j\right) = \begin{cases} \dfrac{1}{\lambda}\left(\prod_{j=1}^{n}\left(1 + \lambda g(x_i)\right) - 1\right), & \lambda \ne 0 \\ \sum_{j=1}^{n} g(x_i), & \lambda = 0 \end{cases} \tag{3}$$

Since $g(X) = 1$, when $\lambda \ne 0$, the value of λ is determined according to the following formula:

$$\lambda + 1 = \prod_{j=1}^{n}\left(1 + \lambda g(x_j)\right) \tag{4}$$

Definition 11. [25] Let $X = \{x_1, x_2, ..., x_n\}$ be a nonempty set, f be a nonnegative real value function defined on X, g be a fuzzy measure defined on X, and the CI of function f on g is defined as follows:

$$\int f dg = \sum_{j=1}^{n}\left[f\left(x_{(j)}\right) - f\left(x_{(j-1)}\right)\right] \cdot g\left(X_{(j)}\right) \tag{5}$$

It can also be expressed as

$$\int f dg = \sum_{j=1}^{n} \left[g\left(X_{(j)}\right) - g\left(X_{(j+1)}\right) \right] \cdot f\left(x_{(j)}\right) \tag{6}$$

where $f\left(x_{(1)}\right) \le f\left(x_{(2)}\right) \le \ldots \le f\left(x_{(n)}\right), f\left(x_{(0)}\right) = 0$, $X_{(j)} = \left\{x_{(j)}, x_{(j+1)}, \ldots, x_{(n)}\right\}$ and $x_{(n+1)} = \varnothing$.

Aggregation characteristics of CI are idempotency, compensation, monotone additivity, etc. In addition, CI is an extension of weighted average and orderly weighted average. As long as the fuzzy measure is additive, the CI degenerates into a weighted average or an ordered weighted average.

2.2.2. PrA operator

Definition 12. [26] Let $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes, then there exists a linear ordered prioritized relationship between attributes, which can be expressed as $C_1 \succ C_2 \succ \cdots \succ C_n$, that is C_j priority ranks are higher than C_k, $\forall j < k$. $C_j(x)$ is the evaluation value of alternative x under attribute C_j, which satisfies $C_j(x) \in [0, 1]$. If

$$P_r A\left(C_j(x)\right) = \sum_{j=1}^{n} w_j C_j(x) \tag{7}$$

where $w_j = T_j / \sum_{j=1}^{n} T_j$, $T_j = \prod_{k=1}^{j-1} C_k(x) \left(j = 2, 3, \ldots, n\right)$, $T_1 = 1$ then P_rA is called the PrA operator.

The PrA operator [26] based on priority measure cannot solve all of priority decision problem. To be specific, for two alternatives, this problem cannot be solved when the satisfaction of their highest priority attribute is the same and the satisfaction of these attributes is the smallest of all priority attributes. The fundamental reason is that the measurement of priority is too strict to compensate for any priority attributes. Based on this, Chen *et al.* [37] proposed a generalized prioritized operator to optimize the prioritized operator [9].

Definition 13. [37] Let $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes, then there exists a linear ordered priority relationship between attributes, which can be expressed as $C_1 \succ C_2 \succ \ldots \succ C_n$. Let $A = \left\{C_{t_1}, C_{t_2}, \ldots, C_{t_l}\right\}$ be a subset of attribute set C, by reordering the elements in subset A, $A = \left\{C_{\sigma(1)}, C_{\sigma(2)}, \ldots, C_{\sigma(l)}\right\}$ can be obtained, of which $\{\sigma(1), \sigma(2), \cdots, \sigma(l)\}$ is an arbitrary sequence of $\{t_1, t_2, \ldots, t_l\}$, satisfying $\sigma(1) < \sigma(2) < \cdots < \sigma(l)$. $c_i(x) \in [0, 1]$ $(i = 1, 2, \ldots, n)$ is the satisfaction of alternative x under attribute C_i. A numerical value of b_i is given for each priority attribute C_i in attribute set C, which satisfies $b_i \in [0, 1]$ and $\sum_{i=1}^{n} b_i = 1$, $b_1 \ge b_2 \ge \ldots \ge b_n$. The generalized prioritized measure $m : P(C) \to [0, 1]$ is defined as follows:

$$m(A) = \sum_{j=1}^{l} b_{\sigma(j)} f_j\left(\sigma(j)\right) \text{ and } m(\varnothing) = 0 \tag{8}$$

where, $P(C)$ is the power set of attribute set C, $f(\cdot)$ is a monotone decreasing function which satisfies $f_j(j) = 1, f_j(n) = 0$ $(j = 1, 2, \ldots, n)$.

If $f_j\left(\sigma(j)\right) = \frac{n+1-\sigma(j)}{n+1-j}$ $(j = 1, 2, \ldots, n)$, then the new prioritized measures are expressed as follows:

$$m(A) = \sum_{j=1}^{l} b_{\sigma(j)} \frac{n+1-\sigma(j)}{n+1-j} \tag{9}$$

Based on this, Chen et al. [37] proposed a generalized PrA operator, i.e.

$$c(x) = Ch_m\left(c_1(x), c_2(x), \ldots, c_n(x)\right) = \sum_{i=1}^{n} c_{(i)}(x) \left(m\left(h_{(i)}\right) - m\left(h_{(i-1)}\right)\right)$$

where $c_{(i)}(x)$ is the *ith* largest value in $\left\{c_{(1)}(x), c_{(2)}(x), \ldots, c_{(n)}(x)\right\}$, $h_{(i)} = \left\{C_{(1)}, C_{(2)}, \ldots, C_{(i)}\right\}$ and $h_{(0)} = \varnothing$, $m\left(h_{(i)}\right)$ $(i = 0, 1, \ldots, n)$ is generalized prioritized measures based on $h_{(i)}$.

In this section, the basic theory of MAGDM based on PHFLTS is studied. Firstly, the related linguistic information representation models are briefly reviewed. In order to overcome the shortcoming of several kinds of linguistic information representation models, the PHFLTS is proposed, and related theories, such as expectation function, deviation function, and ranking method are put forward. Secondly, two kinds of typical functional information integration operators, namely *CI* integration operator and *PrA* operator, are reviewed.

3. PROPORTIONAL HESITANT FUZZY LINGUISTIC PRIORITIZED CHOQUET AGGREGATION OPERATOR

This section focuses on the aggregation of PHFLEs, and proposes the *PHFLP$_r$CA* operator to deal with the MAGDM problems with both priority and correlation.

3.1. *PHFLCA$_\mu$* Operator Based on Fuzzy Measure

Definition 14. Let $\dot{\tilde{H}}_{PH}(\dot{p}) = \left\{\dot{\tilde{h}}^1_{PH}(\dot{p}), \dot{\tilde{h}}^2_{PH}(\dot{p}), \ldots, \dot{\tilde{h}}^n_{PH}(\dot{p})\right\}$ be the PHFLTS, where $\dot{\tilde{h}}^j_{PH}(\dot{p}) = \left\{\left(l_i, \dot{p}^j_i\right) | l_i \in L, \sum_{i=1}^{t} \dot{p}^j_i = 1, i = 1, 2, \ldots, t\right\}$, and μ is fuzzy measure based on $\dot{\tilde{H}}_{PH}(\dot{p})$. Then the *PHFLCA$_\mu$* operator is defined as follows:

$$\begin{aligned} &PHFLCA_\mu\left(\dot{\tilde{h}}^1_{PH}(\dot{p}), \dot{\tilde{h}}^2_{PH}(\dot{p}), \ldots, \dot{\tilde{h}}^n_{PH}(\dot{p})\right) \\ &= \bigoplus_{j=1}^{n} \left(\mu\left(\dot{\tilde{H}}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{\tilde{H}}^{(j+1)}_{PH}(\dot{p})\right)\right) \dot{\tilde{h}}^{(j)}_{PH}(\dot{p}) \end{aligned} \tag{10}$$

where $\dot{\tilde{h}}^{(1)}_{PH}(\dot{p}) \le \dot{\tilde{h}}^{(2)}_{PH}(\dot{p}) \le \ldots \le \dot{\tilde{h}}^{(n)}_{PH}(\dot{p})$ and $\dot{\tilde{H}}^{(j)}_{PH}(\dot{p}) = \left\{\dot{\tilde{h}}^{(j)}_{PH}(\dot{p}), \dot{\tilde{h}}^{(j+1)}_{PH}(\dot{p}), \ldots, \dot{\tilde{h}}^{(n)}_{PH}(\dot{p})\right\}$, $j = 1, 2, \ldots, n$, $\dot{\tilde{H}}^{(n+1)}_{PH}(\dot{p}) = \varnothing$.

Theorem 1. *Let $\dot{\tilde{H}}_{PH}(\dot{p}) = \left\{\dot{\tilde{h}}^1_{PH}(\dot{p}), \dot{\tilde{h}}^2_{PH}(\dot{p}), \ldots, \dot{\tilde{h}}^n_{PH}(\dot{p})\right\}$ be the PHFLTS, where $\dot{\tilde{h}}^j_{PH}(\dot{p}) = \left\{\left(l_i, \dot{p}^j_i\right) | l_i \in L, \sum_{i=1}^{t} \dot{p}^j_i = 1,\right.$*

$i = 1, 2, ..., t\}$, μ *be fuzzy measure based on* $\dot{H}_{PH}(\dot{p})$, *then the result of* $PHFLCA_\mu$ *operator is as follows:*

$$PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) = \left\{\left(l_i, \sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{p}^{(j)}_i\right) | l_i \in L, i = 1, 2, ..., t\right\} \tag{11}$$

Next, the properties of $PHFLCA_\mu$ operator are idempotency, permutation invariance, monotonicity, and boundedness.

Property 1. (Idempotency) Let $\dot{H}_{PH}(\dot{p}) = \{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\}$ be the PHFLTS, where $\dot{h}^j_{PH}(\dot{p}) = \dot{h}_{PH}(\dot{p}) = \left\{(l_i, \dot{p}_i) | l_i \in L, \sum_{i=1}^{t}\dot{p}_i = 1, i = 1, 2, ..., t\right\}$, $j = 1, 2, ..., n$, then

$$PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) = \dot{h}_{PH}(p)$$

Proof: Because of $\dot{h}^j_{PH}(\dot{p}) = \dot{h}_{PH}(\dot{p})$, $\mu(\dot{H}^{(j)}_{PH}(\dot{p})) - \mu(\dot{H}^{(j+1)}_{PH}(\dot{p})) \geq 0$ and $\sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right) = 1$, it is available that

$$PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) = \left\{\left(l_i, \sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{p}_i\right) | l_i \in L\right\} = \dot{h}_{PH}(\dot{p})$$

Property 2. (Commutativity) Let $\left\{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right\}$ be any PHFLTS, and $\left\{\dot{h}^1_{PH}(\dot{p})', \dot{h}^2_{PH}(\dot{p})', ..., \dot{h}^n_{PH}(\dot{p})'\right\}$ is any sequence of $\left\{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right\}$, then

$$PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) = PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p})', \dot{h}^2_{PH}(\dot{p})', ..., \dot{h}^n_{PH}(\dot{p})'\right)$$

Proof: Since $\left\{\dot{h}^1_{PH}(\dot{p})', \dot{h}^2_{PH}(\dot{p})', ..., \dot{h}^n_{PH}(\dot{p})'\right\}$ is an arbitrary sequence of $\left\{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right\}$, CI operator can be regarded as an extended ordered weighted average operator. Based on the above formula, we can find that $PHLCA_\mu$ operators have permutation invariance.

Property 3. (Monotonicity) Let $\left\{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right\}$ and $\left\{\dot{h}^1_{PH}(\dot{q}), \dot{h}^2_{PH}(\dot{q}), ..., \dot{h}^n_{PH}(\dot{q})\right\}$ be any two PHFLTSs, of which $\dot{h}^j_{PH}(\dot{p}) = \left\{\left(l_i, \dot{p}^j_i\right) | l_i \in L, \sum_{i=1}^{t}\dot{p}^j_i = 1, i = 1, 2, ..., t\right\}$, $\dot{h}^j_{PH}(\dot{q}) = \{(l_i, \dot{q}^j_i) | l_i \in L, \sum_{i=1}^{t}\dot{q}^j_i = 1, i = 1, 2, ..., t\}$ and $E(\dot{h}^{(1)}_{PH}(\dot{p})) \leq E(\dot{h}^{(2)}_{PH}(\dot{p})) \leq ... \leq E(\dot{h}^{(n)}_{PH}(\dot{p}))$, $E\left(\dot{h}^{(1)}_{PH}(\dot{q})\right) \leq E\left(\dot{h}^{(2)}_{PH}(\dot{q})\right) \leq ... \leq E\left(\dot{h}^{(n)}_{PH}(\dot{q})\right)$, when $E\left(\dot{h}^{(j)}_{PH}(\dot{p})\right) \geq E\left(\dot{h}^{(j)}_{PH}(\dot{q})\right)$ for all j, then

$$E\left(PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right)\right) \geq E\left(PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{q}), \dot{h}^2_{PH}(\dot{q}), ..., \dot{h}^n_{PH}(\dot{q})\right)\right)$$

Proof: Since $\dot{H}^{(j+1)}_{PH}(\dot{p}) \subseteq \dot{H}^{(j)}_{PH}(\dot{p})$, then $\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right) \geq 0$. Since $\forall j$, $E\left(\dot{h}^{(j)}_{PH}(\dot{p})\right) \geq E\left(\dot{h}^{(j)}_{PH}(\dot{q})\right)$, then

$$\sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{p}^{(j)}_i \geq \sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{q}^{(j)}_i.$$

According to Definition 6,

$$E\left(PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right)\right) \geq E\left(PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{q}), \dot{h}^2_{PH}(\dot{q}), ..., \dot{h}^n_{PH}(\dot{q})\right)\right)$$

then, $PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) \geq PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{q}), \dot{h}^2_{PH}(\dot{q}), ..., \dot{h}^n_{PH}(\dot{q})\right)$.

Property 4. (Boundedness) Let $\left\{\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right\}$ be a PHFLTS, where $\dot{h}^j_{PH}(\dot{p}) = \left\{\left(l_i, \dot{p}^j_i\right) | l_i \in L, \sum_{i=1}^{t}\dot{p}^j_i = 1, i = 1, 2, ..., t\right\}$, $\dot{h}^+_{PH}(\dot{p}) = \{(l_1, 0), (l_2, 0), ..., (l_t, 1)\}$ and $\dot{h}^-_{PH}(\dot{p}) = \{(l_1, 1), (l_2, 0), ..., (l_t, 0)\}$, then

$$\dot{h}^-_{PH}(\dot{p}) \leq PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right) \leq \dot{h}^+_{PH}(\dot{p})$$

Proof: According to Definition 6, we get $E\left(\dot{h}^-_{PH}(\dot{p})\right) = 1$, $E\left(\dot{h}^+_{PH}(\dot{p})\right) = t$.

$$E\left(PHFLCA_\mu\left(\dot{h}^1_{PH}(\dot{p}), \dot{h}^2_{PH}(\dot{p}), ..., \dot{h}^n_{PH}(\dot{p})\right)\right) = \sum_{i=1}^{t} i \cdot \left(\sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{p}_i\right) \leq t\sum_{i=1}^{t}\left(\sum_{j=1}^{n}\left(\mu\left(\dot{H}^{(j)}_{PH}(\dot{p})\right) - \mu\left(\dot{H}^{(j+1)}_{PH}(\dot{p})\right)\right)\dot{p}_i\right) = t$$

The same, we get

$$E\left(PHFLCA_{\mu}\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right)\right) = \sum_{i=1}^{t} i\cdot\left(\sum_{j=1}^{n}\left(\mu\left(\dot{\hat{H}}_{PH}^{(j)}(\dot{p})\right)-\mu\left(\dot{\hat{H}}_{PH}^{(j+1)}(\dot{p})\right)\right)\dot{p}_i\right) \geq \sum_{i=1}^{t}\left(\sum_{j=1}^{n}\left(\mu\left(\dot{\hat{H}}_{PH}^{(j)}(\dot{p})\right)-\mu\left(\dot{\hat{H}}_{PH}^{(j+1)}(\dot{p})\right)\right)\dot{p}_i\right)=1$$

so, $E\left(\dot{\hat{h}}_{PH}^{-}(\dot{p})\right) \leq E\left(PHFLCA_{\mu}\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right)\right) \leq E\left(\dot{\hat{h}}_{PH}^{+}(\dot{p})\right)$

i.e., $\dot{\hat{h}}_{PH}^{-}(\dot{p}) \leq PHFLCA_{\mu}\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) \leq \dot{\hat{h}}_{PH}^{+}(\dot{p})$.

Noted: When the PHFLEs $\dot{\hat{h}}_{PH}^{j}(\dot{p})$ $(j=1,2,...,n)$ are independent of each other, the fuzzy measure μ degenerates into an additive measure, i.e., $\mu\left(\dot{\hat{H}}_{PH}^{(j)}(\dot{p})\right) = \sum_{\dot{\hat{h}}_{PH}^{(j)}(\dot{p})\in\dot{\hat{H}}_{PH}^{(j)}(\dot{p})}\mu\left(\dot{\hat{h}}_{PH}^{(j)}(\dot{p})\right)$, $\forall\dot{\hat{H}}_{PH}^{(j)}(\dot{p}) \in \dot{\hat{H}}_{PH}(\dot{p})$. The $PHFLCA_{\mu}$ operator degenerates to the proportional hesitant linguistic orderly weighted average operator.

3.2. $PHFLP_rWA_m$ Operator Based on Generalized Prioritized Measure

The generalized prioritized measure proposed by Chen et al. [37], can solve the following problems when the satisfaction degree of the highest priority attribute is same and the satisfaction degree is the lowest among all priority attributes. This is because the prioritized measure is too strict to compensate for any priority attributes. Therefore, this section proposes $PHFLP_rWA_m$ operator based on generalized prioritized measure.

Definition 15. Let $C=\{C_1,C_2,...,C_n\}$ be a set of attributes, then there exists a linear ordered priority relationship between attributes, which can be expressed as $C_1 \succ C_2 \succ ... \succ C_n$, i.e., C_j priority ranks are higher than C_k, $\forall j<k$. $\dot{\hat{h}}_{PH}^{j}(\dot{p})$ is the evaluation result of alternative x under attribute C_j, i.e., $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \left\{\left(l_i,\dot{p}_i^j\right)|l_i\in L,\sum_{i=1}^{t}\dot{p}_i^j=1, i=1,2,...,t\right\}$, $\dot{\hat{h}}_{PH}^{j}(\dot{p}) \in \dot{\hat{H}}_{PH}(\dot{p})$, $\dot{\hat{H}}_{PH}(\dot{p}) = \left\{\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right\}$ is the PHFLTS. The $PHFLP_rWA_m$ operator is defined as follows:

$$PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) = \sum_{j=1}^{n}\dot{\hat{h}}_{PH}^{(j)}(\dot{p})\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right) \tag{12}$$

where $\dot{\hat{h}}_{PH}^{(j)}(\dot{p})$ is the *jth* largest element in $\left\{\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right\}$, $H_\tau=\left\{C_{\sigma(1)},C_{\sigma(2)},...,C_{\sigma(\tau)}\right\}$, $\{\sigma(1),\sigma(2),...,\sigma(\tau)\}$ is the sequence of $\{1,2,....,\tau\}$, satisfying if $\varepsilon<k$, $C_{\sigma(\varepsilon)}\succ C_{\sigma(k)}$ and $H_{(0)}=\varnothing$, $m\left(H_j\right)$ $(j=1,2,...,n)$ is generalized prioritized measure.

$$m\left(H_\tau\right)=\sum_{\varepsilon=1}^{\tau}b_{\sigma(\varepsilon)}\frac{n+1-\sigma(\varepsilon)}{n+1-\varepsilon} \tag{13}$$

Theorem 2. *Let $C=\{C_1,C_2,...,C_n\}$ be a set of attributes, then there exists a linear ordered priority relationship between attributes, which can be expressed as $C_1 \succ C_2 \succ ... \succ C_n$, i.e., C_j priority ranks are higher than C_k, $\forall j<k$. $\dot{\hat{h}}_{PH}^{j}(\dot{p})$ is the evaluation result of alternative x under attribute C_j, i.e., $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \left\{\left(l_i,\dot{p}_i^j\right)|l_i\in L,\sum_{i=1}^{t}\dot{p}_i^j=1, i=1,2,...,t\right\}$, $\dot{\hat{h}}_{PH}^{j}(\dot{p}) \in \dot{\hat{H}}_{PH}(\dot{p})$, $\dot{\hat{H}}_{PH}(\dot{p}) = \left\{\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right\}$ is the PHFLTS, then the integration result of $PHFLP_rWA_m$ operator is as follows:*

$$PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) = \left\{\left(l_i,\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)|i=1,2,...,t\right\} \tag{14}$$

Through the above theorem, we can easily find that $PHFLP_rWA_m$ operator has the following properties:

Property 5. (Idempotency) Let $\dot{\hat{H}}_{PH}(\dot{p}) = \left\{\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right\}$ be the PHFLTS, of which $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \left\{\left(l_i,\dot{p}_i^j\right)|l_i\in L,\sum_{i=1}^{t}\dot{p}_i^j=1, i=1,2,...,t\right\}$. If $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \dot{\hat{h}}_{PH}(\dot{p}) = \left\{\left(l_i,\dot{p}_i\right)|i=1,2,...,t\right\}$, $j=1,2,...,n$, then

$$PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) = \dot{\hat{h}}_{PH}(\dot{p})$$

Proof: Because of $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \dot{\hat{h}}_{PH}(\dot{p})$, $m\left(H_j\right)-m\left(H_{j-1}\right)\geq 0$ and $\sum_{j=1}^{n}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)=1$, it is available that

$$PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) = \left\{\left(l_i,\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)|l_i\in L\right\} = \dot{\hat{h}}_{PH}(\dot{p})$$

Property 6. (Monotonicity) Let $\left\{\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right\}$ and $\left\{\dot{\hat{h}}_{PH}^{1}(\dot{q}),\dot{\hat{h}}_{PH}^{2}(\dot{q}),...,\dot{\hat{h}}_{PH}^{n}(\dot{q})\right\}$ be any two PHFLTSs, of which $\dot{\hat{h}}_{PH}^{j}(\dot{p}) = \left\{\left(l_i,\dot{p}_i^j\right)|l_i\in L,\sum_{i=1}^{t}\dot{p}_i^j=1, i=1,2,...,t\right\}$, $\dot{\hat{h}}_{PH}^{j}(\dot{q}) = \left\{\left(l_i,\dot{q}_i^j\right)|l_i\in L,\sum_{i=1}^{t}\dot{q}_i^j=1, i=1,2,...,t\right\}$ and $\dot{\hat{h}}_{PH}^{j}(\dot{p}) \geq \dot{\hat{h}}_{PH}^{j}(\dot{q})$, i.e., the proportional distribution of corresponding linguistic term is $\dot{p}_i^j \geq \dot{q}_i^j$ $(i=1,2,...,t)$, then

$$PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{p}),\dot{\hat{h}}_{PH}^{2}(\dot{p}),...,\dot{\hat{h}}_{PH}^{n}(\dot{p})\right) \geq PHFLP_rWA_m\left(\dot{\hat{h}}_{PH}^{1}(\dot{q}),\dot{\hat{h}}_{PH}^{2}(\dot{q}),...,\dot{\hat{h}}_{PH}^{n}(\dot{q})\right)$$

Proof: Based on Theorem 2, we get

$$PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)=\left\{\left(l_i,\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)|l_i\in L\right\},$$

$$PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{q}),\dot{\tilde{h}}^{2}_{PH}(\dot{q}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{q})\right)=\left\{\left(l_i,\sum_{j=1}^{n}\dot{q}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)|l_i\in L\right\}.$$

According to the Definition 8, the expectation values are

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\right)=\sum_{i=1}^{t}i\cdot\left(\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right),$$

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{q}),\dot{\tilde{h}}^{2}_{PH}(\dot{q}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{q})\right)\right)=\sum_{i=1}^{t}i\cdot\left(\sum_{j=1}^{n}\dot{q}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right).$$

Because $\dot{p}_i^{j}\geq\dot{q}_i^{j}$, we get $\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\geq\dot{q}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)$.

Then $\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\geq\sum_{j=1}^{n}\dot{q}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)$, we can get

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\right)\geq E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{q}),\dot{\tilde{h}}^{2}_{PH}(\dot{q}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{q})\right)\right).$$

Therefore, $PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\geq PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{q}),\dot{\tilde{h}}^{2}_{PH}(\dot{q}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{q})\right)$.

Property 7. (Boundedness) Let $\left\{\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right\}$ be a PHFLTS, where $\dot{\tilde{h}}^{j}_{PH}(\dot{p})=\left\{\left(l_i,\dot{p}_i^{j}\right)|l_i\in L,\sum_{i=1}^{t}\dot{p}_i^{j}=1,i=1,2,\dots,t\right\}$, $\dot{\tilde{h}}^{+}_{PH}(\dot{p})=\{(l_1,0),(l_2,0),\dots,(l_t,1)\}$ and $\dot{\tilde{h}}^{-}_{PH}(\dot{p})=\{(l_1,1),(l_2,0),\dots,(l_t,0)\}$, then

$$\dot{\tilde{h}}^{-}_{PH}(\dot{p})\leq PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\leq\dot{\tilde{h}}^{+}_{PH}(\dot{p})$$

Proof: Based on the Definition 8, we get that

$$E\left(\dot{\tilde{h}}^{+}_{PH}(\dot{p})\right)=t,\ E\left(\dot{\tilde{h}}^{-}_{PH}(\dot{p})\right)=1,$$

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\right)=\sum_{i=1}^{t}i\cdot\left(\sum_{j=1}^{n}\dot{p}_i^{(j)}\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right).$$

Because

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\right)=\sum_{i=1}^{t}i\cdot\left(\sum_{j=1}^{n}\dot{p}_i\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)\leq t\sum_{i=1}^{t}\left(\sum_{j=1}^{n}\dot{p}_i\left(m\left(H_j\right)-m\left(H_{j+1}\right)\right)\right)=t.$$

Similarly,

$$E\left(PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\right)=\sum_{i=1}^{t}i\cdot\left(\sum_{j=1}^{n}\dot{p}_i\left(m\left(H_j\right)-m\left(H_{j-1}\right)\right)\right)\geq\sum_{i=1}^{t}\left(\sum_{j=1}^{n}\dot{p}_i\left(m\left(H_j\right)-m\left(H_{j+1}\right)\right)\right)=1.$$

So, we get $\dot{\tilde{h}}^{-}_{PH}(\dot{p})\leq PHFLP_rWA_m\left(\dot{\tilde{h}}^{1}_{PH}(\dot{p}),\dot{\tilde{h}}^{2}_{PH}(\dot{p}),\dots,\dot{\tilde{h}}^{n}_{PH}(\dot{p})\right)\leq\dot{\tilde{h}}^{+}_{PH}(\dot{p})$.

3.3. $PHFLP_rCA$ Operator

Yager [26] pointed out that prioritized decision problems can be categorized in two forms: (1) one form is strictly ordered priority, i.e., each priority corresponds to one attribute; (2) the other form is weakly ordered priority, i.e., each priority corresponds to one or more attributes. $PHFLP_rWA_m$ operator can only deal with the first kind of priority problems, but it cannot deal with the second kind of MAGDM problems. However, the second kind of MAGDM problems often occur in real life, thus a new integration operator needs proposing to handle the problems of weakly ordered priority, i.e., $PHFLP_rCA$ operator. The operation of $PHFLP_rCA$ operator can be described in two steps. Firstly, the $PHFLCA_g$ operator is based on λ fuzzy measure to get the satisfaction, which is displayed in each priority level. Secondly, the $PHFLP_rWA_m$ operator based on prioritized measure is to get the overall satisfaction, which is displayed in each alternative. Thus the $PHFLP_rCA$ operator proposed in this section considers both the priority relationship among priority levels and the correlation among attributes.

In the MAGDM problems with weakly ordered priority, attribute set $C=\{C_1,C_2,\dots,C_n\}$ is divided into q independent priority levels $H=\{H_1,H_2,\dots,H_q\}$, $H_\tau=\left\{C_1^\tau,C_2^\tau,\dots,C_{n_\tau}^\tau\right\}$, of which n_τ is the number of attributes contained in priority level H_τ. Assuming that there is a priority relation $H_1\succ H_2\succ\dots\succ H_q$ in the independent priority level $\{H_1,H_2,\dots,H_q\}$, when the attribute priority of $k>\varepsilon$ in H_k is higher than that in H_ε. Attribute set $C=\bigcup_{\tau=1}^{q}H_\tau$, numbers of attributes $n = \sum_{\tau=1}^{q} n_\tau$, evaluation value of alternative x under attribute $C_{k_\tau}^{\tau} \in H_\tau$ is PHFLE, expressed as $h_\tau^{k_\tau}(p)$.

$PHFLCA_g$ operator is based on λ fuzzy measure to calculate the satisfaction $h^\tau(p)$, which is displayed in each priority level:

$$PHFLCA_g\left(\dot{\tilde{h}}_\tau^1(\dot{p}), \dot{\tilde{h}}_\tau^2(\dot{p}), \ldots, \dot{\tilde{h}}_\tau^{n_\tau}(\dot{p})\right) = \bigoplus_{j=1}^{n_\tau} \dot{\tilde{h}}_\tau^{(j)}(\dot{p})\left(g\left(H_\tau^{(j)}\right) - g\left(H_\tau^{(j-1)}\right)\right) \tag{15}$$

where $\dot{\tilde{h}}_\tau^{(j)}(\dot{p})$ is the jth largest value in $\left\{\dot{\tilde{h}}_\tau^1(\dot{p}), \dot{\tilde{h}}_\tau^2(\dot{p}), \ldots, \dot{\tilde{h}}_\tau^{n_\tau}(\dot{p})\right\}$, and $H_\tau^{(j)} = \left\{C_\tau^{(1)}, C_\tau^{(2)}, \ldots, C_\tau^{(j)}\right\}$, $H_\tau^{(0)} = \varnothing$, $g\left(H_\tau^{(j)}\right)$ is λ fuzzy measure of $H_\tau^{(j)}$.

According to the above formulas, the ensemble results of $PHFLCA_g$ operators based on λ fuzzy measure is as follows:

$$\dot{\tilde{h}}_\tau(\dot{p}) = PHFLCA_g\left(\dot{\tilde{h}}_\tau^1(\dot{p}), \dot{\tilde{h}}_\tau^2(\dot{p}), \ldots, \dot{\tilde{h}}_\tau^{n_\tau}(\dot{p})\right) = \left\{\left(l_i, \sum_{j=1}^{n_\tau} \dot{p}_\tau^{(j)}\left(g\left(H_\tau^{(j)}\right) - g\left(H_\tau^{(j-1)}\right)\right)\right) \middle| i = 1, 2, \ldots, t\right\} \tag{16}$$

The $PHFLP_rA_m$ operator is based on prioritized measure to calculate the overall satisfaction $\dot{\tilde{h}}(\dot{p})$, which is displayed in each alternative,

$$PHFLP_rA_m\left(\dot{\tilde{h}}_1(\dot{p}), \dot{\tilde{h}}_2(\dot{p}), \ldots, \dot{\tilde{h}}_q(\dot{p})\right) = \bigoplus_{\tau=1}^{q} \dot{\tilde{h}}_{(\tau)}(\dot{p})\left(m\left(h^{(\tau)}\right) - m\left(h^{(\tau-1)}\right)\right) \tag{17}$$

where $\dot{\tilde{h}}_{(\tau)}(\dot{p})$ is τth largest value in $\left\{\dot{\tilde{h}}_{(1)}(\dot{p}), \dot{\tilde{h}}_{(2)}(\dot{p}), \ldots, \dot{\tilde{h}}_{(q)}(\dot{p})\right\}$, $h^{(\tau)} = \left\{C^{(1)}, C^{(2)}, \ldots, C^{(\tau)}\right\}$, $h^{(0)} = \varnothing$, $m\left(h^{(\tau)}\right)$ is generalized prioritized measure.

According to the above formula, the integration result of $PHFLP_rA_m$ operator based on prioritized measure is as follows:

$$PHFLP_rA_m\left(\dot{\tilde{h}}_1(\dot{p}), \dot{\tilde{h}}_2(\dot{p}), \ldots, \dot{\tilde{h}}_q(\dot{p})\right) = \left\{\left(l_i, \sum_{\tau=1}^{q} \dot{p}_{(\tau)}\left(m\left(h^{(\tau)}\right) - m\left(h^{(\tau-1)}\right)\right)\right)\right\} \tag{18}$$

Through the above formulas, it is easy to prove that the operators proposed in this section are idempotent, monotonic and bounded.

4. MAGDM METHOD BASED ON *PHFLPrCA* Operator

4.1. MAGDM Problems with Priority Relations

In a MAGDM problem with both priority and correlation, it is assumed that there are m alternatives, i.e., $\{A_1, A_2, \ldots, A_m\}$. $|G|$ DMs evaluate the alternative according to n attributes, i.e., $\{C_1, C_2, \ldots, C_n\}$. n attributes are divided into q independent priority levels, i.e., $H = \{H_1, H_2, \ldots, H_q\}$. Priority level H_τ contains n_τ attributes, i.e., $H_\tau = \left\{C_1^\tau, C_2^\tau, \ldots, C_{n_\tau}^\tau\right\}$. Assuming that the attributes in the same priority level are interactive, the priority relationship among the q independent priority levels is $H_1 \succ H_2 \succ \ldots \succ H_q$. The attributes in all priority levels constitute the whole set of attributes, $C = \cup_{\tau=1}^{q} H_\tau$. $|G|$ DMs evaluated alternative A_ς under attribute $C_{k_\tau}^\tau$. The evaluation information was hesitant fuzzy linguistic $h_{\varsigma k_\tau e}^\tau$ $(\tau = 1, 2, \ldots, q; k_\tau = 1, 2, \ldots, n_\tau; \varsigma = 1, 2, \ldots, m; e = 1, 2, \ldots, |G|)$. The evaluation information of $|G|$ DMs could be transformed into PHFLEs $\dot{\tilde{h}}_{\varsigma k_\tau}^\tau(\dot{p})$.

4.2. Model of Priority Level in Generalized Prioritized Measure

In order to determine the priority weight, it is necessary to determine the value b_τ $(\tau = 1, 2, \ldots, q)$ in the generalized prioritized measure. Based on O'Hagan's maximum entropy method, a mathematical programming model is established, with predefined priority attitudes as constraints, and with entropy as objective function to determine b_τ $(\tau = 1, 2, \ldots, q)$. Attitude eigenvalue is called Orness measure, which reflects the optimism of DMs. The greater the Orness measure is, the more optimistic the DM will be. Entropy is measured by the degree of discreteness. The greater the degree of discreteness is, the more information will be involved in the process of information integration. Specific models are as follows:

$$\max - \sum_{\tau=1}^{q} b_\tau \ln b_\tau \tag{19}$$

$$s.t. \begin{cases} \sum_{\tau=1}^{q} \dfrac{q-\tau}{q-1} b_\tau = \Omega, & 0.5 \le \Omega \le 1 \\ \sum_{\tau=1}^{q} b_\tau = 1, & b_\tau \in [0, 1] \end{cases}$$

where Ω represents the eigenvalue of preferential attitude. Generally speaking, attributes with higher priority are more important, so the value of Ω is between 0.5 and 1. $-\sum_{\tau=1}^{q} b_\tau \ln b_\tau$ is about the dispersion of $b_\tau (\tau = 1, 2, \ldots, q)$.

4.3. The Steps of MAGDM Method

Priority relation among attributes is a common phenomenon in decision-making problems, as well as, interaction between attributes, i.e., correlation, is another common phenomenon in decision-making process. Therefore, this section proposes a $PHFLP_rCA$ operator-based decision-making method to solve the multi-attribute decision-making problem with weakly ordered priority and association. The framework is shown in Figure 1. The specific steps are as follows:

Step 1: DMs evaluate the alternatives under each attribute to give HFLEs $h_{\varsigma k_\tau e}^\tau$, convert HFLEs $h_{\varsigma k_\tau e}^\tau$ into PHFLEs $\dot{\tilde{h}}_{\varsigma k_\tau}^\tau(\dot{p})$, thus forming the PHFLEs decision matrix $Y = \left[\dot{\tilde{h}}_{\varsigma k_\tau}^\tau(\dot{p})\right]$.

Step 2: Use formula (4) to determine λ_τ (τ=1, 2, ..., q) in each priority level H_τ.

Step 3: Use formula (3) to determine the optimal fuzzy measure $g_{\lambda_\tau}(S_\tau)$ of attribute set $S_\tau \subset \left\{C_1^\tau, C_2^\tau, \ldots, C_{k_\tau}^\tau\right\}$ in each priority level H_τ.

Step 4: Use formula (16) to calculate the satisfaction $\dot{\tilde{h}}_\varsigma^\tau(\dot{p})$ of alternative A_ς (ς=1, 2, ..., m) under each priority level H_τ (τ=1, 2, ..., q).

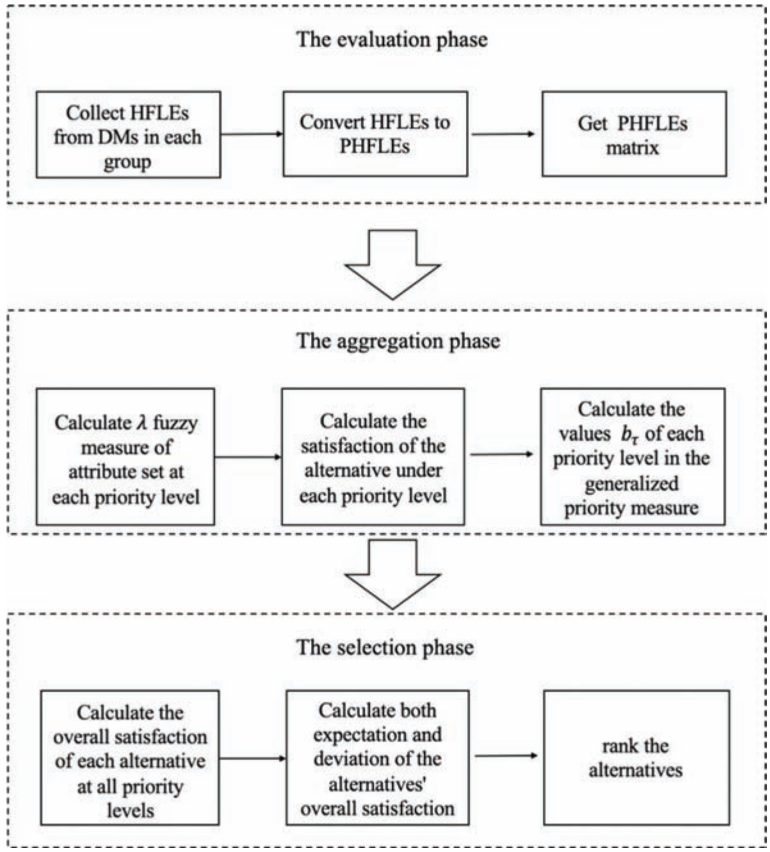

Figure 1 Framework of multiple attribute group decision-making (MAGDM) method based on $PHFLP_rCA$ operator.

Step 5: Use formula (19) to determine b_τ (τ=1, 2, ..., q) of each priority level in the generalized prioritized measure.

Step 6: Use formula (18) to calculate the overall satisfaction $\dot{\tilde{h}}_\varsigma(\dot{p})$ of alternative A_ς ($\varsigma = 1, 2, ..., m$).

Step 7: Use formula (1) and formula (2) to calculate the expectation value $E\left(\dot{\tilde{h}}_\varsigma(\dot{p})\right)$ and deviation $\sigma\left(\dot{\tilde{h}}_\varsigma(\dot{p})\right)$ of the overall satisfaction of each alternative.

Step 8: Rank the alternatives according to the expectations and deviations.

5. CASE STUDY: TEACHING PERFORMANCE EVALUATION OF CHINESE-FOREIGN COOPERATIVE EDUCATION PROJECT OF S COLLEGE

Chinese-foreign cooperative education project is an educational model with the background of globalization. It usually refers to the educational projects that foreign educational institutions and domestic educational institutions have set up in China. Under such background S college carries out many projects with several foreign universities.

However, with the expansion of the projects' scale and the increasing number of students enrolled, the problems of resource integration, personnel training, and scientific research within the S college has gradually emerged. In order to solve the hidden problems within the college, improve the efficiency of teachers' performance and promote the realization of project objectives, S college has the demand that carries out a reasonable evaluation index system. According to the requirement of trinity (teaching ability, scientific research ability, and other ability), the college evaluation committee has established an evaluation index system to combine the characteristics and development goals of Chinese-foreign cooperative education project in S college, as shown in Table 1.

The focus of the evaluation is teachers' teaching ability, so the priority of three elements is *Teaching ability* $(H_1) \succ$ *Scientific research ability* $(H_2) \succ$ *Other abilites* (H_3). It shows that the lack of teaching ability cannot be compensated by the increase of the corresponding scientific research ability. In addition, some correlations are displayed between the specific indicators contained in each element, so the sum of the importance of elements may not be 1. In the process of formulating the evaluation index system, the importance of each indicator is obtained from expert interviews, i.e., in the teaching ability H_1, the importance of three specific indicators are $g_{\lambda_1}\left(\{C_1^1\}\right) = g_{\lambda_1}\left(\{C_3^1\}\right) = 2/3$, $g_{\lambda_1}\left(\{C_2^1\}\right) = 1/3$, respectively; in the scientific research ability H_2, the importance of three specific indicators are $g_{\lambda_2}\left(\{C_1^2\}\right) = g_{\lambda_2}\left(\{C_2^2\}\right) = 1/3$, $g_{\lambda_2}\left(\{C_3^2\}\right) = 2/3$, respectively; in the other abilities H_3, the importance of two specific indicators are $g_{\lambda_3}\left(\{C_1^3\}\right) = g_{\lambda_3}\left(\{C_2^3\}\right) = 2/5$, respectively.

Table 1 Teaching performance evaluation of Chinese-foreign cooperative education project of S college.

Serial Number	Elements	Specific Indicators
1	Teaching ability H_1	Bilingual teaching ability C_1^1 Developing international course ability C_2^1 Cooperative teaching ability C_3^1
2	Scientific researchability H_2	The level of scientific research projects C_1^2 The level of academic papers C_2^2 International research cooperation ability C_3^2
3	Other abilites H_3	Project management ability C_1^3 Cross-cultural ability C_2^3

Table 2 Proportional hesitant fuzzy linguistic elements (PHFLEs) decision matrix.

	A_1	A_2	A_3	A_4
C_1^1	$\{(l_3,0.230),(l_4,0.385),(l_5,0.385)\}$	$\{(l_4,0.462),(l_5,0.538)\}$	$\{(l_2,0.385),(l_3,0.538),(l_4,0.077)\}$	$\{(l_2,0.250),(l_3,0.583),(l_4,0.417)\}$
C_2^1	$\{(l_3,0.357),(l_4,0.429),(l_5,0.214)\}$	$\{(l_4,0.571),(l_5,0.429)\}$	$\{(l_2,0.308),(l_3,0.538),(l_4,0.154)\}$	$\{(l_2,0.214),(l_3,0.571),(l_4,0.214)\}$
C_3^1	$\{(l_3,0.286),(l_4,0.500),(l_5,0.214)\}$	$\{(l_4,0.538),(l_5,0.462)\}$	$\{(l_2,0.357),(l_3,0.571),(l_4,0.072)\}$	$\{(l_2,0.333),(l_3,0.500),(l_4,0.167)\}$
C_1^2	$\{(l_3,0.357),(l_4,0.500),(l_5,0.143)\}$	$\{(l_4,0.500),(l_5,0.500)\}$	$\{(l_2,0.231),(l_3,0.538),(l_4,0.231)\}$	$\{(l_2,0.308),(l_3,0.385),(l_4,0.308)\}$
C_2^2	$\{(l_3,0.072),(l_4,0.571),(l_5,0.357)\}$	$\{(l_4,0.538),(l_5,0.462)\}$	$\{(l_2,0.214),(l_3,0.572),(l_4,0.214)\}$	$\{(l_2,0.333),(l_3,0.250),(l_4,0.417)\}$
C_3^2	$\{(l_3,0.214),(l_4,0.429),(l_5,0.357)\}$	$\{(l_3,0.076),(l_4,0.462),(l_5,0.462)\}$	$\{(l_2,0.231),(l_3,0.461),(l_4,0.308)\}$	$\{(l_2,0.417),(l_3,0.250),(l_4,0.333)\}$
C_1^3	$\{(l_3,0.076),(l_4,0.462),(l_5,0.462)\}$	$\{(l_3,0.071),(l_4,0.571),(l_5,0.357)\}$	$\{(l_2,0.154),(l_3,0.538),(l_4,0.308)\}$	$\{(l_2,0.417),(l_3,0.417),(l_4,0.167)\}$
C_2^3	$\{(l_3,0.286),(l_4,0.428),(l_5,0.286)\}$	$\{(l_4,0.429),(l_5,0.571)\}$	$\{(l_2,0.214),(l_3,0.572),(l_4,0.214)\}$	$\{(l_2,0.333),(l_3,0.417),(l_4,0.250)\}$

Evaluation team is composed of 11 experts. Experts express evaluation values in the form of linguistic words, because the evaluation values are highly uncertain and difficult to express in precise numbers. Experts may be more accustomed to using qualitative words such as "very low," "low," "medium," "high," and "very high." Experts can choose one or more linguistic words from $S = \{s_1 : very\,low, s_2 : low, s_3 : \text{moderate}, s_4 : high, s_5 : very\,high\}$ to evaluate according to their own understanding. When they know nothing about it, they can also give no evaluation value. Therefore, the evaluation results of 11 experts are summarized in Appendix (Table A1).

5.1. Decision-Making Steps

According to the MAGDM method described in the previous section, the teaching performance in S college is evaluated. The specific steps are as follows:

Step 1: The evaluation information given by experts is transformed into PHFLEs, and the PHFLE matrix is constructed, as shown in Table 2.

Step 3: Determine the λ fuzzy measure of each attribute subset under each priority level λ_τ ($\tau = 1, 2, 3$) according to formula (3) and formula (4).

1. For priority level H_1, $g_{\lambda_1}(\{C_1^1\}) = g_{\lambda_1}\left(\{C_3^1\}\right) = 2/3$, $g_{\lambda_1}(\{C_2^1\}) = 1/3$, according to formula (4), $\lambda_1 = -0.879$ can be obtained.
   According to formula (3), $g_{\lambda_1}(\{C_1^1, C_2^1\}) = 0.805$, $g_{\lambda_1}\left(\{C_1^1, C_3^1\}\right) = 0.943$, $g_{\lambda_1}\left(\{C_2^1, C_3^1\}\right) = 0.805$, and $g_{\lambda_1}\left(\{C_1^1, C_2^1, C_3^1\}\right) = 1$ can be obtained.
2. For priority level H_2, $g_{\lambda_2}(\{C_1^2\}) = g_{\lambda_2}(\{C_2^2\}) = 1/3$, $g_{\lambda_2}\left(\{C_3^2\}\right) = 2/3$, according to formula (4), $\lambda_2 = -0.658$ can be obtained.
   According to formula (3), $g_{\lambda_2}(\{C_1^2, C_2^2\}) = 0.594$, $g_{\lambda_2}\left(\{C_1^2, C_3^2\}\right) = 0.854$, $g_{\lambda_2}\left(\{C_2^2, C_3^2\}\right) = 0.854$, and $g_{\lambda_2}\left(\{C_1^2, C_2^2, C_3^2\}\right) = 1$ can be obtained.

3. For priority level H_3, $g_{\lambda_3}\left(\left\{C_1^3\right\}\right) = g_{\lambda_3}\left(\left\{C_2^3\right\}\right) = 2/5$, according to formula (4), $\lambda_3 = 1.25$ can be obtained.
   According to formula (3), $g_{\lambda_3}\left(\left\{C_1^3, C_2^3\right\}\right) = 1$ can be obtained.

Step 4: Use formula (16) to calculate the satisfaction $\dot{\tilde{h}}_\varsigma^\tau(\dot{p})$ of $A_\varsigma\,(\varsigma = 1, 2, 3, 4)$ under priority $C_\tau\,(\tau = 1, 2, 3)$.

Computing the satisfaction of A_1 at a priority level:

$$\dot{\tilde{h}}_1^1(\dot{p}) = \left(g_{\lambda_1}\left(\left\{C_3^1\right\}\right) - g_{\lambda_1}(\{\varnothing\})\right)\dot{\tilde{h}}_{13}^1(\dot{p}) \oplus \left(g_{\lambda_1}\left(\left\{C_2^1, C_3^1\right\}\right) - g_{\lambda_1}\left(\left\{C_3^1\right\}\right)\right) \dot{\tilde{h}}_{12}^1(\dot{p}) \oplus \left(g_{\lambda_1}\left(\left\{C_1^1, C_2^1, C_3^1\right\}\right) - g_{\lambda_1}\left(\left\{C_2^1, C_3^1\right\}\right)\right)\dot{\tilde{h}}_{11}^1(\dot{p})$$
$$= \{(l_1, 0), (l_2, 0), (l_3, 0.263), (l_4, 0.409), (l_5, 0.328)\}$$

$$\dot{\tilde{h}}_1^2(\dot{p}) = \left(g_{\lambda_2}\left(\left\{C_2^2\right\}\right) - g_{\lambda_2}(\{\varnothing\})\right)\dot{\tilde{h}}_{12}^2(\dot{p}) \oplus \left(g_{\lambda_2}\left(\left\{C_2^2, C_3^2\right\}\right) - g_{\lambda_2}\left(\left\{C_2^2\right\}\right)\right) \dot{\tilde{h}}_{13}^2(\dot{p}) \oplus \left(g_{\lambda_2}\left(\left\{C_1^2, C_2^2, C_3^2\right\}\right) - g_{\lambda_2}\left(\left\{C_2^2, C_3^2\right\}\right)\right)\dot{\tilde{h}}_{11}^2(\dot{p})$$
$$= \{(l_1, 0), (l_2, 0), (l_3, 0.147), (l_4, 0.538), l_5(0.315)\}$$

$$\dot{\tilde{h}}_1^3(\dot{p}) = \left(g_{\lambda_3}\left(\left\{C_1^3\right\}\right) - g_{\lambda_3}(\{\varnothing\})\right)\dot{\tilde{h}}_{11}^3(\dot{p}) \oplus \left(g_{\lambda_3}\left(\left\{C_1^3, C_2^3\right\}\right) - g_{\lambda_3}\left(\left\{C_1^3\right\}\right)\right)\dot{\tilde{h}}_{12}^3(\dot{p})$$
$$= \{(l_1, 0), (l_2, 0), (l_3, 0.130), (l_4, 0.426), (l_5, 0.387)\}$$

In the same way, the satisfaction of A_2, A_3, A_4 at each priority level can be obtained as follows:

$$\dot{\tilde{h}}_2^1(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0), (l_4, 0.539), (l_5, 0.461)\};$$

$$\dot{\tilde{h}}_2^2(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.004), (l_4, 0.508), (l_5, 0.487)\};$$

$$\dot{\tilde{h}}_2^3(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.048), (l_4, 0.499), (l_5, 0.396)\};$$

$$\dot{\tilde{h}}_3^1(\dot{p}) = \{(l_1, 0), (l_2, 0.329), (l_3, 0.543), (l_4, 0.127), (l_5, 0)\};$$

$$\dot{\tilde{h}}_3^2(\dot{p}) = \{(l_1, 0), (l_2, 0.228), (l_3, 0.519), (l_4, 0.253), (l_5, 0)\};$$

$$\dot{\tilde{h}}_3^3(\dot{p}) = \{(l_1, 0), (l_2, 0.162), (l_3, 0.517), (l_4, 0.264), (l_5, 0)\};$$

$$\dot{\tilde{h}}_4^1(\dot{p}) = \{(l_1, 0), (l_2, 296), (l_3, 0.524), (l_4, 0.180), (l_5, 0)\};$$

$$\dot{\tilde{h}}_4^2(\dot{p}) = \{(l_1, 0), (l_2, 0.387), (l_3, 0.258), (l_4, 0.355), (l_5, 0)\};$$

$$\dot{\tilde{h}}_4^3(\dot{p}) = \{(l_1, 0), (l_2, 0.370), (l_3, 0.393), (l_4, 0.180), (l_5, 0)\}.$$

Step 5: Use formula (19) to calculate the values $b_\tau\,(\tau = 1, 2, 3)$ in the generalized prioritized measure.

$$\max\{-b_1 \ln b_1 - b_2 \ln b_2 - b_3 \ln b_3\}$$
$$s.t. \begin{cases} b_1 + \frac{1}{2}b_2 + 0 \times b_3 = 0.75 \\ b_1 + b_2 + b_3 = 1 \\ b_1, b_2, b_3 \geq 0 \end{cases}$$

$$b_1 = 0.612, b_2 = 0.276, b_3 = 0.112.$$

Step 6: Use Formula (18) to calculate the overall satisfaction of each alternative at all priority levels.

$$\dot{\tilde{h}}_1(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.227), (l_4, 0.435), (l_5, 0.332)\};$$

$$\dot{\tilde{h}}_2(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.008), (l_4, 0.513), (l_5, 0.472)\};$$

$$\dot{\tilde{h}}_3(\dot{p}) = \{(l_1, 0), (l_2, 0.301), (l_3, 0.537), (l_4, 0.158), (l_5, 0)\};$$

$$\dot{\tilde{h}}_4(\dot{p}) = \{(l_1, 0), (l_2, 0.321), (l_3, 0.461), (l_4, 0.212), (l_5, 0)\}.$$

Step 7: Calculating expectation $E\left(\dot{\tilde{h}}_\varsigma(\dot{p})\right)(\varsigma = 1, 2, 3, 4)$

$E\left(\dot{\tilde{h}}_1(\dot{p})\right) = 4.079$, $E\left(\dot{\tilde{h}}_2(\dot{p})\right) = 4.437$, $E\left(\dot{\tilde{h}}_3(\dot{p})\right) = 2.847$, $E\left(\dot{\tilde{h}}_4(\dot{p})\right) = 2.871$.

Step 8: Rank the alternatives according to the expectation value.

$E\left(\dot{\tilde{h}}_2(\dot{p})\right) > E\left(\dot{\tilde{h}}_1(\dot{p})\right) > E\left(\dot{\tilde{h}}_4(\dot{p})\right) > E\left(\dot{\tilde{h}}_3(\dot{p})\right)$, so $A_2 > A_1 > A_4 > A_3$. That is the performance of the four teachers in S College, A_2 is the best and A_3 is the worst.

5.2. The Impact of Prioritized Attitudinal Character on Decision-Making Results

The influence of prioritized attitudinal character Ω on priority decision results is further analyzed. Firstly, O'Hagan's maximum entropy method is used to calculate the corresponding $b_\tau\,(\tau = 1, 2, 3)$ based on different Ω values. As can be seen from Table 3, Ω can be used to adjust the priority of attributes. As far as $\Omega \in \{0.6, 0.7, 0.75, 0.8, 0.9\}$ is concerned, it can be found from Table 3 and Figure 2 that the $b_\tau\,(\tau = 1, 2, 3)$ value forms a nonsubtractive sequence, i.e., $b_1 \geq b_2 \geq b_3$. Then, the *PHFLP$_r$CA* operator based on the corresponding $b_\tau\,(\tau = 1, 2, 3)$ value is used to rank alternatives. From the ranking results listed in Table 3 and Figure 3, no matter how Ω changes, A_2 is always the best solution. The prioritized attitudinal character Ω can be used to describe the DM's psychology. The greater the value of Ω is, the more optimistic the DM is. On the contrary, the smaller the Ω is, the more pessimistic the DM is. Therefore, DMs can choose an appropriate Ω values according to their own preference and practical problems.

Table 3 Decision-making results corresponding to prioritized attitudinal character.

Ω	$b_\tau\ (\tau=1,2,3)$	$E\left(\dot{\hat{h}}_\varsigma(\dot{p})\right)$	Ranking
0.6	$b_1 = 0.440, b_2 = 0.320, b_3 = 0.240$	$E\left(\dot{\hat{h}}_1(\dot{p})\right) = 4.077, E\left(\dot{\hat{h}}_2(\dot{p})\right) = 4.390, E\left(\dot{\hat{h}}_3(\dot{p})\right) = 2.862, E\left(\dot{\hat{h}}_4(\dot{p})\right) = 2.842.$	$A_2 \succ A_1 \succ A_3 \succ A_4$
0.7	$b_1 = 0.550, b_2 = 0.300, b_3 = 0.150$	$E\left(\dot{\hat{h}}_1(\dot{p})\right) = 4.080, E\left(\dot{\hat{h}}_2(\dot{p})\right) = 4.425, E\left(\dot{\hat{h}}_3(\dot{p})\right) = 2.854, E\left(\dot{\hat{h}}_4(\dot{p})\right) = 2.865.$	$A_2 \succ A_1 \succ A_4 \succ A_3$
0.75	$b_1 = 0.612, b_2 = 0.276, b_3 = 0.112$	$E\left(\dot{\hat{h}}_1(\dot{p})\right) = 4.079, E\left(\dot{\hat{h}}_2(\dot{p})\right) = 4.437, E\left(\dot{\hat{h}}_3(\dot{p})\right) = 2.847, E\left(\dot{\hat{h}}_4(\dot{p})\right) = 2.871.$	$A_2 \succ A_1 \succ A_4 \succ A_3$
0.8	$b_1 = 0.684, b_2 = 0.232, b_3 = 0.084$	$E\left(\dot{\hat{h}}_1(\dot{p})\right) = 4.077, E\left(\dot{\hat{h}}_2(\dot{p})\right) = 4.448, E\left(\dot{\hat{h}}_3(\dot{p})\right) = 2.838, E\left(\dot{\hat{h}}_4(\dot{p})\right) = 2.876.$	$A_2 \succ A_1 \succ A_4 \succ A_3$
0.9	$b_1 = 0.846, b_2 = 0.108, b_3 = 0.046$	$E\left(\dot{\hat{h}}_1(\dot{p})\right) = 4.070, E\left(\dot{\hat{h}}_2(\dot{p})\right) = 4.464, E\left(\dot{\hat{h}}_3(\dot{p})\right) = 2.817, E\left(\dot{\hat{h}}_4(\dot{p})\right) = 2.878.$	$A_2 \succ A_1 \succ A_4 \succ A_3$

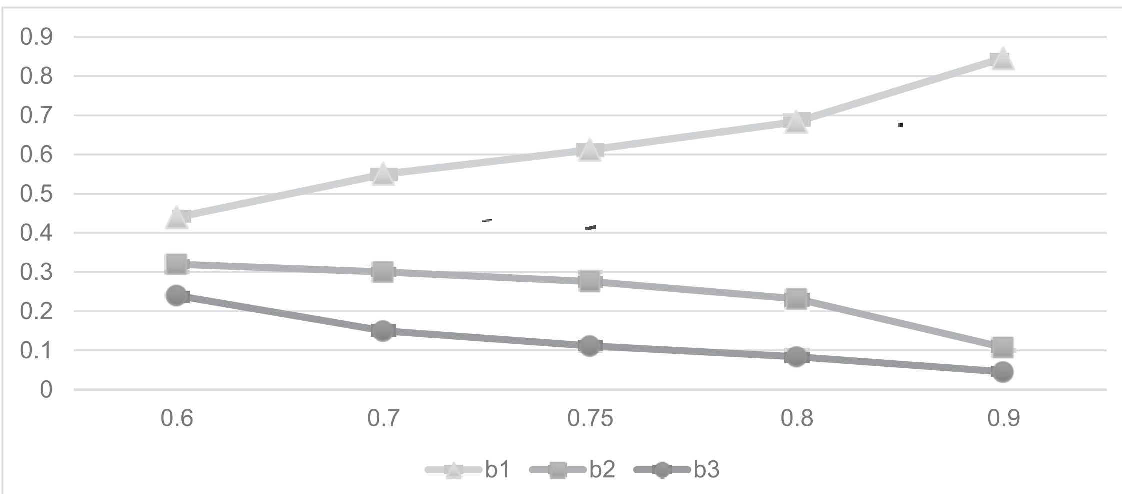

Figure 2 Trend chart of changes.

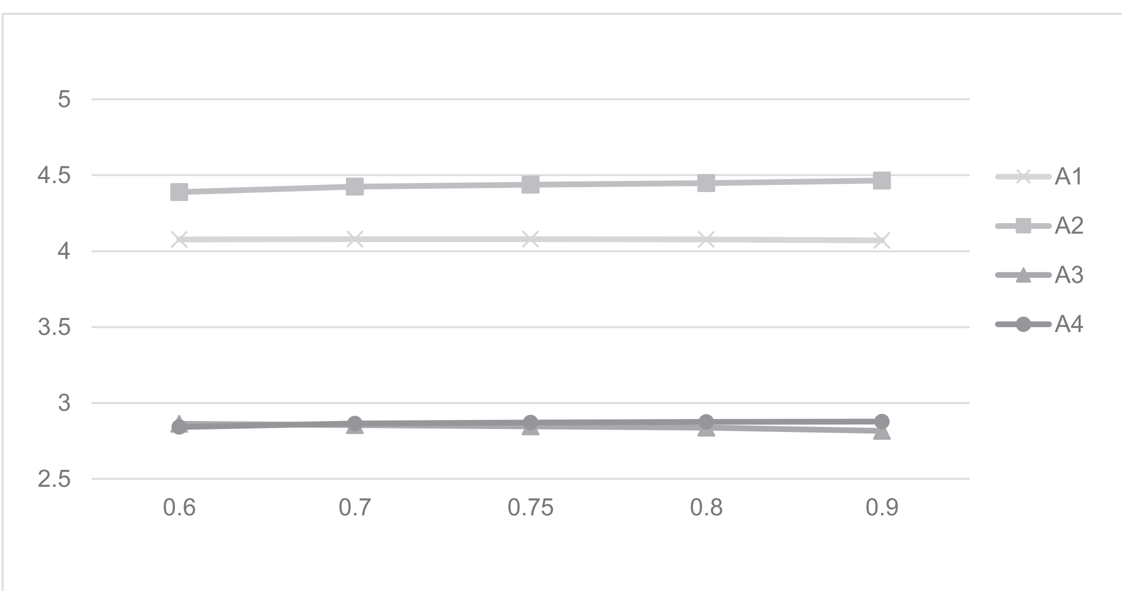

Figure 3 Trend chart of expectation value changes relative to value changes.

5.3. Validity Test of the Method

This subsection focuses on the reliability and validity of the created method by the test criteria [38]. This is because different MAGDM method may lead to different ranking results for the same decision-making problem.

Test criterion 1. Under the condition that attributes' weights keep unchanged, the optimal alternative still maintain its first place when any nonoptimal alternative are substituted by another non-optimal alternative. Thus, if the result meets the above conditions, this MAGDM method is effective.

Among four alternatives, A_3 is a nonoptimal alternative. C_1^3 can C_2^3 are taken place by $C_1^3 = \{(l_2, 0.385), (l_3, 0.538), (l_4, 0.077)\}$, $C_2^3 = \{(l_2, 0.357), (l_3, 0.643)\}$, by using test criterion 1. The overall satisfaction values of each alternative are obtained:

$$\dot{\tilde{h}}_1(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.227), (l_4, 0.435), (l_5, 0.332)\};$$

$$\dot{\tilde{h}}_2(\dot{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.008), (l_4, 0.513), (l_5, 0.472)\};$$

$$\dot{\tilde{h}}_3(\dot{p}) = \{(l_1, 0), (l_2, 0.329), (l_3, 0.534), (l_4, 0.097), (l_5, 0)\};$$

$$\dot{\tilde{h}}_4(\dot{p}) = \{(l_1, 0), (l_2, 0.321), (l_3, 0.461), (l_4, 0.212), (l_5, 0)\}.$$

The expectations of the alternatives are

$E\left(\dot{\tilde{h}}_1(\dot{p})\right) = 4.079$, $E\left(\dot{\tilde{h}}_2(\dot{p})\right) = 4.437$, $E\left(\dot{\tilde{h}}_3(\dot{p})\right) = 2.647$, $E\left(\dot{\tilde{h}}_4(\dot{p})\right) = 2.871$.

Thus, according to the ranking results $A_2 \succ A_1 \succ A_4 \succ A_3$, A_2 is still the best techer in S College. Therefore, the created MAGDM method is verified effectively by test criterion 1.

Test criterion 2. An effective MAGDM method should be proved transitive property, which will be demonstrated in test criterion 3.

Test criterion 3. If the same method is used to solve sub-problems, which are divided from the original MAGDM problem, the ranking result should be the same as the original MAGDM problem.

According to criteria 2 and 3 the initial MAGDM problem is divided to two sub-problems, $\{A_1, A_2, A_4\}$ and $\{A_2, A_3, A_4\}$. These two sub-problems are solved by the created MAGDM method, so the ranking results of them are $A_2 \succ A_1 \succ A_4$ and $A_2 \succ A_4 \succ A_3$. The integration of the above ranking results is $A_2 \succ A_1 \succ A_4 \succ A_3$, that is the same ranking result of original MAGDM problem, which proved transitive property. Therefore, the created MAGDM method is verified effectively by test criterion 2 and 3.

5.4. Advantages of the Method

To further illustrate the advantages of the proposed method, the *PHFLP$_r$CA* operator-based MAGDM method proposed in this paper is compared with other two decision-making methods [12], which are *PHFLWA* operator and *PHFLOWA* operator. The two methods mentioned above are used to rank the alternatives of the cases in this paper. The ranking results are shown in the following table:

Compared with the other two methods, this method has the following advantages:

1. The created method is flexible. By changing the Ω value, the result of decision-making problem can be solved more flexibly. When $\Omega = 0.6$, the ranking results of the proposed method are slight differences with those of other two methods (shown in Table 4); when $\Omega = 0.8$, the ranking results of the proposed method are consistent with them. This shows that the proposed method is more flexible than the other two methods in dealing with decision-making problems.
2. The created method can solve the decision-making problems with both priority and correlation. The existing two methods in this section have the same shortcoming, i.e., they do not consider the relationship between attributes, and cannot deal with the priority relationship between different levels. However, according to the actual situation, this paper can solve the decision-making problems of priority and correlation (complementary relationship, redundant relationship, independent relationship) among attributes, whereas the above two methods are not suitable for dealing with such problems.
3. The created method owns a simple calculation process. Both methods 1 and 2 aggregate indicators by means of two times. If there are more than two indicators involved, they have to aggregate them one by one, which results in a complicated calculation process. On the contrary, the created method can aggregate indicators by means of only one time, no matter how many indicators are. Fewer steps make the created method simpler than methods 1 and 2.

In this section, seven steps are applied in teaching performance evaluation to obtain the best alternative. And the best alternative won't change by adjusting Ω, thus DMs can choose an appropriate Ω values according to their own preference and practical problems. Moreover, this method passed validity test and is proved to own many advantages by compared with other two methods.

6. CONCLUSION

In order to solve teaching performance evaluation of Chinese-foreign cooperative education project, this paper mainly studies the PHFLE information aggregation operator with priority and correlation function and its application. Firstly, the *PHFLCA$_g$* operator is proposed for MAGDM problems with correlations among attributes (redundancy, complementarity, and independence). Secondly, the *PHFLP$_r$CA* operator is proposed for the case that attributes have both priority and correlation. In this paper, the properties of correlation operators are briefly introduced while corresponding operators are proposed. Thirdly, based on the *PHFLP$_r$CA* operator, a new MAGDM method is proposed. In order to verify the effectiveness of the proposed method, this paper chooses the typical MAGDM problem on teaching performance evaluation of Chinese-foreign cooperative education project in S College, which has priority and correlation among its attributes. Then this paper illustrates the role of the proposed method in solving the problem. Finally, the advantages of this method are illustrated by comparing with other two methods.

Table 4 Ranking results of different methods.

Methods		Ranking Values (R)	Ranking Results
Method 1 (*PHFLWA*)		$R(A_1) = 2.568, R(A_2) = 2.944, R(A_3) = 1.084, R(A_4) = 1.404.$	$A_2 \succ A_1 \succ A_4 \succ A_3$
Method 2 (*PHFLOWA*)		$R(A_1) = 2.751, R(A_2) = 3.234, R(A_3) = 1.006, R(A_4) = 1.009.$	$A_2 \succ A_1 \succ A_4 \succ A_3$
Our method	$\Omega = 0.6$	$R(A_1) = 4.077, R(A_2) = 4.390, R(A_3) = 2.862, R(A_4) = 2.842.$	$A_2 \succ A_1 \succ A_3 \succ A_4$
	$\Omega = 0.8$	$R(A_1) = 4.077, R(A_2) = 4.448, R(A_3) = 2.838, R(A_4) = 2.878.$	$A_2 \succ A_1 \succ A_4 \succ A_3$

The limitations of this paper are shown in following two aspects, which will be optimized in our future research. The first limitation is the LTS supposed in balanced and symmetric environment, however, in real life the LTS is unbalanced and asymmetric sometimes. The second limitation is that the DMs adopt the same LTS, however, DMs often choose different LTSs by their experience and preference. In the future PHFLE will be applied into unbalanced and multi-granular environment.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

AUTHORS' CONTRIBUTIONS

Conceptualization, Lei Wang and Peide Liu; methodology, Lili Rong and Fei Teng; formal analysis, Lei Wang and Lili Rong; writing-original draft preparation, Lei Wang and Fei Teng; writing-review and editing, Lei Wang and Peide Liu; supervision, Peide Liu; funding acquisition, Lei Wang, Lili Rong, Fei Teng and Peide Liu.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Nos. 71471172 and 71801142), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), the Natural Science Foundation of Shandong Province (No. ZR2020QG002), the Social science planning project of Shandong Province (No. 20CSDJ23), the Scientific research project of Shandong TV University (Nos. 2019KD002 and 2020JXYJ001Z), Key projects of Art Science in Shandong Province (Nos. ZD202008366 and ZD202008492).

REFERENCES

[1] J. Chen, H. Hsieh, Q. Do, Evaluating teaching performance based on fuzzy AHP and comprehensive evaluation approach, Appl. Soft Comput. 28 (2015), 100–108.

[2] Z.S. Chen, M. Li, W.T. Kong, K.S. Chin, Evaluation and selection of hazmat transportation alternatives: a PHFLTS-and TOPSIS-integrated multi-perspective approach, Int. J. Environ. Res. Pub. Health. 16 (2019), 4116.

[3] L. Zhou, H. Li, K. Sun, Teaching performance evaluation by means of a hierarchical multifactorial evaluation model based on type-2 fuzzy sets, Appl. Intell. 46 (2016), 34–44.

[4] M. Kuzmanovic, G. Savic, B. Andric Gusavac, D. Makajic-Nikolic, B. Panic, A conjoint-based approach to student evaluations of teaching performance, Expert Syst. Appl. 40 (2013), 4083–4089.

[5] B. Dutta, Á. Labella, R.M. Rodríguez, L. Martínez, Aggregating interrelated attributes in multi-attribute decision-making with ELICIT information based on Bonferroni mean and its variants, Int. J. Comput. Intell. Syst. 12 (2019), 1179–1196.

[6] Á. Labella, R.M. Rodríguez, L. Martínez, Computing with comparative linguistic expressions and symbolic translation for decision making: ELICIT information, IEEE Trans. Fuzzy Syst. 28 (2020), 2510–2522.

[7] S.H. Xiong, Z.S. Chen, K.S. Chin, A novel MAGDM approach with proportional hesitant fuzzy sets, Int. J. Comput. Intell. Syst. 11 (2018), 256–271.

[8] P. Liu, Y. Wang, Intuitionistic fuzzy interaction hamy mean operators and their application to multi-attribute group decision making, Group Decis. Negot. 28 (2019), 197–232.

[9] P. Liu, W. Liu, Scaled Prioritized Operators Based on the Linguistic Intuitionistic Fuzzy Numbers and Their Applications to Multi-Attribute Decision Making, Int. J. Fuzzy Syst. 20 (2018), 1539–1550.

[10] X. Liang, P. Liu, Z. Wang, Hotel selection utilizing online reviews: a novel decision support model based on sentiment analysis and dl-vikor method, Technol. Econ. Dev. Econ. 25 (2019), 1139–1161.

[11] X. Liang, F. Teng, Y. Sun, Multiple group decision making for selecting emergency alternatives: a novel method based on the ldwpa operator and ld-mabac, Int. J. Environ. Res. Pub. Health. 17 (2020), 2945.

[12] Z.S. Chen, K.S. Chin, Y.L. Li, Y. Yang, Proportional hesitant fuzzy linguistic term set for multiple criteria group decision making, Inf. Sci. 357 (2016), 61–87.

[13] Z.S. Chen, Y. Yang, X.J. Wang, K.S. Chin, K.L. Tsui, Fostering linguistic decision-making under uncertainty: a proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and andness optimization models, Inf. Sci. 500 (2019), 229–258.

[14] R.M. Rodríguez, Á. Labella, G. De Tré, L. Martínez, A large scale consensus reaching process managing group hesitation, Knowl. Based Syst. 159 (2018), 86–97.

[15] J. Huang, X.Y. You, H.C. Liu, S.L. Si, New approach for quality function deployment based on proportional hesitant fuzzy linguistic term sets and prospect theory, Int. J. Prod. Res. 57 (2019), 1283–1299.

[16] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inf. Sci. 8 (1975), 199–249.

[17] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-II, Inf. Sci. 8 (1975), 301–357.

[18] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-III, Inf. Sci. 9 (1975), 43–80.

[19] R.M. Rodriguez, L. Martinez, F. Herrera, A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets, Inf. Sci. 241 (2013), 28–42.

[20] H. Liao, Z. Xu, X.J. Zeng, Hesitant fuzzy linguistic vikor method and its application in qualitative multiple criteria decision making, Fuzzy Syst. IEEE Trans. 23 (2015), 1343–1355.

[21] H. Wang, Z. Xu, Total orders of extended hesitant fuzzy linguistic term sets: definitions, generations and applications, Knowl. Based Syst. 107 (2016), 142–154.
[22] C. Wei, R.M. Rodriguez, L. Martinez, Uncertainty measures of extended hesitant fuzzy linguistic term sets, IEEE Trans. Fuzzy Syst. 26 (2018), 1763–1768.
[23] S.M. Yu, J. Wang, J.Q. Wang, L. Li, A multi-criteria decision-making model for hotel selection with linguistic distribution assessments, Appl. Soft Comput. 67 (2017), 741–755.
[24] G. Zhang, Y. Dong, Y. Xu, Consistency and consensus measures for linguistic preference relations based on distribution assessments, Inf. Fusion. 17 (2014), 46–55.
[25] M. Grabisch, Fuzzy integral in multicriteria decision making, Fuzzy Sets Syst. 69 (1995), 279–298.
[26] R.R. Yager, Prioritized aggregation operators, Int. J. Approx. Reason. 48 (2008), 263–274.
[27] Z.S. Chen, X.L. Liu, R.M. Rodríguez, X.J. Wang, K.S. Chin, K.L. Tsui, L. Martínez, Identifying and prioritizing factors affecting in-cabin passenger comfort on high-speed rail in China: a fuzzy-based linguistic approach, Appl. Soft Comput. 95 (2020), 106558.
[28] P. Liu, Y. Wang, Interval neutrosophic prioritized owa operator and its application to multiple attribute decision making, J. Syst. Sci. Compl. 29 (2016), 681–697.
[29] P. Liu, Y. Li, J. AntucheviČiene, Multi-criteria decision-making method based on intuitionistic trapezoidal fuzzy prioritised owa operator, Technol. Econ. Dev. Econ. 22 (2016), 453–469.
[30] P. Liu, J. Liu, Some q-rung orthopai fuzzy bonferroni mean operators and their application to multi-attribute group decision making, Int. J. Intell. Syst. 33 (2018), 2341–2363.
[31] M. Sugeno, Fuzzy measure and fuzzy integral, Trans. Soc. Instr. Control Eng. 8 (1972), 218–226.
[32] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, A model of consensus in group decision making under linguistic assessments, Fuzzy Sets Syst. 78 (1996), 73–87.
[33] R.M. Rodriguez, L. Martinez, F. Herrera, Hesitant fuzzy linguistic term sets for decision making, IEEE Trans. Fuzzy Syst. 20 (2012), 109–119.
[34] Q. Pang, H. Wang, Z. Xu, Probabilistic linguistic term sets in multi-attribute group decision making, Information Sciences, 369 (2016), 128–143.
[35] M. Grabisch, I. Kojadinovic, P. Meyer, A review of methods for capacity identification in Choquet integral based multi-attribute utility theory: applications of the Kappalab R package, Eur. J. Oper. Res. 186 (2008), 766–785.
[36] M. Grabisch, The application of fuzzy integrals in multicriteria decision making, Eur. J. Oper. Res. 89 (1996), 445–456.
[37] L. Chen, Z. Xu, X. Yu, Prioritized measure-guided aggregation operators, IEEE Trans. Fuzzy Syst. 22 (2014), 1127–1138.
[38] X. Wang, E. Triantaphyllou, Ranking irregularities when evaluating alternatives by using some electre methods, Omega. 36 (2008), 45–63.

APPENDIX

Table A1 Information on performance evaluation of four teachers in S college.

	A_1	A_2	A_3	A_4
C_1^1	$\{l_4\}, \{l_5\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_4, l_5\}, \{l_3, l_4\}$	$\{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_2, l_3\}, \{l_2, l_3\}$	$\{l_3\}, \{l_3\}, \{l_2\}, \{l_4\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_2, l_3\}$
C_2^1	$\{l_4\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_3\}, \{l_3\}, \{l_4, l_5\}, \{l_3, l_4\}, \{l_4, l_5\}$	$\{l_4\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_2\}, \{l_2\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_3\}, \{l_3\}, \{l_2, l_3\}, \{l_3, l_4\}$	$\{l_3\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_4\}, \{l_2, l_3\}, \{l_3, l_4\}, \{l_3, l_4\}$
C_3^1	$\{l_3\}, \{l_4\}, \{l_3\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_3\}, \{l_4, l_5\}, \{l_4, l_5\}, \{l_3, l_4\}$	$\{l_5\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_3\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_2, l_3\}, \{l_2, l_3\}, \{l_2, l_3\}$	$\{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_2\}, \{l_3\}, \{l_3, l_4\}$
C_1^2	$\{l_3\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_3, l_4\}, \{l_3, l_4\}, \{l_4, l_5\}$	$\{l_5\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}$	$\{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_3\}, \{l_3\}, \{l_3, l_4\}, \{l_2, l_3\}$	$\{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2, l_3\}, \{l_3, l_4\}$
C_2^2	$\{l_4\}, \{l_4\}, \{l_3\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_4\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_3\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_2, l_3\}, \{l_3, l_4\}, \{l_3, l_4\}$	$\{l_2\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_4\}, \{l_4\}, \{l_3, l_4\}$
C_3^2	$\{l_3\}, \{l_4\}, \{l_5\}, \{l_3\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}, \{l_3, l_4\}, \{l_4, l_5\}$	$\{l_5\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_3, l_4\}, \{l_4, l_5\}$	$\{l_2\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_4\}, \{l_2, l_3\}, \{l_3, l_4\}$	$\{l_3\}, \{l_2\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_4\}, \{l_2\}, \{l_4\}, \{l_2\}, \{l_3, l_4\}$
C_1^3	$\{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_3\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_4\}, \{l_5\}, \{l_5\}, \{l_4\}, \{l_4\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_3, l_4\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_3, l_4\}, \{l_3, l_4\}$	$\{l_2\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_4\}, \{l_2\}, \{l_3\}, \{l_2\}, \{l_2\}, \{l_3, l_4\}$
C_2^3	$\{l_3\}, \{l_4\}, \{l_3\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_3\}, \{l_4, l_5\}, \{l_4, l_5\}, \{l_3, l_4\}$	$\{l_5\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4\}, \{l_5\}, \{l_4, l_5\}, \{l_4, l_5\}, \{l_4, l_5\}$	$\{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_2\}, \{l_3\}, \{l_2\}, \{l_4\}, \{l_2, l_3\}, \{l_2, l_3\}, \{l_3, l_4\}$	$\{l_2\}, \{l_3\}, \{l_3\}, \{l_4\}, \{l_3\}, \{l_3\}, \{l_2\}, \{l_2\}, \{l_4\}, \{l_2\}, \{l_3, l_4\}$