Developed Bi-Objective Predictive Control Modeling for Max-Plus Linear System

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ABSTRACT
Known to that MPC (Model Predictive Control) for MPL (Max Plus Linear System) is a practical approach to design optimal input sequences for a specific class of discrete event systems. Discrete event systems (DES) are dynamical which evolve in time by the occurrence of events at possibly irregular time intervals. The development of control acquired control problem for supply chain management by developing a mathematical model for a multi-echelon system. A combination of model predictive control and economic model predictive control called Bi-objective Model Predictive Control. Adaptive Weighted Sum or called AWS method is used to design a bi-objective optimization problem by combining these two control and the method can effectively solve the bi-objective optimization problem. The AWS method could stabilize a system with more cost-effective inputs when it is compared with model predictive control and show that the control can stabilize a wider range of initial state when it is compared to economic model predictive control. In this paper, we do development Bi-objective Model Predictive Control for Max Plus Linear system. Based MPC for MPL systems, we model Bi-objective model predictive control by introducing maximal additive algebraic theory to developing bi-objective predictive control an optimal strategy. Therefore the model predictive control based on the max-plus theory has important theoretical and practical significance for systems with guaranteed stability, thoroughness, and robustness.
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1. INTRODUCTION

The idea of posing control problem as problem of constrained optimization is not new [1]. In fact, all the "classical theory of Optimal Control, as developed between, say, 1955 and 1970, was driven by problems of constrained optimization arising out of the needs of the aerospace industry, particularly by military needs in flight, and by the problems of launching, guiding and landing space vehicles. In some ways, this theory solved an extremely wide range of problems. Suppose that the "plant" being controlled has an input vector u and a state vector x, and has nonlinear behavior governed by the vector differential equation

$$\frac{dx}{dt} = f(x,u),$$

Also suppose that the control objective is to minimize a "cost function" (or "value function") which has the form

$$V(x,u,t) = \int_0^T \ell(x(t),u(t),t)dt + F(x(T))$$

Where $\ell(x(t),u(t),t)$ is some function which is never negative, and that the control input is constrained to be in some set $u(t) \in U$. Model predictive control (MPC) is a proven technology for the control of multivariable systems in the presence of input, output, and state constraints and is capable of tracking pre-scheduled reference signals. Usually MPC uses linear or nonlinear discrete-time models. Extended MPC to class of discrete systems that can be described by a model that is "linear" in the max-plus algebra [2]. The max-plus linear (MPL) system is state-space description for a certain class of discrete-event-systems that are linear in the max-plus algebra, MPC method for MPL systems has been proposed.
(MPC) is a rolling horizon optimization control method with guaranteed stability properties. Boom and Schutter consider the stability of MPC for these (MPL) systems with guaranteed stability [3]. They show that with this end-point constraint the optimized cost function can be seen as a Lyapunov function for the system and can thus be used to prove stability. MPC is a popular controller design technique in the process industry. In previous research, Bi-objective MPC application in a numerical simulation can be

used as a controller for supply chain management without delay. Various studies have discussed control theory for inventory management in the supply chain is an important research domain, although stability studies for predictive tracking control have been carried out, but many of these control applications do not take into account stability in the supply chain. Bi-objective predictive control model for inventory management in the supply chain, research conducted by Widowati [4]. The bi-objective predictive access model in supply chain management without delay. On the other hand, economic model predictive control is a control strategy that purely aims to minimize economic costs without considering the stability of the system during its performance. Adaptive Weighted Sum (AWS) method is used to design a biobjective optimization problem by combining these two control strategies and weighing each of the respective strategy based on a subjective perspective. The acquired control is then compared to model predictive control and economic model predictive control in a numerical simulation. Based on the results from the simulation, it can be seen that the control obtained through AWS method could stabilize a system with more cost-effective inputs when it is compared with model predictive control. The results also show that the control can stabilize a wider range of initial state when it is compared to economic model predictive control.

In this paper, we developing a bi-objective predictive control model for max-plus linear systems. In order to improve the robustness and stability of the model predictive control system, we combine with max-plus theory. The goal of design optimization is to seek the best design that minimize the objective function by changing design variables while satisfying design constraints. During design optimization, one often needs to consider several design criteria or objective function simultaneously. We do development Bi-objective MPC for MPL system. Based MPC for MPL systems, we model Bi-objective model predictive control by introducing maximal additive algebraic theory to developing bi-objective predictive control an optimal strategy. Therefore the model predictive control based on the max-plus theory has important theoretical and practical significance for systems with guaranteed stability, thoroughness, and robustness.

2. METHODS

The method used to developed Bi-objective Predictive Control for Max-Plus Linear system is approached by designing technique for the control systems, as follows.

a. Known:
   1. Bi-objective Model Predictive Control
   2. Max-Plus Algebra and Max-Plus Linear system
b. Identify MPC for MPL systems:
   1. Derive an output prediction equation using the control process model,
   2. Determine an optimal control sequence in the future based on the predictive equation,
   3. Apply the Receding Horizon algorithm.
c. Develop Bi-objective Model Predictive Control for MPL system by using MPC for MPL system.

2.1. *Mathematical Modelling*

Model bi-objective predictive control design has been shown by several researchers ([5], [4]). The method used in combining the two functions of the control objective is the Adaptive Weighted Sum (AWS) method. The proposed using adaptive weighted sum method focuses on unexplored regions by using a priori weight selections and by specifying additional inequality constraints. It is demonstrated that the adaptive weighted sum method produces well-distributed solutions, finds pareto optimal solutions in non-convex regions, can be a potential liability of normal boundary intersection, and neglects non-pareto optimal solutions [6]. The bi-objective predictive control model consists of a function an economic objectives and tracking. In here, we only present the highlights of the bi-objective predictive control model. Introducing a bi-objective stage cost, which is an economic stage cost and a tracking stage cost. The economic cost for implementing input u from state x is given by $\ell_E(x,u)$. The optimal steady-state problem for the economic cost is defined as:

$$min_{x,u}\ell_E(\boldsymbol{x}(j),\boldsymbol{u}(j)) = q^T\boldsymbol{x}(j) + r^T\boldsymbol{u}(j) \quad (1)$$

s.t.

$x = Ax + Bu + B_d d_s,$

$x(j) \in \mathbb{X}$,
$u(j) \in \mathbb{U},$

where q^T and r^T are vectors which represent the effect of the current state and input to the economic cost of the system. While the tracking stage cost for implementing input u from state x is given by $\ell_T(x,u;z_t)$, which modify the cost function in the terminal penalty formulation by adding a terminal penalty (penalize deviations) from a chosen steady-state $z_t = (x_t, u_t)$. The optimal steady-state problem for the tracking cost as follows:

$$min_{x,u}\ell_T(\boldsymbol{x}(k),\boldsymbol{u}(k);z_t) = (\boldsymbol{x}(k)-\boldsymbol{x}_t)^T Q(\boldsymbol{x}(k)-\boldsymbol{x}_t) + (\boldsymbol{u}(k)-\boldsymbol{u}_t)^T R(\boldsymbol{u}(k)-\boldsymbol{u}_t) \quad (2)$$

s.t.

$x = Ax + Bu + B_d d_s,$

$x(k) \in \mathbb{X}$,
$u(k) \in \mathbb{U}$,

in which matrices Q and R are positive semi-definite matrices which guides and maintain states and inputs to their respective steady-state. The parameter $\omega \in [0,1]$ is a relative weighting given to the economic costs and the tracking cost. Then acquired the bi-objective stage cost is of the form:

$$\ell(x,u) = \frac{\omega}{s_E}\ell_E(x,u) + \frac{(1-\omega)}{s_T}\ell_T(x,u;z_t)$$

(3)

where $\ell_E(x,u)$ and $\ell_T(x,u;z_t)$ are two objective function to be mutually minimized, the scalling parameters are s_E and s_T are obtained with consider the utopia and nadir points of the individual stage costs $\ell_E(x,u)$ and $\ell_T(x,u;z_t)$. [6]. Denote $z = (x,u)$, then solve

$z_T = (x_T, u_T) = \arg min_{x(j),u(j)}\, \ell_T(x(j),u(j);z_t)$,
$z_E = (x_E, u_E) = \arg min_{x(j),u(j)}\, \ell_E(x(j),u(j))$,
with
$J_{Utopia} = J_U = (\ell_E(z_E), \ell_T(z_T;z_t))$,
$J_{Nadir} = J_N = (\ell_E(z_T), \ell_T(z_E;z_t))$,

therefore

$$\begin{aligned}(s_E, s_T) &= J_N - J_U \\ &= (\ell_E(z_T), \ell_T(z_E;z_t)) \\ &\quad - (\ell_E(z_E), \ell_T(z_T;z_t)).\end{aligned}$$

Now the steady state problem can be written in detail as follows:

$$min_{x,u}\, \frac{\omega}{s_E}(q^T x + r^T u) + \frac{(1-\omega)}{s_T}\left((x-x_t)^T Q(x-x_t) + (u-u_t)^T R(u-u_t)\right)$$

(4)
with constraints
$x = Ax + Bu + B_d d_s$,

$x(j) \in \mathbb{X}$,
$u(j) \in \mathbb{U}$.

From the steady state problem can be obtained steady state $(x_s, u_s; d)$ which represents the policy of the system. The following bi-objective predictive controls are designed with final constraints to ensure system stability. In model mathematics, the control of two optimization-based goals is formulated as.

$$min_{u(0),u(1),\dots,u(N-1)} V_N(x_0, u(0), u(1), \dots, u(N-1))$$

(5)

with constraints,

$x(0) = x_0$,

$x(j+1) = Ax(j) + Bu(j) + B_d d_s$,

$x(j) \in \mathbb{X}$,
$u(j) \in \mathbb{U}$,
$x(N) = x_s$,

$j \in \{0, 1, 2, \dots, N-1\}$,

with objective function $V_N(x_0, u(0), u(1), \dots, u(N-1))$ is defined as the sum of the combined cost functions:

$$V_N(x_0, u(0), u(1), \dots, u(N-1)) = \sum_{j=0}^{N-1}\left(\frac{\omega}{s_E}\ell_E(x(j),u(j)) + \frac{(1-\omega)}{s_T}\ell_T(x(j),u(j);z_t)\right),$$

(6)

with consider the following linear model

$$x(j+1) = Ax(j) + Bu(j) + B_d d_s,$$

(7)

in which $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the manipulated input, and $d_s \in \mathbb{R}^d$ is the disturbance to the system. Assumed that the system (A,B) is stabilizable. The states and inputs are constrained as follows:

$x(j) \in \mathbb{X}$,
$u(j) \in \mathbb{U}$.

Theorem 1 [5]. (Lyapunov function with terminal constraint)

Let the system (A,B) be stabilizable. Let the constraint set $\mathbb{X}$ is convex and closed, there exists $(x_s, u_s;\, d_s)$ is a unique solution and the multiplier λ_s is such that $(x_s, u_s;\, d_s)$ uniquely solves, and the system $x^+ = Ax + Bu + B_d d_s$ is strictly dissipative with respect to the supply rate $s(x,u) =$

$\ell(x,u)-\ell(x_s,u_s)$ and storage function $\lambda(x)=\lambda'_s x$ hold. Then the steady state solution of the close loop system $x^+ = Ax+Bu+B_d d_s$ is asymptotically stable with χ_N as the region of attraction. The Lyapunov function is

$$\tilde{V}(x) \coloneqq V_N^0(x) + \lambda'_s[x-x_s] - N\ell(x_s,u_s)\ ,$$

in which $V_N^0(x)$ is the optimal cost function of (5).

2.2. Max-Plus Linear Systems

Before, we proposed about Max-Plus Linear system. We will present about Max-Plus Algebra. The basic operations of the max-plus algebra are maximization and addition, which will be represented by $\oplus$ and $\otimes$, respectively:

$$x \oplus y = \max(x,y) \text{ and } x \otimes y = x + y$$

For $x, y \in \mathbb{R}_\varepsilon \overset{\text{def}}{=} \mathbb{R} \cup \{-\infty\}$. Define $\varepsilon = -\infty$. The structure ($\mathbb{R}_\varepsilon$,$\oplus$,$\otimes$) is called the max-plus algebra by Baccelli et al., 1992 [7]. The operation $\oplus$ and $\otimes$ are called the max-plus-algebraic addition and max-plus-algebraic multiplication, respectively. Since many properties and concepts from linear algebra can be translated to the max-plus algebra by replacing $+$ by $\oplus$ and $\times$ by $\otimes$.

The matrix $\varepsilon_{m\times n}$is the $m\times n$ max-plus -algebraic zero matrix: $(\varepsilon_{m\times n})_{ij} = \varepsilon$ for all i,j; and E_n is the $n\times n$ max-plus-algebraic identity matrix: $(E_n)_{ii} = 0$ for all i and $(E_n)_{ii} = \varepsilon$ for all i,j with $i \neq j$. If $A,B \in {\mathbb{R}_\varepsilon}^{m\times n}, C \in {\mathbb{R}_\varepsilon}^{n\times p}$ then

$$(A \oplus B)_{ij} = \left(a_{ij} \oplus b_{ij}\right) = \max(a_{ij}, b_{ij}),$$
$$(A \otimes C)_{ij} = \bigoplus_{k=1}^{n} \left(a_{ik} \otimes c_{kj}\right) = \max_k (a_{ik} + c_{kj})$$

For all i,j. Note the analogy with the conventional definitions of matrix sum and product. The max-plus -algebraic matrix power of $A \in {\mathbb{R}_\varepsilon}^{n\times n}$ is defined as follows: $A^{\otimes^0} = E_n$ and $A^{\otimes^k} = A \otimes A^{\otimes^{k-1}}$ for $k = 1, 2, 3, \dots$.

The MPL system represents the behavior of the discrete-event systems described by state-space equations that are similar to the ones in modern control theory. DES with only synchronization and no concurrency can be modeled by a max-plus-algebraic model of the following form Bacelli et al., 1992[7]:

$$x(k+1) = A_k \otimes x(k) \oplus B_k \otimes u(k) \quad (8)$$
$$y(k) = C_k \otimes x(k) \quad (9)$$

The index k is the event counter, which indicates the number of event occurance from initial state. $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^m$, dan $y(k) \in \mathbb{R}^l$ are the state variables, control inputs, and controlled outputs, respectively. Moreover, $A_k \in {\mathbb{R}_\varepsilon}^{n\times n}, B_k \in {\mathbb{R}_\varepsilon}^{n\times m}$ dan $C_k \in {\mathbb{R}_\varepsilon}^{l\times n}$, where m is the number of inputs and l the number of outputs. Note the analogy of the description (8),(9) with the state space model (8),(9) for plus time linear systems. an important difference with the description (8),(9) is that now the components of the input, the output and the state are event times, and that the counter k in (8),(9) is an event counter (and event occurance instants are in general not equidistant), whereas in (8)(,9) k increases at each clock cycle. A discrete event system that can be modeled by (8),(9) will be called max-plus linear time invariant discrete event system or max-plus linear (MPL) system [2]. Note that for MPL systems, the sequences are non-decreasing functions. This because the MPL system input is time so it applies:

$$u(k) \leq u(k+1) \quad (10)$$

For each $k \geq 0$. In construct design MPC for MPL systems without constraint, state (10) will using. Then, made changes coordinates for MPL systems (8)-(10). The largest eigenvalue matrices A_k is λ_{max} finite, then based on theorem (), there is a matrices invertible $P \in {\mathbb{R}_\varepsilon}^{n\times n}$ so matrices $\tilde{A} = P^{\otimes^{-1}} \otimes A_k \otimes P$ fulfilled $[\tilde{A}]_{ij} \leq \lambda_{max}$ for each $i,j = 1,2,\dots,n$.

Based on the existence of the P matrices, the change in coordinates for the MPL system (8)-(10) becomes:

$$\tilde{A} = P^{\otimes^{-1}} \otimes A_k \otimes P \implies A_k = P \otimes \tilde{A} \otimes P^{\otimes^{-1}} \quad (11)$$
$$\tilde{x}(k) = P^{\otimes^{-1}} \otimes x(k) \Longrightarrow x(k) = P \otimes \tilde{x}(k) \quad (12)$$
$$\tilde{B} = P^{\otimes^{-1}} \otimes B_k \Longrightarrow B_k = P \otimes \tilde{B} \quad (13)$$
$$\tilde{C} = C_k \otimes P \Longrightarrow C_k = \tilde{C} \otimes P^{\otimes^{-1}} \quad (14)$$
$$\tilde{y}(k) = y(k) \quad (15)$$
$$\tilde{u}(k) = u(k) \quad (16)$$

then, substitution equations (11)-(16) to (8),(9) was obtained:

$$P \otimes \tilde{x}(k+1) = P \otimes \tilde{A} \otimes P^{\otimes^{-1}} \otimes P \otimes \tilde{x}(k) \oplus P \otimes \tilde{B} \otimes \tilde{u}(k)$$
$$= P \otimes \tilde{A} \otimes \tilde{x}(k) \oplus P \otimes \tilde{B} \otimes \tilde{u}(k) \qquad = P \otimes (\tilde{A} \otimes \tilde{x}(k) \oplus \tilde{B} \otimes \tilde{u}(k)\ )$$

(17)
and

$$\tilde{y}(k) = \tilde{C} \otimes P^{\otimes^{-1}} \otimes P \otimes \tilde{x}(k) = \tilde{C} \otimes \tilde{x}(k)$$

Next, in the equation (17) the two segments multiplied by $P^{\otimes^{-1}}$ from the left are acquired

$$P^{\otimes^{-1}} \otimes P \otimes \tilde{x}(k+1) = P^{\otimes^{-1}} \otimes P \otimes (\tilde{A} \otimes \tilde{x}(k) \oplus \tilde{B} \otimes \tilde{u}(k))$$

$$\tilde{x}(k+1) = \tilde{A} \otimes \tilde{x}(k) \oplus \tilde{B} \otimes \tilde{u}(k)$$

thus acquired the system

$$\tilde{x}(k+1) = \tilde{A} \otimes \tilde{x}(k) \oplus \tilde{B} \otimes \tilde{u}(k)$$

(18)

$$\tilde{y}(k) = \tilde{C} \otimes \tilde{x}(k)$$

(19)

The next step is to normalize the system (18)-(19) by subtracting the state vector $\tilde{x}$, input $\tilde{x}$, dan output $\tilde{y}$ with $\rho k, \rho$ vector with entries > 0, and substracting each matrix entry $\tilde{A}$ with ρ, as follows :

$$x(k) = \tilde{x}(k) - \rho k,$$
$$u(k) = \tilde{u}(k) - \rho k,$$
$$y(k) = \tilde{y}(k) - \rho k,$$

$[A]_{ij} = [\tilde{A}]_{ij} - \rho$, untuk $i, j \in \{1,2,3,\ldots,n\}$

$$B = \tilde{B}$$
$$C = \tilde{C}$$

then the normalization system is obtained

$$x(k+1) = A_k \otimes x(k) \oplus B_k \otimes u(k)$$

(20)

$$y(k) = C_k \otimes x(k)$$

(21)

Remark 1.[2] For plus time linear systems the influence of noise is usually modeled by adding an extra noise term to the state and/oroutput equation. For MPL models the entries of the system matrix correspond to production times or transportation times. So, instead of modeling noise (i.e. variation in the processing times) by adding an extra max-plus-algebraic term in (20) or (21), noise should rather be modeled as an additive term to these system matrices. However, this would not lead to a nice model structure. Therefore, we will use the max-plus linear model (20), (21) as an approximation of a discrete event system with uncertainty and/or modeling errors when we extend the MPC framework to MPL systems. this also motivates the use of a receding horizon strategy when we define MPC for MPL systems, since then we can regularly update our model of the system as new measurements become available.

2.3. The Model Predictive Control (MPC) for MPL Systems

The MPC design method can be applied to various kind of system models. Especially, in case of an application to the MPL systems, the derivation of an output. Prediction equation straightforwardly can be done because the linear property satisfy multiplication between a matrix and a vector over, the max-plus algebra. Therefore, this research focuses on the MPC method. In this section, we will introduce the MPC systems (Schutter, and van den Boom, 2001) [2]. Lets start with the derivation the predictive equations by using repeatedly, the state variables at the future events counter $k+1, \ldots k+N$ can be straightly calculated as follows.

$$\begin{cases} x(k+1) = A_k \otimes x(k) \oplus B_k \otimes u(k+1) \\ x(k+2) = A_{k+1} \otimes A_k \otimes x(k) \oplus A_{k+1} \otimes B_k \otimes u(k+1) \oplus B_{k+1} \otimes u(k+2) \\ \vdots \\ x(k+N) = A_{k+N-1} \otimes \ldots \otimes A_k \otimes x(k) \oplus A_{k+N-1} \otimes \ldots \otimes A_{k+1} \otimes B_k \\ \quad \otimes u(k+1) \oplus A_{k+N-1} \otimes \ldots \otimes A_{k+2} \otimes B_{k+1} \\ \quad \otimes u(k+2) \oplus \ldots \oplus B_{k+N-1} \otimes u(k+N) \end{cases}$$

In [2] too, we have shown that prediction of future values of $y(k)$ for the system (20)-(21) can be done by successive substitution, leading to the expression

$$\widetilde{y}(k) = \widetilde{C} \otimes x(k) \oplus \widetilde{D} \otimes \tilde{u}(k)$$

where $\widetilde{C}$ and $\widetilde{D}$ are given by

$$\widetilde{C} = \begin{bmatrix} C \otimes A \\ C \otimes A^{\otimes^2} \\ \vdots \\ C \otimes A^{\otimes^{N_p}} \end{bmatrix}$$

$$\widetilde{D} = \begin{bmatrix} C \otimes B & \varepsilon & \cdots & \varepsilon \\ C \otimes A \otimes B & C \otimes B & \cdots & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ C \otimes A^{\otimes^{N_p-1}} \otimes B & C \otimes A^{\otimes^{N_p-2}} \otimes B & \cdots & C \otimes B \end{bmatrix}$$

and $\tilde{u}(k), \tilde{y}(k)$ are defined as

$$\tilde{y}(k) = \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+N_p) \end{bmatrix}, \qquad \tilde{u}(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N_p-1) \end{bmatrix}$$

The MPC formulation that will be carried out is the MPC formulation for the normalized MPL system. Where $\hat{y}(k+j)$ denotes the predictionof $y(k+j)$ based on knowledge at event step k and N_p is the prediction horizon. The MPC problem for MPL systems is called the MPL-MPC problem is formulated as follows [2]:

$$\min_{\tilde{u}(k),\tilde{y}(k)} J\big(\tilde{u}(k), \tilde{y}(k)\big) = \min_{\tilde{u}(k),\tilde{y}(k)} J_{out}\big(\tilde{y}(k)\big) + \lambda J_{in}\big(\tilde{u}(k)\big) \tag{22}$$

Subject to,

$$\tilde{y}(k) = \widetilde{C} \otimes x(k) \oplus \widetilde{D} \otimes \tilde{u}(k), \tag{23}$$
$$E(k)\tilde{u}(k) + F(k)\tilde{y}(k) \leq h(k), \tag{24}$$
$$\Delta u(k+j) \geq 0 \quad \text{for } j = 0,1,\dots,N_p-1, \tag{25}$$
$$\Delta^2 u(k+j) \geq 0 \quad \text{for } j = N_c,\dots,N_p-1, \tag{26}$$

where $\Delta u(k) = u(k) - u(k-1)$ and $\Delta^2 u(k) = \Delta u(k) - \Delta u(k-1) = u(k) - 2u(k-1) + u(k-2)$, equation (24) reflects constraints on the input and output event separation times or maximum due dates for the output events, equation (25) guarantees a nondecreasing input signal and equation (26) is due to the control horizon N_c.

Theorem 2 [2]. Let the mapping $\tilde{y} \longrightarrow F(k)\tilde{y}$ be a monotonically non-decreasing function of $\tilde{y}$. let $(\tilde{u}^*, \tilde{y}^*)$ be an optimal solution of the relaxed MPL-MPC problem. If we define $\tilde{y}^\# = \widetilde{C} \otimes x(k) \oplus \widetilde{D} \otimes \tilde{u}^*$ then $(\tilde{u}^*, \tilde{y}^\#)$ is an optimal solution of the original MPL-MPC problem.

So, known that the MPL-MPC problem can be recast as a convex problem. In the general case, the closed-loop system (consisting of the MPL process with the MPL-MPC controller) will not be an MPL system, but it will be pieceswise affine in the state $x(k)$ and reference $r(k)$. Stability means that all signals in this system should remain bounded.

Definition 1[8]. A discrete event system is called stable if all its buffer levels remain bounded.

Based on [3], The MPL-MPC problem is stable. The existence of a solution of the MPL-MPC problem at event step k problem can be verified by solving the system, which describes the feasible set of the problem.

3. RESULTS AND DISCUSSION

As well as in design MPC for MPL systems, in design bi-objective model predictive control for MPL system, we assume that $x(k)$, the state at event step k, can be measured or estimated using previous measurements. We can then use (20)-(21) to estimate the evolution of the output of the system for the input sequence $u(k), \dots, u(k+N_p-1)$. Has been explained in the previous section about bi-objective model predictive control, the steady state problem can be written in detail as follows :

$$min_{u(0),u(1),\dots,u(N-1)} V_N(x_0, u(0), u(1), \dots, u(N-1); x_0)$$

with constraints

$x(0) = x_0,$
$x(k+1) = A_k \otimes x(k) \oplus B_k \otimes u(k)$,
$x(k) \in \mathbb{X},$
$u(k) \in \mathbb{U},$
$x(N) = x_s,$
$k \in \{0, 1, 2, \ldots, N-1\},$

in which the cost function

$V_N(x_0, u(0), u(1), \ldots, u(N-1); x_0) = \sum_{j=0}^{N-1}(\frac{\omega}{s_E}\ell_E(x(k), u(k)) + \frac{(1-\omega)}{s_T}\ell_T(x(k), u(k); z_t)),$
where the control horizon is denoted by N.

We can get the prediction equation in a vector form expressed as,

$$y(k) = C \otimes x(k) \oplus D \otimes u(k)$$

where

$$y(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+N-1) \end{bmatrix}, u(k) = \begin{bmatrix} u() \\ u(k+1) \\ \vdots \\ u(k+N-1) \end{bmatrix},$$

where C and D are given by

$$C = \begin{bmatrix} C_{k+1}A_k \\ C_{k+2}A_{k+1}A_k \\ \vdots \\ C_{k+N}A_{k+N-1} \ldots A_k \end{bmatrix} = \begin{bmatrix} C \otimes A \\ C \otimes A^{\otimes^2} \\ \vdots \\ C \otimes A^{\otimes^N} \end{bmatrix}$$

$$D = \begin{bmatrix} C \otimes B & \varepsilon & \cdots & \varepsilon \\ C \otimes A \otimes B & C \otimes B & \cdots & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ C \otimes A^{\otimes^{N-1}} \otimes B & C \otimes A^{\otimes^{N-2}} \otimes B & \cdots & C \otimes B \end{bmatrix}$$

The next, we will derive an optimal input using the output prediction equation. Now let the desired reference signals be given as

$$R(k+1) = \begin{bmatrix} r(k+1) \\ r(k+2) \\ \vdots \\ r(k+N) \end{bmatrix}, r(k+i) = \begin{bmatrix} r_1(k+i) \\ r_2(k+i) \\ \vdots \\ r_q(k+i) \end{bmatrix}$$

then, it can be considered that the desired control inputs $U(k+1)$ for the given reference signals is the solution of the following equation.

$$R(k+1) = Cx(k) \oplus \mathrm{Du}(\mathrm{k}+1)$$

Theory in the max-plus algebra, the solution by solving the transformed linear equation given as

$$\mathrm{Du}(k+1) = R(k+1) \oplus \mathrm{Cx(k)}$$

This equation has the form a linear equation in the max-plus algebra. It implies that the desired input using $R(k+1) = Cx(k) \oplus \mathrm{Du}(\mathrm{k}+1)$ can be obtained by solving the linear equation. The solution from equation $\mathrm{Du}(k+1) = R(k+1) \oplus \mathrm{Cx(k)}$ is expressed by utilizing the greatest subsolution method as follows:

$$u(k+1) = \mathrm{D}^T \odot \{R(k+1) \oplus C\mathrm{x(k)}\}$$

The input to the system are determined by utilizing the Receding Horizon method. Namely, the first element of $u(k+1)$, in equation $u(k+1) = D^T \odot \{R(k+1) \oplus \mathrm{Cx(k)}\}$ is only applied to the controlled system as shown in the following way.

$$u(k+1) = [e_p, \epsilon_{pp}, \epsilon_{pp}, \ldots, \epsilon_{pp}]u(k)$$

The input after the $(k+1)$-th step are determined by equation
$u(k+1) = [e_p, \epsilon_{pp}, \epsilon_{pp}, \ldots, \epsilon_{pp}]u(k+1)$ as the event counter increases. Thus, a feedback control against changes of the internal state can be realized.

4. CONCLUSION

The main contribution of this paper is the research about bi-objective model predictive control with develop model bi-objective MPC for MPL systems, where is introduced maximal additive algebraic theory to developing bi-objective predictive control an optimal strategies. In design bi-objective model predictive control for MPL system, we acquire the explicit form of bi-objective model predictive control for MPL systems. Based on systems , we can solve the optimization problem to achieve optimal result with controls consider bi-objective function. Furthermore, bi-objective model predictive control to stabilize the MPL systems.

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