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Simulation of the Automatic Balancing Process of A Rotor, Rigidly Fixed in the Housing on Elastic Supports

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***Abstract* — The process of automatic balancing of an unbalanced rotor was simulated using four pendulums mounted in pairs on the shaft on both sides of the rotor. The rotor is rigidly fixed in the housing, which is mounted on elastic supports. Studies have shown that the pendulums partially or fully compensate the unbalance of the rotor within the range of its rotational frequencies. By selecting the stiffness characteristics of the housing supports and mass-and-inertia parameters of the housing, the rotor becomes a zone of stable operation of the auto-balancing device. It has been established that the degree of compensation for the rotor unbalance by pendulums of equal mass and length essentially depends on the relative position of the center of rotor masses, the center of the housing masses and the center of rigidity of the elastic supports of the housing and the distance between the pendulums in one pair.**

***Keywords* — rotor, housing, elastic supports, pendulum, automatic balancing, center of masses, stability zone.**

I. INTRODUCTION

1. One of the efficient ways to reduce the dynamic loading of rotors due to changes in the unbalance during operation is automatic balancing. As far as is known, auto-balancing devices are divided into two types: active and passive. Both of these types have their advantages and disadvantages. The advantage of active balancing systems is that they are highly precise and operate at any speed of rotation of the rotor. Their disadvantage is considerable complexity and cost, as well as low reliability [1], [2]. On the contrary, passive devices are simple, reliable and inexpensive. Their disadvantage is that they do not provide equilibrium at rotor speeds below critical speeds [3]. Most of the scientific works in this area is associated with the study of passive auto-balancers for an individual rotor [4-14], where the balancing process is related to the effect of self-centering. However, in other works [15], [16], [17], [18], studies have shown that additional zones of stable operation of auto-balancers can be provided by adding overbalances [15] or by choosing parameters for rotor supports [16], or due to the choice of stiffness characteristics of the supports of the housing in which the rotor is installed [17], [18]. In these cases, the zones of stable operation of the auto-balancer are not in the zone of supercritical rotor speeds, but in the superresonant zones of the attached mass or housing on elastic supports, in which the rotor is installed. It is also formulated by I.I. Blekhman [19] in the generalized principle of auto-balancing. The essence of this principle is that for a rotor with auto-balancer in the region of the rotor spinning frequencies above the highest frequency of free oscillations of a mechanical system, there is a tendency to weaken oscillations. In the area of rotational frequencies below the smallest frequency of free oscillations, there is a tendency to increased oscillations; and within the intermediate frequency ranges there may be intervals in which there is a tendency for either weakening of the oscillations or their amplification.

The purpose of these studies is to simulate the process of automatic balancing of the rotor installed in the housing in the superresonance zone of the housing on elastic supports.

II. EQUATIONS OF MOTION FOR A MODEL OF A ROTOR WITH PENDULUM AUTO-BALANCERS, RIGIDLY FIXED IN A HOUSING ON ELASTIC SUPPORTS

To study the process of formation and establishment of the auto-balancing process, let us use a dynamic model of a rotor with pendulum auto-balancers installed in a housing on elastic weightless supports, which is shown in Fig. 1. This model was used in [18] to develop a method of calculating the parameters and zones of stable operation of pendulum auto-balancers. In this work, it is shown that the stable operation of the auto-balancing device can be achieved by selecting the mass-and-inertia parameters of the housing and the stiffness properties of the housing supports, so that the operating rotor speed falls within the stability zone of the auto-balancing device. The model was a massive housing, fixed on a immovable base with the help of weightless elastic supports. A rigid rotor is installed in the housing in its own bearings.

On the rotor shaft, four pendulums of equal mass m and length l each are suspended in pairs on both sides of the rotor to compensate for dynamic unbalance. The motion of the model will be considered with respect to the fixed coordinate system *Oxyz* with the origin coinciding with the center of rotor masses in the static equilibrium position of the housing.

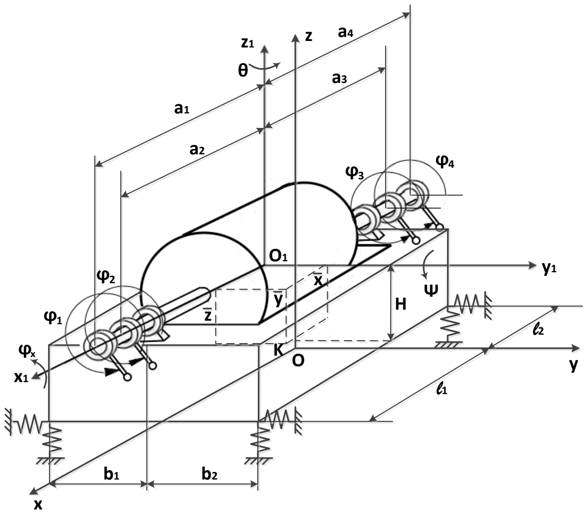

Fig. 1. Dynamic model of a rotor with pendulum auto-balancers, mounted in the housing on elastic supports

Since in most real structures there is virtually no housing movement along the rotor axis. Then to describe the rotor movement with the housing, we choose the following generalized coordinates: y, z are linear displacements of a point O_1. O_1 is the intersection of the rotor axis with the plane passing through its center of masses perpendicular to the axis of rotation. φ_x, θ, ψ are angular displacements of the housing together with the rotor around the axes x_1, y_1, z_1. Axes x_1, y_1, z_1 at the initial moment of time are parallel to the axes x, y, z. The position of the pendulums is determined by the angles φ_1, φ_2, φ_3, φ_4. The positive directions of reference for these angles are shown in Fig. 1.

While deriving the equations of motion of the model, we adopt the following asusmptions. First, we will assume that the resistance to movement of the model is of the nature of "viscous" friction, that is, proportional to the generalized rotor speeds, and the resistance to rotation of the pendulums is proportional to their relative speeds. Second, we suppose that the main axes of inertia of the housing are parallel to the axes x_1, y_1, z_1, which are in turn parallel to the axes x, y, z. Third, we will assume that the housing supports are isotropic, and the engine has sufficient power to speed up the rotor with a constant angular acceleration ε. We shall accept the following notation. M_p, A, C are the mass, equatorial and polar moments of inertia of the rotor. M_k, J_x, J_y, J_z are the mass and moments of inertia of the housing. e, δ, $\bar{\varepsilon}$ are unbalance parameters; c_y, c_x are the stiffness coefficients of the supports in the vertical and horizontal direction. $\bar{x}$, $\bar{y}$, $\bar{z}$ are constant coordinates of the center of masses of the housing in the reference system O_1, x_1, y_1, z_1; a_1, a_2, a_3, a_4 are distances from the center of masses of the rotor to the pendulum supports on the rotor shaft. b_1, b_2, l_1, l_2 are geometric dimensions of the location of the supports (Fig. 1). β_0 is the coefficient of resistance to the relative rotation of the pendulums.

Setting up expressions for the kinetic and potential energies, the Rayleigh function and the generalized force of resistance to the relative rotation of the pendulums and applying the Lagrange equation of the second kind, we obtain the equations of motion of the model during speed-up and in steady motion in the matrix form:

$$[A^*]\{\ddot{q}\} + [B^*]\{\dot{q}\} + [K^*]\{q\} = \{F\}, \qquad (1)$$

where $\{q\} = \{y, z, \theta, \psi, \varphi_x, \varphi_1, \varphi_2, \varphi_3, \varphi_4\}^T$.

$$[A^*] = \begin{bmatrix} [A] & [A_1] \\ [A_1]^T & [A_2] \end{bmatrix}$$

$$[A] = \begin{bmatrix} M^* & 0 & m_1 & 0 & -m_2 \\ 0 & M^* & 0 & m_1 & m_3 \\ m_1 & 0 & J_z^* & m_4 & m_5 \\ 0 & m_1 & m_4 & J_4^* & m_6 \\ -m_2 & m_3 & m_5 & m_6 & J_x \end{bmatrix}$$

$$[A_1] = ml \begin{bmatrix} -\sin\varphi_1; & -\sin\varphi_2; & -\sin\varphi_3; & -\sin\varphi_4 \\ \cos\varphi_1; & \cos\varphi_2; & \cos\varphi_3; & \cos\varphi_4 \\ -a_1\sin\varphi_1; & -a_2\sin\varphi_2; & a_{31}\sin\varphi_3; & a_4\sin\varphi_4 \\ a_1\cos\varphi_1; & a_2\cos\varphi_2; & -a_3\cos\varphi_3; & -a_4\cos\varphi_4 \end{bmatrix}$$

$$[A_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[B^*] = \begin{bmatrix} [B] & [B_1] \\ [B_1]^T & [B_2] \end{bmatrix}$$

$[B_1]$; $[B_2]$ are zero matrices with dimension 5x4 and 4x4, respectively.

$$\{F\} = \left\{ \begin{matrix} M_p e\dot{\varphi}^2\cos\varphi + M_p e\ddot{\varphi}^2\sin\varphi + ml\sum_{k=1}^{4}\dot{\varphi}_k^{\,2}\cos\varphi_k; \\ M_p e\dot{\varphi}^2\sin\varphi - M_p e\ddot{\varphi}^2\cos\varphi + ml\sum_{k=1}^{4}\dot{\varphi}_k^{\,2}\sin\varphi_k; \\ (A-C)\delta\dot{\varphi}^2\cos(\varphi-\varepsilon) + (A-C)\delta\ddot{\varphi}\sin(\varphi-\varepsilon) - C\dot{\varphi}\dot{\psi} - \\ - C\dot{\varphi}\dot{\psi} + ml\sum_{k=1}^{n} a_k\sigma_k\,\dot{\varphi}_k\cos\varphi_k; \\ (A-C)\delta\ddot{\varphi}\sin(\varphi-\varepsilon) + A\delta\,\ddot{\varphi}\cos(\varphi-\varepsilon) - C\dot{\varphi}\dot{\theta} + \\ + ml\sum_{k=1}^{n} a_k\sigma_k\,\dot{\varphi}_k\sin\dot{\varphi}_k; \\ 0; \\ k_1(\dot{\varphi}-\dot{\varphi}_1) - mg\cos\varphi_1; \\ k_1(\dot{\varphi}-\dot{\varphi}_2) - mg\cos\varphi_2; \\ k_1(\dot{\varphi}-\dot{\varphi}_3) - mg\cos\varphi_3; \\ k_1(\dot{\varphi}-\dot{\varphi}_4) - mg\cos\varphi_4; \end{matrix} \right\}$$

where $\delta_k = 1$ with k = 1, 2; $\delta_k = -1$ with k = 3, 4

$$[B] = \begin{bmatrix} \mu_1 & 0 & \mu_6 & 0 & \mu_8 \\ 0 & \mu_2 & 0 & \mu_7 & 0 \\ \mu_6 & 0 & \mu_3 & 0 & \mu_9 \\ 0 & \mu_7 & 0 & \mu_4 & 0 \\ \mu_8 & 0 & \mu_9 & 0 & \mu_5 \end{bmatrix}$$

$$[K^*] = \begin{bmatrix} [K] & [K_1] \\ [K_1]^T & [K_2] \end{bmatrix};$$

$$[K] = \begin{bmatrix} c_1 & 0 & c_2 & 0 & c_1 H \\ 0 & c_3 & 0 & c_4 & c_5 \\ c_2 & 0 & c_6 & 0 & c_2 H \\ 0 & c_4 & 0 & c_7 & c_8 \\ c_1 H & c_5 & c_2 H & c_8 & c_9 \end{bmatrix}$$

$[K_1]$; $[K_2]$ are zero matrices with dimension 5x4 and 4x4, respectively.

While compiling matrices $[A]$; $[K]$ the following notation was used:

$$M^* = M_p + M_k + 4m \; ; \qquad J_x^* = C + M_k(\bar{y}^2 + \bar{z}^2) + J_x$$

$$J_z^* = A + M_k(\bar{x}^2 + \bar{y}^2) + J_z + m\sum_{k=1}^{4} a_k^2$$

$$J_y^* = A + M_k(\bar{x}^2 + \bar{z}^2) + J_y + m\sum_{k=1}^{4} a_k^2$$

$m_1 = \bar{x} \cdot M_к$; $m_2 = -\bar{z} \cdot M_к$; $m_3 = \bar{y} \cdot M_к$; $m_4 = \bar{y} \cdot \bar{z} \cdot M_к$;
$m_5 = -\bar{x} \cdot \bar{z} \cdot M_к$; $m_6 = \bar{x} \cdot \bar{y} \cdot M_к$; $c_1 = 4\, c_y$;
$c_2 = 2\, c_y(l_1 - l_2)$; $c_3 = 4\, c_z$; $c_4 = 2\, c_z(l_1 - l_2)$;
$c_5 = 2\, c_z(b_2 - b_1)$; $c_6 = 2\, c_y(l_1^2 + l_2^2)$; $c_7 = 2\, c_z(l_1^2 + l_2^2)$;
$c_8 = (l_1 b_2 - l_2 b_2 + + l_2 b_1 - l_1 b_1)$;
$c_9 = 4\, c_y H^2 + 2\, c_z(b_1^2 + b_2^2)$.

In {F} - φ, $\dot{\varphi}$, $\ddot{\varphi}$ are respectively the angle of rotation, angular velocity and angular acceleration of the rotor; in [B] - μ_1, $\mu_2 \dots \dots \mu_9$ are damping coefficients.

III. SIMULATION OF THE PROCESS OF AUTO-BALANCING OF THE ROTOR, RIGIDLY FIXED IN A HOUSING ON ELASTIC SUPPORTS

The numerical integration of the system of differential equations (2) was carried out by the fourth-order Runge – Kutta method, but the calculation algorithm included the inversion at each step of integrating the matrix [A]. The main results of the simulation of the auto-balancing process of an unbalanced rotor rigidly fixed in the housing, which in turn is mounted on an immovable base with the help of four vertical and four horizontal elastic supports, are shown in Fig. 2-5.

Fig. 2 shows the laws of speed-up of the rotor (1), the first pair of pendulums (2) and the second pair of pendulums (3). The first pair of pendulums consists of the first and second pendulums, the second pair consists of the third and fourth pendulums. It can be seen that the laws of speed-up of pendulums in one pair are the same. Initially, the angular velocities of the pendulums lag behind the angular velocity of the rotor, and then, when the angular velocities of the pendulums become equal to the angular velocity of the rotor, the pendulums begin to "search" for their place. Here the angular velocities of the pendulums differ from the angular velocities of the rotor, and then, when the pendulums find their place, the angular velocities become the same.

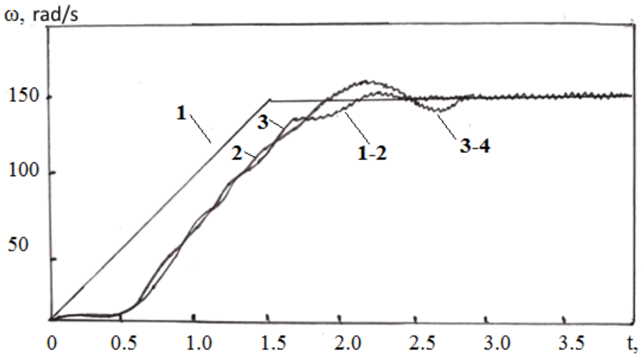

Fig. 2. The laws of variation of the angular velocity of the rotor (1) and two pairs of pendulums: of the first and second in one pair (2), of the third and fourth in the second pair (3)

Fig. 3 shows how the position of the pendulums changes during the balancing process with respect to the unbalance vector in the first pair (a) and in the second pair (b). It is seen that the pendulums in the first pair were established opposite to the unbalance vector, but they almost did not move apart $\alpha_1 = 182{,}7^\circ$; $\alpha_2 == 178{,}5^\circ$; The pendulums in the second pair were not only established opposite to the unbalance vector, but also moved apart: $\alpha_3 = 131{,}4^\circ$; $\alpha_4 = 228{,}7^\circ$.

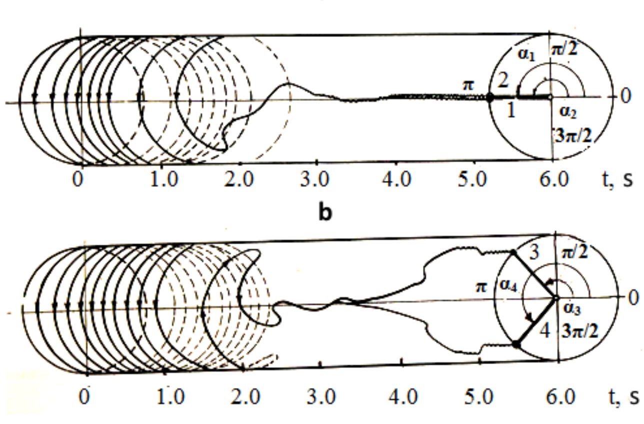

Fig. 3. The trajectories of the pendulums with respect to the unbalance vector: a) of the first pair of pendulums (first pendulum and second pendulum); b) of the second pair of pendulums (third pendulum and fourth pendulum)

However, these values differ from those calculated by formulas 8 of [18]. Based only on the parameters of unbalance and mass-and-inertia parameters of the rotor and pendulums, the values of the installation angles of the pendulums should be: $\alpha_1 = \alpha_3 == 137^\circ$; $\alpha_2 = \alpha_4 = 223^\circ$. The reason for this discrepancy is the location of the center of masses of the rotor, the center of masses of the housing and the center of rigidity of the housing supports relative to each other. Because of this discrepancy in the studied model, the amplitudes of oscillations of the pendulum supports in one pair differ tenfold from those of the other pair, the pendulums in the first pair are not moved apart, and only partial balancing occurs. It should be noted that this occurs with the pendulums of the same mass and length. However, even in this case, the amplitudes of oscillations of the center of masses of the rotor, and hence the loads in the rotor supports with auto-balancer (Fig. 4) are significantly smaller than those of a rotor without auto-balancer (Fig. 5). For example, for the model under study, the amplitude of oscillation of the center of masses of the rotor without auto-balancer A = 0.22 mm, and for a rotor with auto-balancer A = 0.07 mm. In the same figures one can see that the amplitudes of oscillations of the center of masses of the rotor and the load in the rotor supports with auto-balancer, when going through the resonant zones, significantly exceed the amplitudes of the oscillations and the load in the rotor supports without auto-balancer.

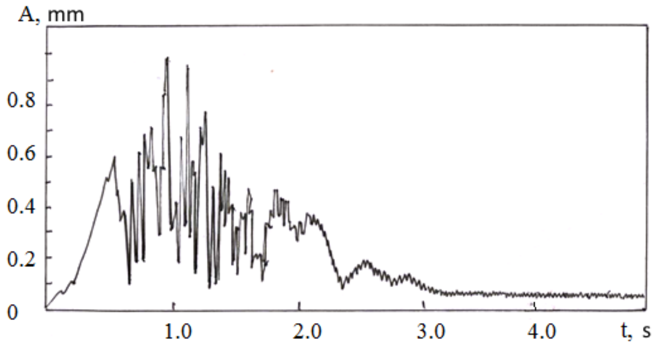

Fig. 4. Oscillations of the center of masses of the rotor with auto-balancer

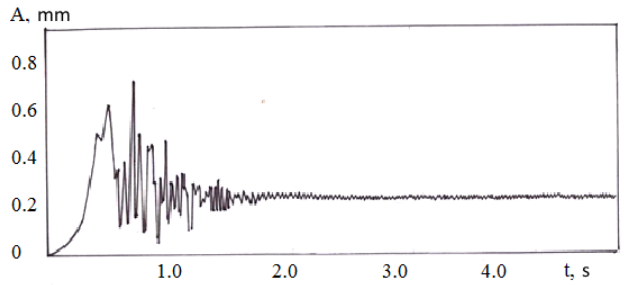

Fig. 5. Oscillations of the center of masses of the rotor without auto-balancers

Separate studies of the influence of geometrical dimensions $\bar{x}$, $\bar{y}, \bar{z}$ on the quality of balancing showed that changes $\bar{y}, \bar{z}$ have little effect on the rotor oscillation amplitudes, and if $\bar{x}$ is within the range of $0 < \bar{x} < 0.09$ m, then the amplitudes of oscillations and loads in the rotor supports can be 25-30 times smaller than that of a rotor without auto-balancer. The amplitudes of oscillations and loads in the supports of the rotor with auto-balancer are also affected by the distance between the pendulum supports in one pair. For example, if the distance between the pendulum supports in one pair is equal to 1 cm, then the balancing error will be 10%.

IV. CONCLUSIONS

The process of automatic balancing of an unbalanced rotor was simulated using four pendulums mounted in pairs on the shaft on both sides of the rotor. The rotor is rigidly fixed in the housing, which, in turn, is mounted on elastic supports. The study has shown that, in fact, the pendulums partially or fully compensate the rotor unbalance within the range of its rotational frequencies, which, due to selecting the stiffness characteristics of the housing supports and mass-and-inertia parameters of the housing according to a method in [18], becomes a zone of stable operation of the auto-balancing device.

The degree of the rotor unbalance compensation in the housing with the help of pendulums of the same mass and length is significantly influenced by the relative position of the center of masses of the rotor, the center of masses of the housing, the center of rigidity of the housing supports and the distance between the pendulums in one pair. Since the transition of the unbalanced rotor with auto-balancer through the resonant zones is accompanied by high oscillation amplitudes and high loads in the rotor bearings, it becomes expedient to develop locking devices so that automatic balancing begins after the rotor speed-up.

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