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DISTRIBUTION OF THE WEALTH OF THE RICHEST PERSONS IN THE WORLD

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Abstract

The aim of this paper is to examine the probability distribution of wealth of the richest persons in the world based on estimates from the CEOWORLD magazine's rich list for March 2019. Since one can safely assume that there are a tiny number of people out of the whole world's population in this list, we basically deal with the very right tail of the wealth distribution, which should according to the Pickands-Balkema-de Haan theorem follow a generalized Pareto distribution. We discuss in this paper not only different estimates of this distribution with an emphasis on the shape parameter per se but also their behavior throughout bootstrap samples. Among the main findings is the observation based on the maximum to sum plot and parametric estimates that there is high probability of infinite variance. This could have a serious impact on estimates of inequality measures. The whole distribution follows nearly a Pareto distribution, whereas the very right tail is closer to an exponential. The bootstrap technique confirms that maximum likelihood estimates are almost normally distributed, but they overestimate variance. Estimates via the method of L-moments diverge from the normal distribution. The correlation of the parameter estimates is moderately negative, which is demonstrated in a simultaneous confidence region.

Key words

CEOWORLD magazine's Rich List, Pickands-Balkema-de Haan theorem, generalized Pareto distribution, parameter estimates, bootstrap

JEL classification

C13, C15

1. Introduction

Wealth and income distributions are widely debated and researched topic both in scientific and popular literature. They are implicated in discussions of general wealth of populations as well as in discussions of economic equality and fairness with perceived need to re-distribute. One of the most popular measurements of the wealth and income distributions is Gini coefficient, other measures are used in evaluating the right tail of the income or wealth distributions. Many of the currently used measures of inequality have estimates based on assumption of the finite variance and don't fare well under heavy-tailed distributions (Cowell and Flachaire, 2007, Fontanari et al., 2018), so the discussion whether income distribution has finite variance is of high importance in economics.

The right tail is a vague term describing the part of the distribution higher than some specified threshold. In the context of the wealth distribution, the right tail has the meaning of the distribution of the wealth among sufficiently rich people. Since wealth distribution is usually heavy-tailed (for the definition of heavy-tailed distribution see e.g. Foss et al, 2013), its right tail has properties important for the whole distribution. Among most popular heavy-tailed distributions used in modelling of income or wealth distributions are log-normal and Pareto distributions. There is however difference between log-normal and Pareto distribution in the speed of decline of the density, which was recently discussed in connection with the wealth distribution e.g. in Jaguelski et al. (2017) or Campolieti (2018).

The heaviness of the distribution can be estimated as a tail index via several methods, some specific, e.g. Pickands or Hill estimators. More general method is an estimate of specific distribution and calculating tail index from this.

In this paper we have an aim to take a different angle of view – to start with the description of the basics of extreme value theory (EVT) and move onto its relevance to the discussion of the distribution of the wealth of the richest people in the world. Then we continue with estimating this distribution and inference within two methods used for the estimates – maximum likelihood and L-moments.

2. Data and software

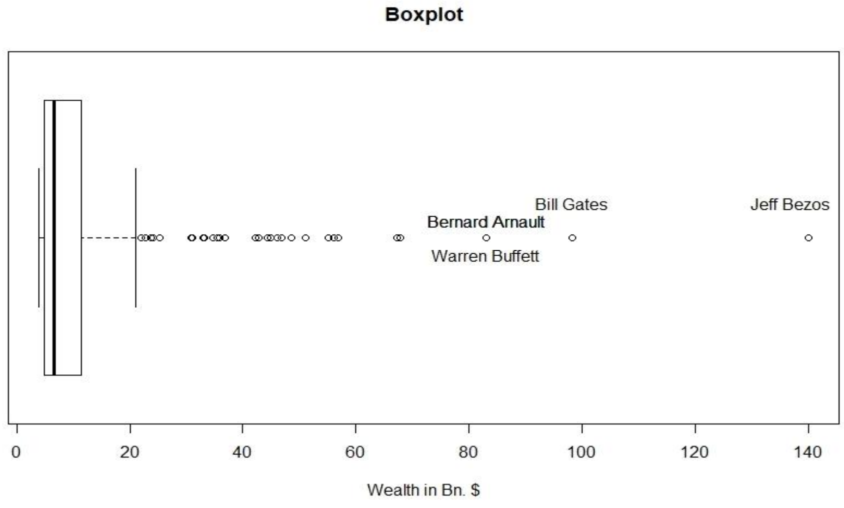
We use data obtained from the CEO World magazine's Rich List[1], that contains estimated net worth of wealth of the 500 richest persons in the world. For the sake of simplicity, we truncated the data at 4 billion dollars, so in other words we use the list of people having (estimated) more than 4 billion dollars – there are 483 such persons in the dataset.

Table 1: Descriptive statistics of the wealth of 483 richest persons in the world

Statistics	Value in billions of $
Minimum	4.01
First quartile	4.88
Median	6.57
Third Quartile	11.40
Maximum	140.00
Mean	10.75
Variance	162.30

Source: the authors.

Figure 1: Boxplot of the wealth of 483 richest persons in the world



Source: the authors.

Table 1 contains basic descriptive statistics and Figure 1 shows the distribution of the wealth as a boxplot. The four richest persons are pointed out. Summary statistics and figures point out toward heavy-tailed distribution as expected.

Software used throughout the paper is R (R Core Team, 2018) with packages extRemes (Gilleland and Katz, 2016) for parameter estimates and ellipse (Murdoch and Chow, 2018) for simultaneous confidence region estimate.

[1] Available at https://ceoworld.biz/2018/05/30/rich-list/ (accessed on 2019-03-06).

3. Extreme value theory

EVT is a branch of mathematic statistics dealing with the probability distribution of the extremes. There are two different, even though connected, approaches based on two different theorems. EVT is used in many different fields such as insurance, hydrology, meteorology or finance.

First theorem used in EVT is Fisher-Tippet-Gnedenko theorem (Gnedenko, 1943) basically stating, that if the distribution of maximas of independent identically distributed random variables (e.g. maximas in random samples from the same distribution) converges, it converges to the generalized extreme value distribution (GEVD), that can be differed to Fréchet, Weibull or Gumbel distributions.

Second theorem used in EVT is Pickands-Balkema-de Haan theorem (de Haan and Balkema, 1974; Pickands, 1975) stating following:

"*Let X1, X2, … be a sequence of independent and identically distributed random variables with distribution function F. Random variables for which X > μ has excess distribution function*

$$F_\mu(y) = P(X - \mu \le y \,|\, X > \mu)\,, \tag{1}$$

where X is a random variable, μ is given threshold, y = x – μ are excesses. Then

$$F_\mu(y) \xrightarrow{\mu\to\infty} G_{\xi,\sigma}(y)\,, \tag{2}$$

where Gζ,σ(y) is cumulative distribution function of generalized Pareto distribution (GPD) written in formula 3.

$$G_{\mu,\xi,\sigma}(x) = \begin{cases} 1-\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi} & \text{for } \xi \ne 0\ , \\ 1-\exp\left(-\dfrac{x-\mu}{\sigma}\right) & \text{for } \xi = 0\ . \end{cases} \tag{3}$$

The three parameters of the GPD are threshold (sometimes referred to as a location) parameter μ, scale parameter σ and shape parameter ζ. Shape parameter basically decide whether probability density decreases slower or faster than exponential distribution and if its value is 0, then the right tail is exponentially distributed. When ζ > 0, then the right tail decreases faster than the exponential distribution and it can be classified as heavy-tailed. With ζ > 0 and μ = σ/ζ, the GPD is equivalent to Pareto distribution with parametrization $x_m = \sigma/\zeta$ and α = 1/ζ with following cumulative density function

$$F_{x_m,\alpha} = 1-\left(\frac{x_m}{x}\right)^{\alpha}. \tag{4}$$

More generally 1/ζ can be viewed as an estimate of tail index mentioned above.

GPD has finite expectation if ζ < 1, finite variance if ζ < 0.5, finite skewness if ζ < 1/3 and finite kurtosis if ζ < 1/4.

There is a more general relation between underlying distribution, its maxima distributions per Fisher-Tippet-Gnedenko theorem and its right tail behavior expressed as a GPD. These are expressed as domains of attraction of the GEVD and some of well-known distributions that have specific domains of attraction are mentioned in Table 2.

Table 2: Domains of attraction for some distributions

Situation	Domain of attraction	Tail behavior	Distribution examples
$\zeta > 0$	Fréchet	Polynomial, possibly Pareto	Pareto, Cauchy, Student's
$\zeta < 0$	Weibull	Finite end-point	Uniform, Beta
$\zeta = 0$	Gumbel	Exponential	Normal, log-normal, exponential, gamma

Source: the authors.

The "true" distribution of the wealth of the richest persons is usually discussed to be truncated lognormal (Campolieti, 2018) or Paretian (Jaguelski et al., 2017; or Capehart, 2014) or it can be modelled by different functions, e.g. by Lorenz curve (Wang and You, 2016).

Starting from the EVT and more specifically Pickands-Balkema-de Haan theorem we can view wealth of an individual person following unknown underlying probability distribution and the world population as a sample from this distribution. Then following this logic, we can view the very right tail of the wealth distribution as an excess distribution that will follow GPD if the threshold is sufficiently high. We think that having 483 richest persons from the world's population exceeding 7.5 billion people and the threshold of 4 billion dollars is sufficiently high and this approach is justifiable not only on empirical but also on theoretical ground.

3.1. Estimates of GPD

When estimating GPD, researchers usually stand before several possible issues. First of them is selection of an appropriate estimation method, method of inference and within these methods, selection of the sufficiently high threshold. There are methods specific to the GPD, such as de Haan method (Simiu and Heckert, 1996) and there are more general approaches, e.g. maximum likelihood estimate (MLE), method of probability weighted moments (PWM) or method of L-moments (MLM).

In this paper we chose MLE and MLM – first because of its well-known nature and asymptotic normality of the parameter estimates, where we use inverse Hessian matrix as an estimate of covariance matrix of parameter estimates. If $\zeta > 0.5$, the MLE produces unbiased estimates, but its asymptotic properties are questionable (due to infinite variance). Second one because of its robustness and possibility to obtain unbiased estimates under the condition that the first moment is finite, that is if $\zeta < 1$.

Because as we will see further in chapter 4 the point estimates of ζ are higher than 0.5 but lower than 1, we use bootstrapping methods to show the properties of the parameter estimates and its 95 % confidence intervals and compare them to the estimates obtained via normal approximation within MLE. We also discus correlation of the parameter estimates within MLE and even within MLM. We use 200 000 bootstrap samples.

Another issue is usually threshold selection. As for this we follow standard graphical procedure, that is evaluating the figure plotting mean over threshold against the threshold and parameter estimates against the threshold. We discuss some possible implications but use the original 4 billion dollars as a threshold, nevertheless.

4. Results

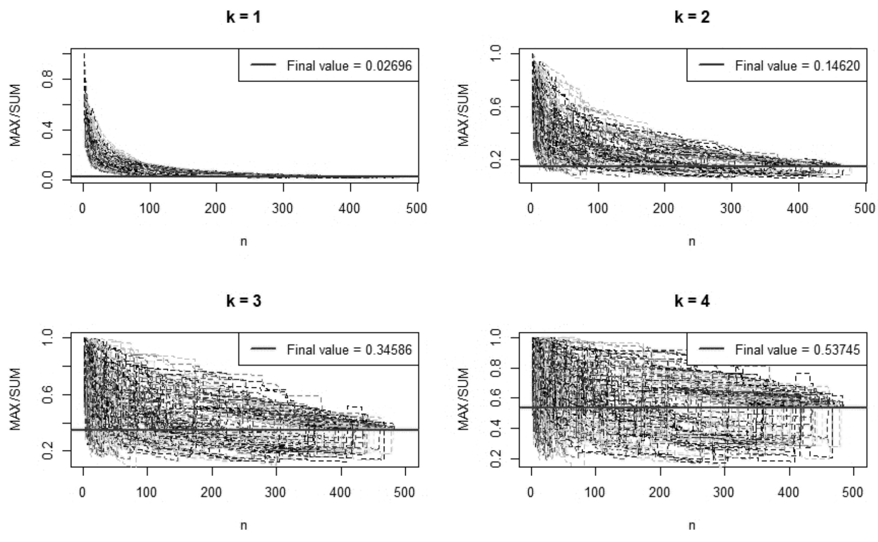
4.1. Maximum to Sum plot

As a first step in evaluating distribution of the top wealth we start with Maximum to Sum plot (Cirillo and Taleb, 2016) as a tool to evaluate existence of moments of the underlying distributions. For a sequence $(X_1, X_2, \ldots, X_n)$ of nonnegative independent identically distributed random variables, if for $k = 1, 2, 3,\ldots$, holds $E(X^k) < \infty$, then follows

$$R_n^k = M_n^k / S_n^k \xrightarrow{n\to\infty} 0,$$
$$\text{where } S_n^k = \sum_{i=1}^{n} X_i^k \text{ and } M_n^k = \max\left(X_1^k, ..., X_n^k\right). \quad (5)$$

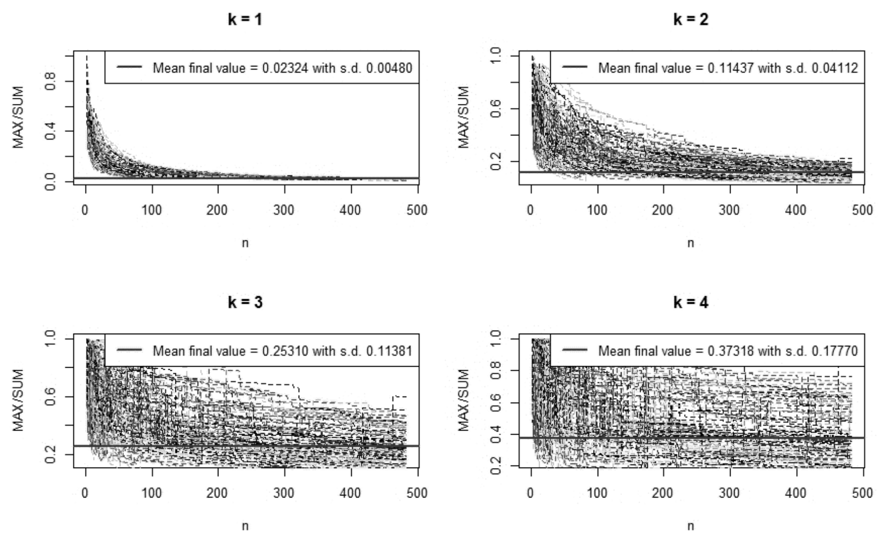
Figure 2 shows 100 reshuffled paths of the ratio of maximum to sum from the original data for first 4 moments – since the path is dependent on the order in the sample and since there is no clear way to sort them (like e.g. in time series data) we used this approach to visualize the data. Figure 3 shows 100 possible paths based on 100 bootstrap samples. In our opinion it is clear, that the ratio converges to 0 for k = 1 and does not converge to 0 for k = 3 and 4. As for the second moment, the ratio seems to converge rather slowly and so the second moment may not be finite. This suggests, that ζ is higher than 1/3 as third moment is not finite and possibly higher than 0.5 as second moment is possibly not finite. The bootstrap method has its limit here in the fact that all moments are finite in the sample and hence the mean final values in Figure 3 are substantially lower than final values in Figure 2.

Figure 2: Maximum to Sum plot for 100 different paths



Source: the authors.

Figure 3: Maximum to Sum plot for 100 bootstrap distributions



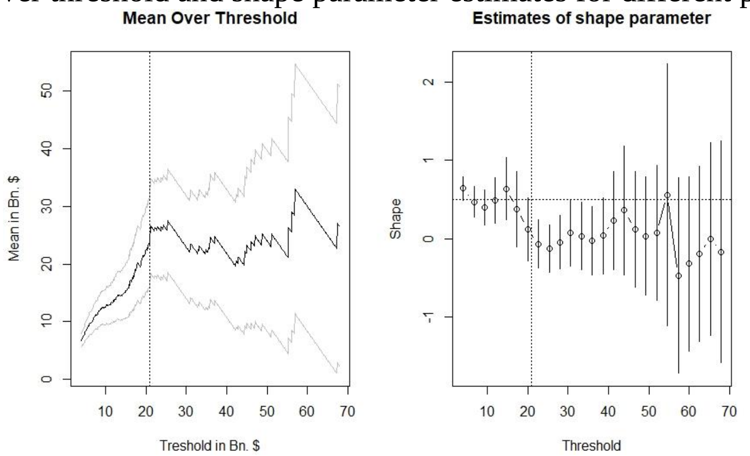
Source: the authors.

4.2. Relation of mean and parameter estimates to selected threshold

Figure 4 shows mean of the observations over chosen threshold plotted against thresholds from 4 to 68 billion along with 95 % confidence intervals for this mean, so called sample mean excess plot. In theory (see e.g. Tanaka and Takara, 2002) the threshold should be selected at the interval, where there is approximately stable value of the mean. Given this figure the threshold would be selected at 21 billion (dotted line) leaving 34 highest observations as the basis for the right tail estimate.

Similarly, threshold is often being determined by plotting estimated parameters, especially of shape, against the threshold, with similar recommendation – looking for the region of stable parameter estimate. Figure 4 then shows estimated shape parameter against threshold with dotted line marking threshold of 21 billion. Estimate for the shape parameter is stable up to approximately 15 billion mark and then decreases, so this area also passes subjective nature of this approach with 4 billion being acceptable value. Parameters for this graphical tool were estimated via MLE.

Figure 4: Mean over threshold and shape parameter estimates for different possible thresholds



Source: the authors.

Shape parameter estimate is approximately 0 at the threshold of 21 billion suggesting exponential distribution as an appropriate distribution for this very right tail, whereas it is estimated to be more than 0.6 for the threshold of 4 billion. All the parameter estimates via MLE and MLM are in Table 3. Higher threshold naturally leads to fewer observations and thus higher variance of parameter estimates in MLE. Correlation of the parameter estimates in MLE is approximately –0.5960.

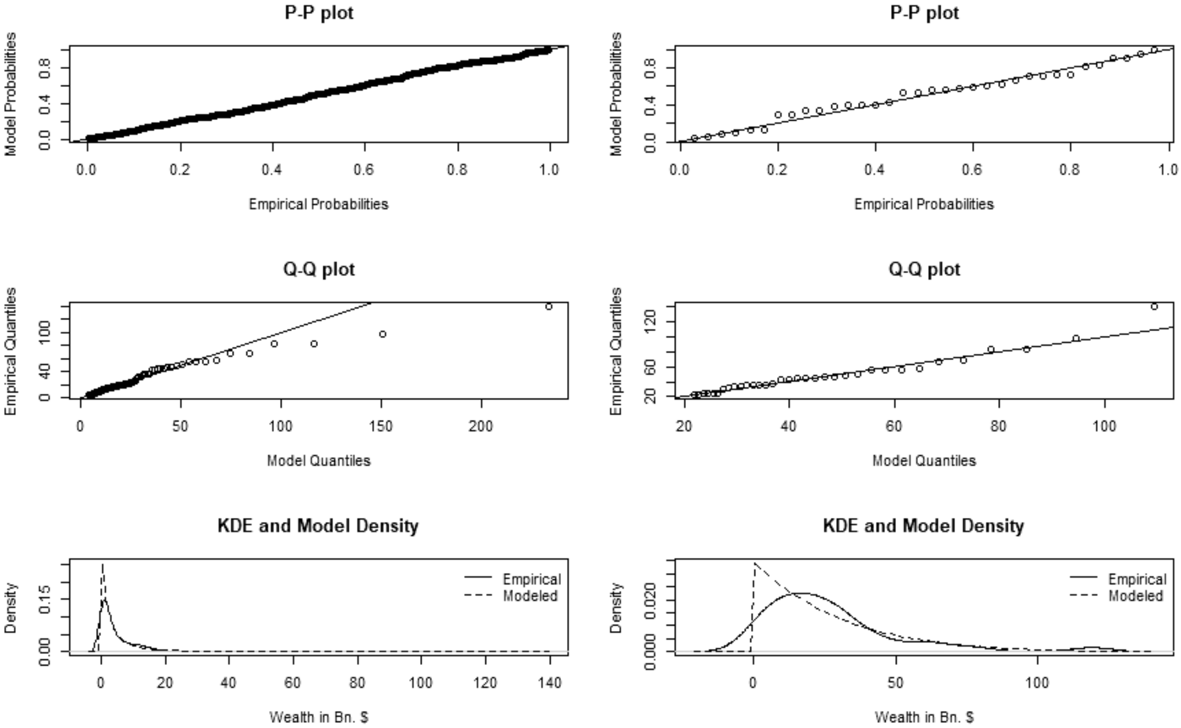
Table 3: Parameter estimates

Method	μ	ζ	s.d. of ζ	σ	s.d. of σ
MLE	4	0.6337	0.0786	2.9606	0.2550
MLE	21	–0.0907	0.1479	29.1673	6.5891
MLM	4	0.5244	NA	3.2110	NA
MLM	21	–0.1629	NA	31.1217	NA

Source: the authors.

Figure 5 contains P-P plots, Q-Q plots and model estimated densities plotted with kernel density estimates. The assessment shows that both models estimate observations well except for the highest values in the model with threshold of 4 billion dollars. Highest observation was estimated to be almost 250 billion whereas true value is 136 billion (140 minus threshold 4, i.e. plots start at 0 = threshold). Under scrutiny this overestimation can be seen in Figure 4 as well. All this information suggests that whereas majority of the observations in the right tail follow approximately Pareto distribution (since 2.9606/0.6337 = 4.67, close to threshold 4), the very right part is closer to exponential distribution.

Figure 5: Evaluation of the model quality (threshold of 4 billion $ on the left side)



Source: the authors.

4.3. Bootstrap estimates

To evaluate MLE we can use their asymptotic normality and estimated covariance matrix, that yields estimate of variance of the parameter estimates. However, since the underlying distribution probably does not have finite variance, we decided to confront these inductions with 200 000 bootstrap samples and find out distribution and several other information from these. Moreover, to make same basic induction about MLM estimates we used the same technique. Sampling was done from the original data (denoted as $^{(1)}$ in the tables) and then from the estimated models (denoted as $^{(2)}$ in the tables). Since data have finite variance and models do not, we were also looking for the difference in the estimated characteristics.

Table 4 shows original estimates with those from bootstrap samples. Estimate of the shape parameter seems to be slightly overestimated and scale parameter underestimated, which seems logical given their negative correlation. Standard deviations seem slightly overestimated whereas correlation seems underestimated (overestimated in absolute value) in comparison with the second bootstrap procedure (bootstrapping from the model).

Table 4: Estimates based on bootstrapping for MLE

Parameter	Estimate	Estimated 95% C.I.	B.mean est.$^{(1)}$	B. 95% C.I.$^{(1)}$	B. mean est.$^{(2)}$	B. 95% C.I.$^{(2)}$
ζ	0.6337	(0.480; 0.788)	0.6304	(0.494; 0.787)	0.6296	(0.485; 0.777)
σ	2.9606	(2.461; 3.460)	2.9770	(2.485; 3.537)	2.9775	(2.524; 3.486)
s.d.(ζ)	0.0786	NA	0.0785	(0.071; 0.086)	0.0743	(0.067; 0.082)
s.d.(σ)	0.2550	NA	0.2560	(0.217; 0.300)	0.2447	(0.210; 0.283)
Corr.	–0.5960	NA	–0.5969	(–0.640; –0.556)	–0.5553	(–0.606; –0.507)

Source: the authors.

Table 5: Estimates based on bootstrapping for MLM

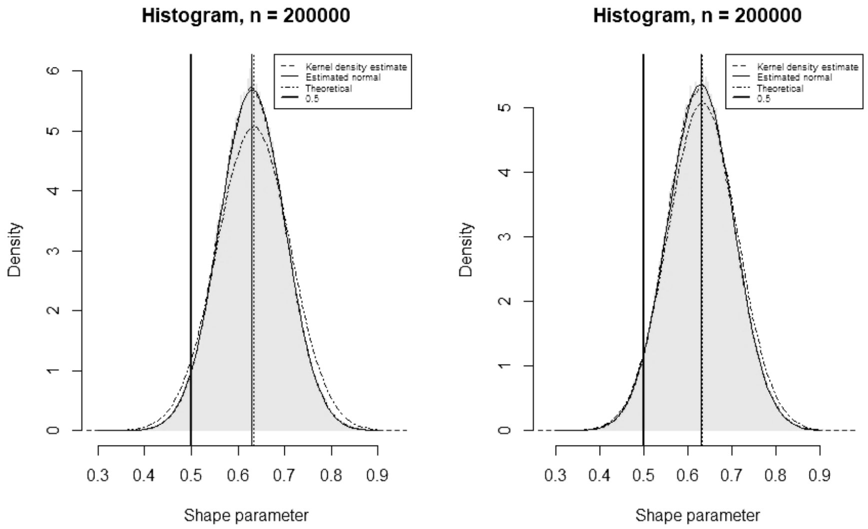
Parameter	Estimate	Estimated 95% C.I.	B.mean est.(1)	B. 95% C.I.(1)	B. mean est.(2)	B. 95% C.I.(2)
ζ	0.5244	NA	0.5184	(0.435; 0.591)	0.5076	(0.374; 0.680)
σ	3.2110	NA	3.2390	(2.767; 3.769)	3.2479	(2.728; 3.788)

Source: the authors.

Table 5 contains estimates of parameters obtained via MLM and calculated bootstrap confidence intervals both from the data and from the model. Shape parameter seems to be slightly overestimated, especially when compared to the bootstrap samples from the model, whereas opposite seems to be true for the scale parameter – the opposite of what have been seen in the MLE. Accuracy of both methods (MLE and MLM) as measured by the width of the confidence intervals seems to be comparable. Only one outstanding exception is in the confidence interval for shape parameter obtained from bootstrap samples from data – interval for the MLM estimate seems narrow in comparison to others, but also to the one obtained from the bootstrap samples from the MLM model. This difference can be partially due to the difference in variance in the sample (finite) and in the model (infinite).

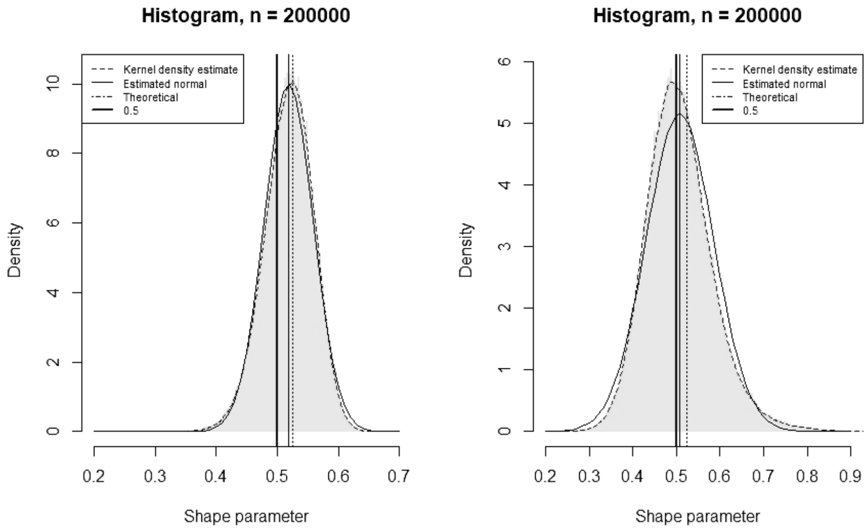
Histograms for the estimates of shape parameter based on 200 000 bootstrap samples are in Figure 6 for the MLE and in Figure 7 for the MLM. MLE estimates follow normal distribution well but have distribution with lower variance than estimated by the original model – the higher difference is seen in the bootstrap from the original sample than in the bootstrap from the model (s.d. 0.0700 and 0.0745 vs. 0.0786). Despite this difference estimated confidence intervals are similar as seen in Table 4. In MLM estimates in the bootstrap samples from the original sample we can see there is slight divergence from the normality – estimated normal distribution is moved to the left when compared to the kernel density estimate. In the estimates from the model the divergence is much more pronounced with higher modus and higher variance for normal estimate than for bootstrap estimate (s.d. 0.0399 vs. 0.0775 in the bootstrap from sample than from model). This difference is also visible in the estimated confidence intervals in Table 5.

Figure 6: Histogram of bootstrap shape parameter estimates, from sample and model, MLE



Source: the authors.

Figure 7: Histogram of bootstrap shape parameter estimates, from sample and model, MLM



Source: the authors.

Maximum likelihood estimates suggest, that the distribution of top incomes does not have finite variance whereas estimates done via method of L-moments do not provide clear evidence on any of the possibilities.

Table 6 holds correlation estimates obtained through different methods. There is original estimate from the MLE model, then there are means of correlations of 200 000 bootstrap samples and following are inter-sample correlations – correlations calculated in the 200 000 bootstrap estimates for both estimation methods. With the one exception correlations are in the range from –0.55 to –0.62, that suggests low to moderate negative linear relationship between estimates of the parameters.

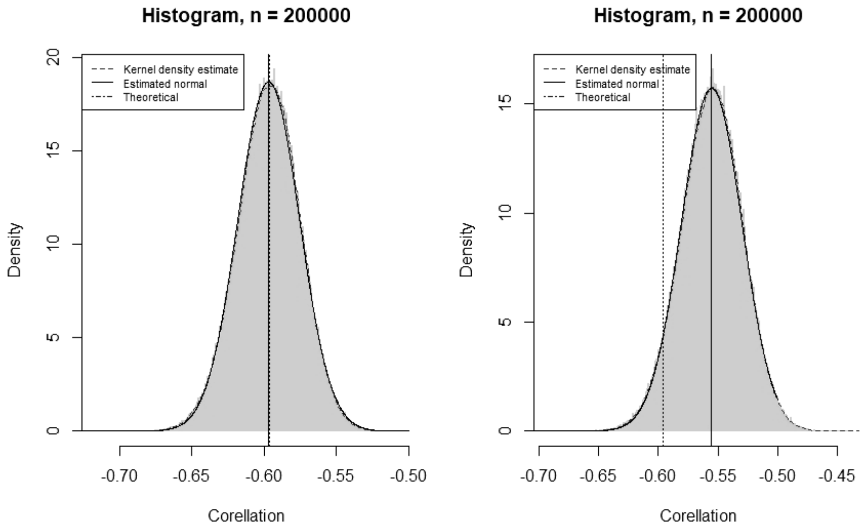
Table 6: Correlations

Source	MLE
MLE	–0.5960
Mean MLE bootstrap (1)	–0.5969
Inter-sample MLE bootstrap (1)	–0.6184
Mean MLE bootstrap (2)	–0.5553
Inter-sample MLE bootstrap (2)	–0.5545
Inter-sample MLM bootstrap (1)	–0.4390
Inter-sample MLM bootstrap (2)	–0.6191

Source: the authors.

Figure 8 contains estimated correlations from the bootstraps from the sample and from the model respectively. They follow normal distribution well. The difference in means from these two bootstrapping methods as seen in Table 6 (rows 2 and 4) are clearly visible.

Figure 8: Histogram of estimated correlation of parameters, from sample and model, MLE



Source: the authors.

4.4. Simultaneous regions of the parameter estimates

With the estimate of covariance matrix and under the assumption of normality one can estimate parameters not only individually or simultaneously using Bonferroni correction, but also as a simultaneous region, which is powerful tool to evaluate estimates. As we showed in the chapter 4.3, the estimates have approximately normal distribution but with lower variance than estimated by MLE, with the exception for the MLM based on the model bootstrap.

For the plot of the simultaneous region of parameter estimate we use original MLE, bootstrap estimates from the sample and from the model for the MLE as well as for the MLM to see the difference between these. Characteristics are in Table 7, note that s.d. for the bootstraps are not the same as in Table 4 – these are inter-sample standard deviations among 200 000 bootstrap estimates and those for the shape parameter were stated in the text above.

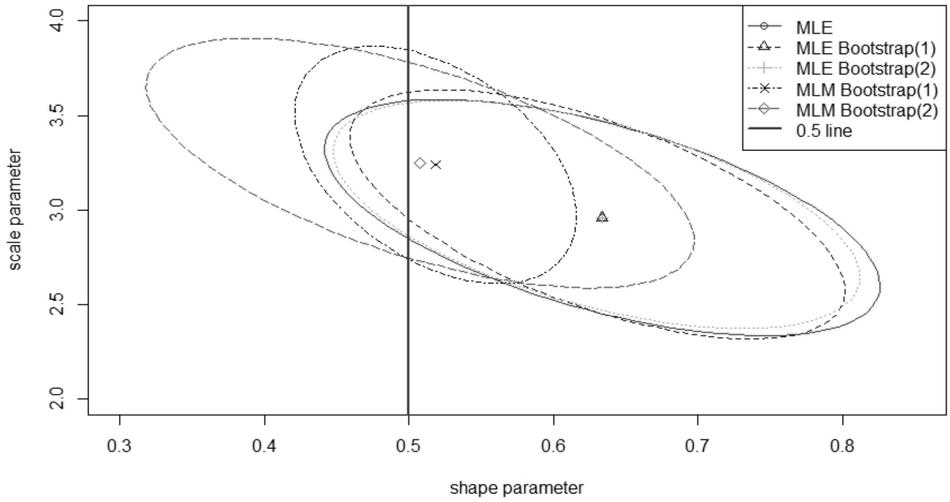
Table 7: Values used for the simultaneous region estimate

Source	ζ	s.d. (ζ)	σ	s.d.(σ)	Cov(ζ, σ)
MLE	0.6337	0.0786	2.9606	0.2550	–0.01165
MLE bootstrap$^{(1)}$	0.6304	0.0700	2.9770	0.2689	–0.01165
MLE bootstrap$^{(2)}$	0.6296	0.0745	2.9775	0.2459	–0.01016
MLM bootstrap$^{(1)}$	0.5184	0.0399	3.2393	0.2561	–0.00449
MLM bootstrap$^{(2)}$	0.5076	0.0775	3.2479	0.2693	–0.01292

Source: the authors.

Figure 9 confirms, that all three MLE – from the original sample, based on the bootstrap samples from the sample and from the initial estimate, have very similar confidence regions, whereas for the MLM estimates bootstrap confidence region estimates based on the original sample and based on the initial MLM estimate differs – the former being less corelated and with lower variance of the estimate of shape parameter.

Figure 9: Simultaneous confidence regions, different estimates



Source: the authors.

5. Conclusion

The paper summarized very basics of extreme value theory and its theoretical implications for the modelling of the wealth of the richest persons that are usually not discussed in the literature. It then moved onto the visual introspection of (non)existence of the first four moments and connected it to the estimated value of shape parameter of the generalized Pareto distribution. The discussion of the quality of the estimates done by maximum likelihood and L-moments methods follows and is finished by presentation of estimated confidence regions for shape and scale parameters.

Among main findings belong the fact, that the distribution may not have finite variance and that it is quite close to Pareto distribution except for the very right tail that is closer to exponential distribution. The possible non-existence of finite variance would have serious consequences for the standard inequality measures, but even if the variance is finite, the heavy-tailed distribution needs specific techniques for estimating these.

Maximum likelihood estimate is strongly in favor of infinite variance and it is normally distributed even though with lower variance than estimated. L-moment estimates are on the other hand indecisive about variance finiteness, have comparable variance of parameter estimates except for the bootstrapping done from the sample, but it may be due to the fact, that the sample have finite variance unlike the model. Parameter estimates are negatively correlated.

Findings about the nature of the distribution of the wealth of the richest persons should be replicated in different sets, that is from diverging countries and from different time periods. Among methodological issues belong the difference in the estimates via different methods and selection of the proper one. Their direct comparison by bootstrapping is not straightforward, as they share sample, which does not necessarily have same properties as the underlying distribution, and they do not share models (estimated parameters that are used to build bootstrap samples are different). This also warrant further research.

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