Research of Multi-objective Optimization with Time Restriction

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Abstract. Most of the operating strategies for modern companies are depending on scientific management methods. The research of 0-1 program can provide valuable and efficient solutions for companies. The purpose of this paper is analyzing the properties between teams and projects like quality, remuneration, cost, and revenue. Confirming the relationship between those properties by linear regression and Back Propagation Neural Network. Then, calculate the best Assignment scheme with a time restriction to provide a reference for companies.

Statement of Problem

As the development of management theories and the progress of social technology, efficient management method become an important factor for the company. Improving the whole production and assigning teams with an efficient model to have to promote efficiency for companies.

In recent years, great achievements of multi-objective assignment have been made in China. 0-1 program model can increase resource utilization. There are some related researches like literature [1] used ant colony optimization, provide an optimized model by data of time, cost and quality. [2] Use Genetic Algorithm to offer options for different cooperative partners by two properties: task fitness and task coordination rate. [3] Realized task allocations by minimized Euclidean distance of ideal points. [4] Analyzed Hungarian method, then solved the problem of staff assignment. Most of those mentioned literature refers scientific Multi-objective assignment method, build mathematic model then provide solutions. When considering degrees of importance for different variables, they use AHP or Entropy method. However, for practical problems, the degrees of importance are not constant but changing by the value of those variables. According to the above analysis, this paper survey the relationship between variables and the objective function, then calculate the solution.

Model Creation

Parameter Introduction

I means the set of tasks, i=1,2,…m;
J means the set of teams, j=1,2,…n;
t_{ij} means the period if assign team j to perform task i;
w_{ij} means the remuneration for team j to perform task I;
q_{ij} means the quality of task i if assigning team j to perform task i;
c_{ij} means the cost of team j to perform task i;
r_{ij} means the revenue if assign team j to perform task I;
v means the time restriction;
x_{ij} is 0-1 decision variable, if assign team j to perform task i, x_{ij}=1. If not, x_{ij}=0.

Designing Model

$$\text{Maxf} = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}\, x_{ij} \tag{1}$$

s.t.

$$\text{Max}\{t_{ij}x_{ij}, 1 \le i \le m, 1 \le j \le n\} \le v \quad (2)$$

$$\sum_{j=1}^{n} x_{ij} \le 1, i = 1,2, \dots m \quad (3)$$

$$\sum_{i=1}^{m} x_{ij} \le 1, j = 1,2, \dots n \quad (4)$$

x_{ij}=0 or 1; i=1,2....m; j=1,2…n (5)

Equation (1) is the objective function, and it shows maximize the total revenue. Equation (2) to (5) is the restriction. (2) Means all the task should be finished no longer than v days. Equation (3) means each task is performed by one team. Equation (4) means each team can only perform one task. Equation (5) means the range of decision variable.

Solution of the Model

Testing Hypothesis of Linear Relationship

Using 'a' to show the quantity of the company's historical data of projects. Each data have 'h' number of variable Z_h(h=1,2,3…b, b is 3 in this paper) and R_g(g=1,2,3…a) means the value of the project. z_{gh} means the value of variable Z_h in project 'g'. Using linear regression for variables and the objective function of those historical data. If the condition of fitting looks well, it can be assumed that the relationship between objective function and variables is linear. If not, the relationship is nonlinear.

Analyzing Nonlinear Variable

For the condition that the relationship between objective function and variables is nonlinear, using *Matlab* to calculate. Choosing some of R_g and z_{gj} as a test set, then, create a BP Neural Network called 'net' for the other R_g and z_{gj}. Putting the z_{gj} of the test set into 'net' and compare those results with R_g of the test set. If the standard deviation is low, it can be assumed that the 'net' fit the nonlinear relationship.

Forecasting Revenue

When confirming the relationship between R_g and z_{gj}, put the information of each task(w_{ij}, q_{ij}, c_{ij}) into the linear regression or BP Neural Network. Then getting the array W, Q, and C which means the contribution of quality, remuneration, and cost for revenue. At last, calculating and forecasting the revenue as array R. The formula is:

$$R = W + Q + C \quad (6)$$

Assigning with Time Restriction

When getting the array R, maximize total revenue with time restriction array T. Common method are Hungarian methods and ant colony optimization. This paper decides to use Excel Solver to get the result directly. Finally, the result is the best assignment decision.

Example

Background Introduction

The company has 7 teams. Managers need to arrange them into 7 tasks. The time, remuneration, and cost for each team to perform each task are shown by matrixes. Time matrix and quality matrix are shown in table 1. The remuneration matrix and cost matrix are shown as table 2 and table 3. The time restriction is 25 days. There are 100 history data about similar tasks. Forecasting the revenue and making a decision to maximize the total revenue. For two conditions of the relationships between variables and objective function, there are two kinds of solutions. The calculation procedure of the two methods given above is briefly described below.

Table 1. Table of Time and Score of quality

	Time (day)								Score of quality						
	i_1	i_2	i_3	i_4	i_5	i_6	i_7		i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	3.1	3.7	2.6	3.5	2.2	2	2.7	j_1	90	98	70	90	78	91	84
j_2	4	3.8	2.1	3	2.9	2.8	2.3	j_2	93	90	89	87	73	91	77
j_3	2.5	3.6	3.8	3.3	2	3.8	3.5	j_3	75	92	77	98	94	82	75
j_4	2.1	3.7	2.4	2.3	3.5	2.5	2.7	j_4	79	83	98	91	93	84	95
j_5	3.7	3.3	2.1	3.2	2.7	3.3	3.1	j_5	77	75	98	97	98	86	85
j_6	3.9	3.9	3.6	3.1	2.7	2.2	2.2	j_6	78	74	76	81	78	77	78
j_7	2.8	3.9	3.1	2.5	3.8	2.1	2	j_7	76	81	81	96	77	75	92

Table 2. Remuneration for each team

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	56309	63244	64605	53440	58455	68696	53709
j_2	63936	66673	52606	61633	63695	60811	51885
j_3	60963	59820	63323	67880	52307	51969	66692
j_4	63756	66542	63463	62176	51158	65443	55921
j_5	60445	64169	51125	57464	62164	58590	62344
j_6	55432	66648	68773	53741	51258	56792	61258
j_7	51083	52192	61149	67318	54846	61881	57244

Table 3. Cost table

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	120974	129460	136529	117231	136380	138313	117464
j_2	113825	111733	117542	117030	112386	123675	125482
j_3	132347	120720	129731	109015	111542	124359	128730
j_4	124707	118791	120183	101403	132465	130992	101801
j_5	108665	137683	136362	130878	120607	136059	119964
j_6	111943	110253	122162	112899	122709	113120	132554
j_7	119943	116180	121719	135406	106042	136371	101075

Condition 1: Linear Relationship

Using linear regression for 100 history data. When the fitness is well, record the coefficient of quality, remuneration and cost are 1952.447, 2.929 and 1.022. Multiply those three coefficients with matrixes of quality, remuneration, and cost. Then add them together we can get the table of forecasting revenue, shown as table 4.

Combine table 4 with time matrix, using Excel Solver to get the best decision in 25 days. Shown in table 5:

Condition 2: Nonlinear Relationship

Using linear regression for 100 history data. When the fitness is not good, it can be assumed that the relationship between variables and objective function is nonlinear. Thus, using Matlab to calculate. Choosing 10 data randomly as a test set. Creating and training a BP Neural Network for the other 90 data. Called the BP Neural Network as ‘net’. Put test variables into ‘net’, compare the results with test objective function, the result is shown in Figure 1. The Standard Deviation is low. So, ‘net’ can describe the nonlinear relationship.

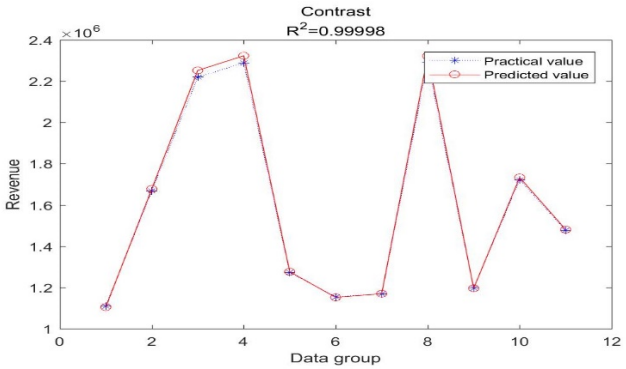

Figure 1. The result of 'net'

Put all the data of quality matrix, remuneration matrix, and cost matrix into 'net'. The result is the revenue of different tasks and teams. Shown in table 6.

Combine table 7 and time matrix, using Excel Solver to get the best decision. Shown in table 7.

Table 4. Revenue of linear relationship

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	464285	508890	465432	452056	462886	520239	441367
j_2	485175	485197	447979	469991	443950	482184	430552
j_3	460253	478214	468397	501574	450733	439413	473336
j_4	468435	478359	500050	463420	466799	489562	453316
j_5	438437	475096	480447	491457	496679	478573	471167
j_6	429057	452372	474672	430938	427834	432291	467186
j_7	420590	429755	461650	522994	419357	467054	450591

Table 6. Revenue of Nonlinear relationship

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	1223630	1495703	1135623	1169739	1135829	1510218	1083707
j_2	1372876	1367111	1143526	1249361	1076456	1319468	986643
j_3	1121306	1309618	1178185	1512478	1200589	1052901	1207640
j_4	1195406	1281264	1468183	1275559	1237141	1317377	1241901
j_5	1062620	1195414	1334212	1389466	1447496	1250521	1240776
j_6	1007423	1135268	1236665	1027160	982067	1017957	1166964
j_7	944396	1015294	1169178	1555249	968708	1151529	1211487

Table 5. Decision of linear relationship

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	0	0	0	0	0	1	0
j_2	1	0	0	0	0	0	0
j_3	0	1	0	0	0	0	0
j_4	0	0	1	0	0	0	0
j_5	0	0	0	0	1	0	0
j_6	0	0	0	0	0	0	1
j_7	0	0	0	1	0	0	0

Table 7. Decision of nonlinear relationship

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
j_1	0	0	0	0	0	1	0
j_2	0	1	0	0	0	0	0
j_3	0	0	0	0	0	0	1
j_4	1	0	0	0	0	0	0
j_5	0	0	0	0	1	0	0
j_6	0	0	1	0	0	0	0
j_7	0	0	0	1	0	0	0

Conclusion

The assignment problem is an important part of Multi-objective optimization. Companies should maximize their revenue as this method. This paper uses the Multi-objective assignment model with a time restriction, refer to linear regression and BP Neural Network, and avoid the traditional thinking like constant weight and normalization. Clearer and more practical objective function make this paper more valuable. This paper provides a model and method for the company to arrange teams and tasks.

Reference

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