

# Modeling the Road Roughness for Automotive Dynamics Simulation

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**Abstract.** It is meaningful to investigate the road roughness model and simulation approach for modern vehicle suspension design and development. Mathematical modeling of the road roughness was systematically carried out in this study, which includes modeling the Power Spectral Density (PSD) of road roughness in terms of spatial frequency, modeling the PSD of road roughness in terms of frequency, and modeling the road roughness function in the time domain. Simulations of road roughness and road classification were then performed using the established frequency and time domain models, finally the road models were incorporated into the dynamics simulation of a luxury car with air suspension. Simulation results show that the road model and simulation approach are convincing and accurate. The road theory and simulation approach obtained in this work could be useful and instructive for vehicle suspension development.

## Introduction

Vehicle dynamics simulation plays an important role in modern vehicle design and development, especially in the process before any prototype is made. However, the simulation which is based on mature theory and data should be convincing and accurate enough to promote any decisions.

In automotive suspension development, whether in the conventional suspension [1], or in the active [2, 3] suspension, or in modern air suspension [4-7], road input is crucial in vehicle vibration and suspension performance simulation, so it is meaningful to investigate the road roughness model and simulation approaches. In current automotive theories [8-10], although there are discussions about road roughness and its model, the theory and approach appear non-systematic or non-specific.

In this study, mathematical modeling of the road roughness for vehicle system dynamics simulation was systematically carried out, which includes modeling the Power Spectral Density (PSD) of road roughness in terms of spatial frequency, modeling the PSD of road roughness in terms of frequency, and modeling the road roughness function in the time domain. Simulation of road roughness and road classification were then performed using the established frequency and time domain models, finally the road models were incorporated into the dynamics simulation of a luxury car with air suspension. Simulation results show that the road model and simulation approach are convincing and accurate. The road theory and simulation approach obtained in this work could be useful and instructive for vehicle suspension development.

## Mathematical Modeling

### Power Spectral Density of Road Roughness in Terms of Spatial Frequency

In term of spatial frequency, the power spectral density of road roughness  $G_q(n)$  can be formulated by

$$G_q(n) = G_q(n_0) \left( \frac{n}{n_0} \right)^{-W} \quad (1)$$

where  $n$  is the spatial frequency,  $n_0$  is the reference spatial frequency and usually  $n_0=0.1 \text{ m}^{-1}$ ,  $W$  is the frequency index and usually  $W=2$ .

Taking the first order derivative of Eq. (1) to obtain the velocity power spectral density of road roughness, in terms of  $G_q(n)$

$$G_{\dot{q}}(n) = (2\pi n)^2 G_q(n) \quad (2)$$

Thus, substituting Eq. (1) to Eq. (2) to obtain

$$G_{\dot{q}}(n) = (2\pi n_0)^2 G_q(n_0) \quad (3)$$

Eq. (3) is a constant and usually called “the white noise”, which is very useful in vehicle system dynamics simulations.

### **Power Spectral Density of Road Roughness in Terms of Frequency**

When an automobile is running at a speed of  $v$  m/s on a road surface with a spatial frequency of  $n$ , the power spectrum density of road roughness is defined as

$$G_q(n) = \lim_{\Delta n \rightarrow 0} \frac{\sigma_{q \square \Delta n}^2}{\Delta n} \quad (4)$$

where  $\sigma_{q \square \Delta n}^2$  is the Root Mean Square (RMS) of power spectrum density at the spatial frequency range  $\Delta n$ , for the frequency in the time domain can be described as  $f=vn$  Hz, so the power spectrum density in terms of frequency  $f$  is

$$G_q(f) = \lim_{\Delta f \rightarrow 0} \frac{\sigma_{q \square \Delta n}^2}{\Delta f} \quad (5)$$

Thus, substituting  $f=vn$  Hz, Eqs. (4) and (1) into Eq. (5) to obtain

$$G_q(f) = \frac{1}{v} G_q(n) = \frac{1}{v} G_q(n_0) \left( \frac{n}{n_0} \right)^{-2} = G_q(n_0) n_0^2 \frac{v}{f^2} \quad (6)$$

Similarly, taking the first order derivative of Eq. (6) would obtain the velocity power spectral density of road roughness, in terms of  $G_{\dot{q}}(f)$

$$G_{\dot{q}}(f) = (2\pi f)^2 G_q(f) = 4\pi^2 G_q(f_0) n_0^2 v \quad (7)$$

### **Road Roughness Function in the Time Domain**

For numerical simulations of vehicle system dynamics are usually performed in the time domain, so we have to try to obtain the concrete functions of road roughness in terms of time, from the above statistical models.

Introducing the circular frequency  $\omega=2\pi f$  to obtain the velocity power spectral density of road roughness in terms of  $\omega$

$$G_{\dot{q}}(\omega) = \frac{2\pi G_q(n_0) n_0^2 v}{\omega^2} \quad (8)$$

If assuming  $G_q(\omega)$  is the response of a white noise with magnitude of 1, i.e.,

$$G_{\dot{q}}(\omega) = |H(j\omega)|^2 G_w(\omega) \quad (9)$$

where  $G_w(\omega)$  is the power spectral density of white noise,  $H(j\omega)$  is the system frequency response function, then uniting Eqs. (8) and (9) to get

$$H(j\omega) = \frac{2\pi n_0 \sqrt{G_q(n_0)} v}{j\omega} \quad (10)$$

According to its definition, the system frequency response function  $H(j\omega)$  is

$$H(j\omega) = \frac{q(t)}{w(t)} \quad (11)$$

where  $q(t)$ ,  $w(t)$  are the road roughness and the white noise functions in the time domain, respectively. So uniting Eqs. (10) and (11) and taking the Laplace Transform to get

$$sq(s) = 2\pi n_0 \sqrt{G_q(n_0)} vW(s) \quad (12)$$

Continuing to take the Inverse Laplace Transform of Eq. (12) to get the road roughness function in the time domain

$$\dot{q}(t) = 2\pi n_0 \sqrt{G_q(n_0)} vw(t) \quad (13)$$

Similarly, the white noise road roughness function in the time domain can be deduced as

$$\dot{q}(t) = -2\pi f_0 q(t) + 2\pi n_0 \sqrt{G_q(n_0)} vw(t) \quad (14)$$

## Simulation

### Road Roughness Simulation

Using the mathematical models established in the second Section, road roughness can be simulated, and one of the results are shown by Fig. 1. Fig. 1(a) demonstrates the power spectral density of road roughness in terms of spatial frequency  $n$ , Fig. 1(b) demonstrates the road roughness function in terms of time  $t$  and with a RMS value of 0.0229 m.

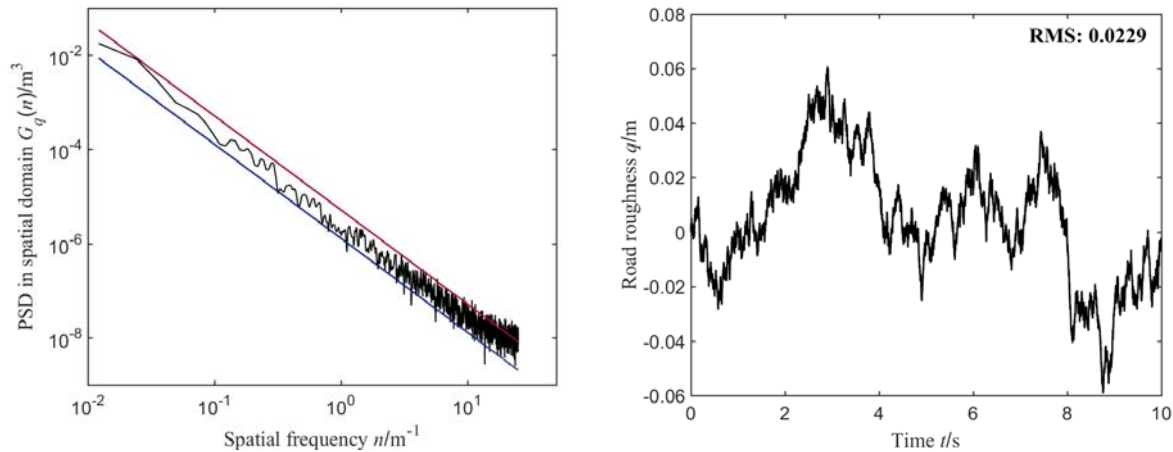


Figure 1. Simulating the road roughness: (a) power spectral density in terms of spatial frequency, (b) the time domain profile

Therefore, by using the model and simulations, roads can be classified according to its roughness. Fig. 2 summarizes the A-H classes of road, in which A class is the best road with high quality flat surface, and H class is the worst road.

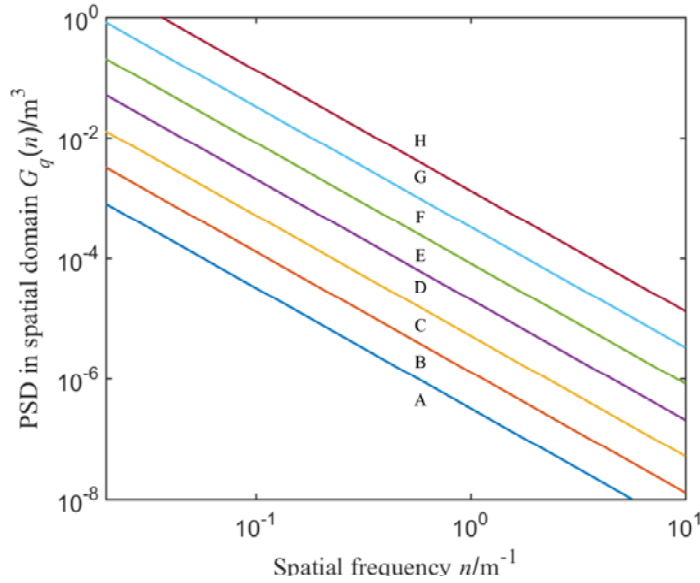


Figure 2. Road classification according to the roughness

### Incorporation of Road Roughness Model to Automotive Dynamics Simulation

As illustrated by Fig. 3, the road model was incorporated into the dynamics simulation of a luxury car with air suspension.

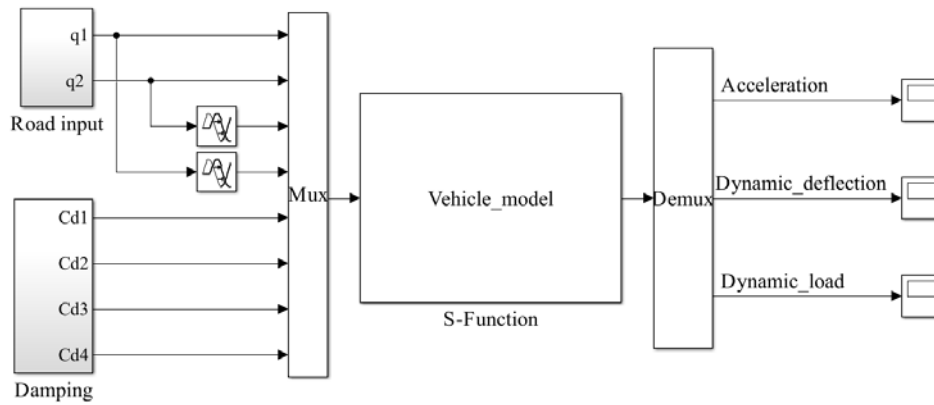


Figure 3. Incorporating the road model to a full automotive dynamics simulation model

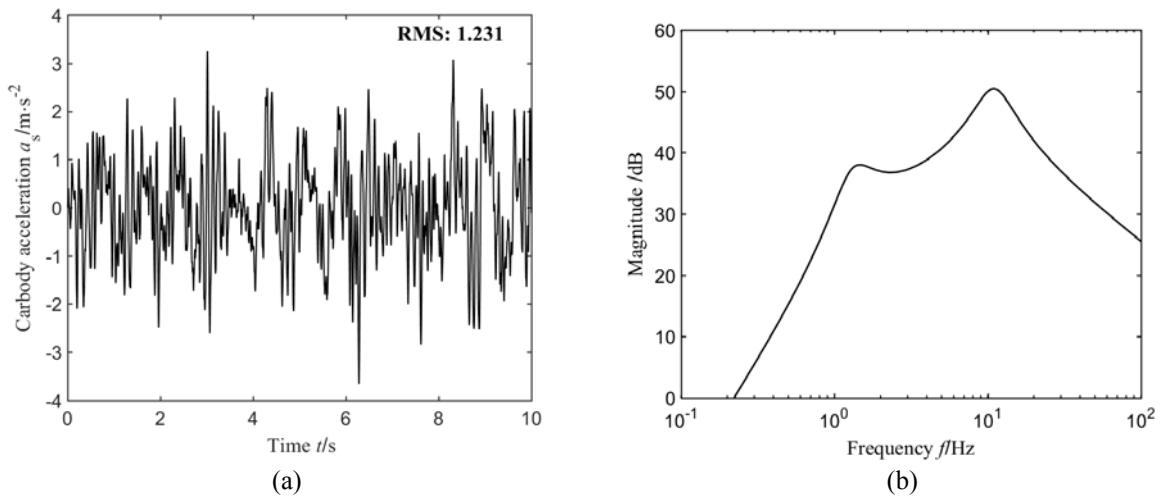


Figure 4. Vehicle response of class C road roughness at a speed of 20 km/h: (a) Car-body acceleration, (b) Magnitude-frequency characteristics of car-body acceleration

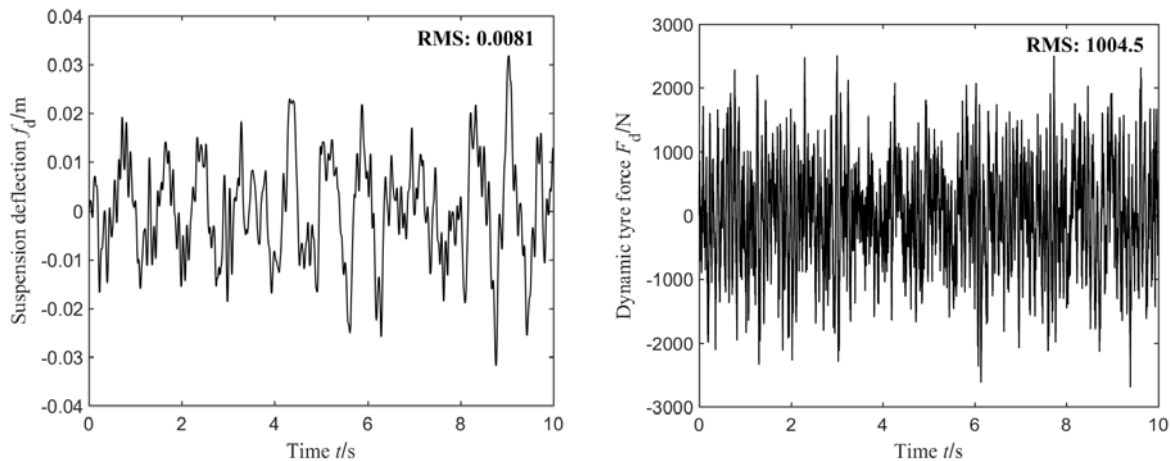


Figure 5. Vehicle response of class C road roughness at a speed of 20 km/h: (a) Suspension deflection, (b) Dynamic tyre force

In the simulation, the class C road values are used and the vehicle speed is 20 km/h. Fig. (4) shows the car-body acceleration and its magnitude-frequency characteristics, Fig. (5) shows the suspension deflection and dynamic tyre force.

### Concluding Remarks

(1) Road input is crucial in vehicle vibration and suspension performance simulation, so it is meaningful to investigate the road roughness model and simulation approach for modern vehicle suspension design and development.

(2) Mathematic models of the road roughness could be the PSD in terms of spatial frequency, the PSD in terms of frequency, and the road roughness function in the time domain. The models are interconnected and all useful for specific simulations.

(3) Simulation results show that the road model and simulation approach performed in this work are convincing and accurate, so the road model and simulation approach could be useful and instructive for vehicle suspension development.

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