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Trajectory Optimization of Unmanned Aerial Vehicle's Ascending Phase based on h-p Adaptive Pseudospectral Method

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Abstract. The pseudospectral method is one of the most effective methods to optimize the trajectory of UAV, however, the initial guess of the initial state is mainly based on the engineer's engineering experience, lacking of actual data support. Based on h-p adaptive pseudospectral method, this paper takes fuel economy as optimization index, then proposes four initial guessing strategies to focus on the influence of initial track angle guessing on simulation results quantitatively. The simulation results show that the selection of the initial track angle guess has a great influence on the optimization results. For the model used in the example, when the initial track angle is 20, the attitude and optimization index of UAV are better. The content of this paper has certain reference significance for practical engineering applications.

Keywords: Trajectory optimization; Adaptive pseudospectral method; Unmanned aerial vehicles; Minimum fuel consumption; Initial trajectory angle guess.

1. Introduction

In recent years, with the continuous development of aerospace technology, unmanned aerial vehicles have become a hot research topic in the aerospace field due to their special dual-use advantages [1]. With the continuous updating of products, the performance requirements of the new generation of UAVs are constantly improving. Especially for military UAVs, they are required to have penetration, stealth and long endurance functions. This leads to the new generation of UAVs flying in the air, the coupling is strengthened, time-varying is accelerated, and uncertainties are increased, which makes the solution and planning of the trajectory of the new generation of UAVs more important.

At present, pseudospectral method is mainly used to solve this kind of trajectory planning problem. The trajectory optimization problem of unmanned aerial vehicle can be described as: the vehicle flies according to the optimal control law under the constraint conditions of state quantity and path, so that the objective function of the vehicle is optimal. Therefore, in essence, the trajectory optimization problem of unmanned aerial vehicle can be transformed into the optimal control problem. The numerical method of optimal control and the nonlinear programming algorithm mainly include direct shooting method, collocation method and pseudospectral method, especially the pseudospectral method developed rapidly in recent years, which has been widely used in complex optimal control problems due to its high accuracy and fast convergence speed, and has become one of the effective methods to solve the trajectory optimization problem of unmanned aerial vehicles[2-5].

However, when using pseudospectral method to optimize the trajectory of an aircraft, it is necessary to provide an initial guess of the initial state. Under different initial guesses, the results of the trajectory optimization will often be different, or even produce great differences. In engineering practice, it is often necessary to optimize the trajectory of the UAV first, then design the controller to track the planned trajectory accurately. The different trajectories directly lead to the design method of the subsequent controller, and even have a series of effects on the controllability judgment of the whole UAV. Therefore, the importance of initial guess is self-evident. At present, the initial guess and estimation mainly depends on the practical experience of engineers, which is not only subjective, but also time-consuming and labor-consuming, and difficult to achieve the desired results.

Many literatures have studied the application of pseudospectral method, and continuously expanded the theoretical system and application scope of pseudospectral method [6]. Darby proposed an h-p adaptive method to deal with the nonuniform problem of constraint destruction in segments when constraint destruction is most serious for the trajectory optimization problem where there are discontinuities in the control quantity. Without reducing the accuracy of the solution, this method greatly reduces the solution time and has caused extensive research and discussion [7]; Guanghao Zhang proposed an h-p pseudospectral method based on detection points and optimized the reentry trajectory of the vehicle. According to the maximum allowable error, the method adaptively adjusts the number of discrete subintervals and the number of distribution points in the interval with a dual channel optimization strategy, then adaptively sets the detection points according to the strength of the no-fly zone constraint, and disperses the optimal control problem at the distribution points and detection points, and finally obtains the result through nonlinear programming[8]; Wenjie Qiu, and others based on h-p adaptive pseudospectral method optimized the multi-stage trajectory of the aircraft with consideration of velocity constraint, trajectory stage switching point constraint and trajectory end parameter constraint. Simulation results show that h-p adaptive pseudospectral method can effectively solve the trajectory optimization problem of multi-stage boost-glide aircraft, and the optimization result is superior to the maximum lift-drag ratio glide flight trajectory[9]; Hongyu Zhou, and others used h-p pseudospectral method to design a new generation of reusable launch vehicle for horizontal take-off and landing. The document divides the trajectory of the ascending segment into two segments according to the power difference and uses global search method to determine the best time for power switching, and obtains good simulation results[10]; Boyuan Zhang, and others based on h-p adaptive pseudospectral method have optimized the trajectory of quad-rotor UAV formation, and have given the safety zone strategy of collision avoidance constraint between quad-rotor UAVs, which provides an effective method for solving the trajectory optimization problem of quad-rotor UAV formation[11].

Similarly, the above-mentioned studies have optimized various practical problems under relatively subjective initial state guesses. In many literatures, there is a relative lack of attention to initial guesses. Only Yong Enmi et al. took advantage of the advantages of Gauss pseudospectral method, proposed a serial and piecewise optimization strategy with initial value estimation, and constructed an initial guess generator (IGG), to design variables. calculates the approximate optimal solution, and then finds the more accurate solution under multi-node by interpolation [12].

Based on an h-p adaptive Radau pseudospectral method (hp-RPM), this algorithm is applied to the fuel-saving climbing problem of UAV in the ascent phase. The difference of attitude and trajectory of unmanned aerial vehicle (UAV) under different initial guessing strategies for initial trajectory angle is studied in detail. Through comparison, a better initial guess for initial trajectory angle is obtained. Under this initial guess, the trajectory optimization of UAV is carried out, and some reference planning suggestions for the overall trajectory planning of UAV are given.

2. Mathematical Model for Trajectory Optimization of Ascending Segment

2.1 Dynamic Equation

In this paper, the following mathematical model can be established by using the three-degree of freedom equation of the aircraft:

It can be assumed that: 1) the instantaneous balance assumption is adopted to establish the particle trajectory equation; 2) the pitch angle control scheme is adopted to only consider the motion of the aircraft in the longitudinal plane. Based on the above assumptions, the dynamics equation of the aircraft is as follows:

$$\begin{cases} \dfrac{dV}{dt} = (Pcos\alpha - D)/m - \mu sin\theta/(R_e + y)^2 \\ \dfrac{d\theta}{dt} = \dfrac{(Psin\alpha + L)}{mV} + cos\theta(\dfrac{V}{R_e + y} - \dfrac{\mu}{V(R_e + y)^2}) \\ \dfrac{dx}{dt} = Vcos\theta \\ \dfrac{dy}{dt} = Vsin\theta \\ \dfrac{dm}{dt} = -m_0 \end{cases} \tag{1}$$

Where, $[V, x, y, m]^T$ represents the speed, track inclination, lateral flight distance, flight altitude and mass of the unmanned aerial vehicle. P represents the thrust of the booster rocket engine, α is the angle of attack of the aircraft, D is the flight resistance of the aircraft, L is the lift of the aircraft, μ is the gravity parameter of the earth, and R_e is the radius of the earth.

The aerodynamic force of an unmanned aerial vehicle can be expressed by the following formula:

$$\begin{cases} D = \dfrac{1}{2}V^2 C_D S \\ L = \dfrac{1}{2}V^2 C_L S \end{cases} \tag{2}$$

Among them, S is the reference area of the aircraft and generally takes the largest circle section of the aircraft. D is the resistance of the aircraft and L is the lift force of the aircraft during flight. ρ is the atmospheric density at different heights. C_D is the drag coefficient of the aircraft and C_L is the lift coefficient of the aircraft. The calculation formula is as follows:

$$\begin{cases} C_D = C_{D0} + C_{D1}\alpha + C_{D2}e^{C_{D3}M_a} \\ C_L = C_{L0} + C_{L1}\alpha + C_{L2}e^{C_{L3}M_a} \end{cases} \tag{3}$$

The vehicle is launched at zero length and boosted by solid rocket motor at a specific angle of launch. After reaching a certain height and speed, the solid rocket motor shuts down and the turbojet engine works and enters the ascending stage. During the climbing process, the reduced speed of turbojet engine is constant at 0.98, and the thrust value is interpolated through the thrust table.

2.2 Constraints and Boundaries.

The process boundary can be described as follows:

$$\begin{cases} V_{\min} \le V \le V_{\max} \\ \theta_{\min} \le \theta \le \theta_{\max} \\ x_{\min} \le x \le x_{\max} \\ y_{\min} \le y \le y_{\max} \\ m_{\min} \le m \le m_{\max} \end{cases} \tag{4}$$

The overload can be described as follows:

$$\begin{cases} n = \dfrac{\sqrt{L^2 + D^2}}{mg_0} \\ n \le n_{\max} \end{cases} \tag{5}$$

Where, g_0 is the sea level gravity acceleration and n_{max} is the maximum allowable overload.

The control constraint be described as follows:

$$\alpha_{\min} \le \alpha \le \alpha_{\max} \tag{6}$$

The terminal constraint be described as follows:

$$\begin{cases} y(t_f) = y_f \\ v(t_f) = v_f \\ \theta(t_f) = \theta_f \end{cases} \tag{7}$$

2.3 Objective Function

The fuel consumption of the aircraft is smaller during the ascending phase, which can make the saved fuel provide longer power for the subsequent segment, thus increasing the flight distance of the aircraft. Therefore, the minimum fuel consumption in the ascending section is selected as the optimization objective, which can be expressed by the following formula:

$$\min J = -m_f / 2 \tag{8}$$

Where, m_f is the terminal mass of the aircraft is represented.

3. hp-RPM Algorithm Design

3.1 hp-RPM Fundamentals

The basic idea of pseudospectral method is to discretize a continuous problem and fit different discrete points with different functions to obtain the solution of the problem. When hp-RPM is used to solve the trajectory optimization problem of unmanned aerial vehicle, the essence is to solve the *Bloza* optimal control. The solution method is to use *Lagendre-Gauss-Radau* (LGR) collocation points to discretize the optimal problem, transform it into *NLP* problem, and then solve the problem [13].

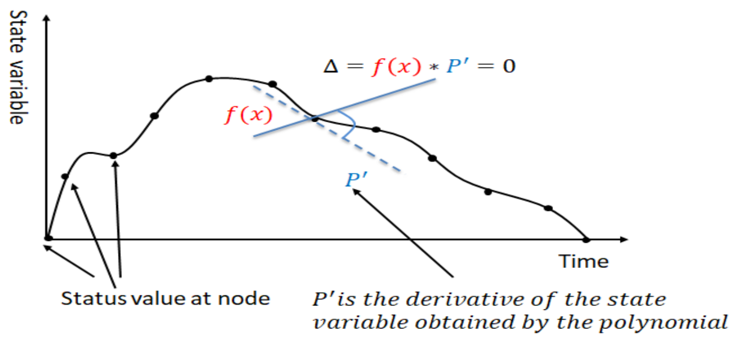

Figure 1. Schematic diagram of pseudospectrum method.

Assuming that the whole-time interval is divided into k subintervals, expressed by $[t_{k-1}, t_k]$, where, $k = 1,2,3..K$, any time subinterval t_k is transformed into the following form:

$$\tau(k) = \frac{2t(k) - (t_k + t_{k-1})}{t_k - t_{k-1}} \tag{9}$$

In the transformed time interval, the state equation, the control equation and the objective equation are described in discrete form by using independent variables. The different ways of selecting matching points are the main differences between different pseudospectral methods. Radau pseudospectral method uses LGR points as matching points. For the kth subinterval, $N^{(k)}$ LGR nodes are selected to form $N^{(k)}$ order *Lagrange* interpolation polynomial, which is used as the basis function to approximate the state quantity and control quantity, and the continuous optimal control problem is discretely converted into a multi-interval *NLP* problem. The matching points consist of zero points of N and $N-1$ order *Lagrange* polynomials. The formula for matching points is as follows:

$$\begin{cases} P_0(\tau)=1 \\ P_n(\tau)=\dfrac{1}{2^n n!}\dfrac{d^n}{d^n}\left[(\tau^2-1)^n\right] \end{cases} \tag{10}$$

3.2 Transformation of Optimal Control Problem

The performance index, state variable and control quantity of the optimal control problem are expressed by the transformed time interval, which can be expressed by the following formula [14].

$$J=\left(x_0^{(1)},t_0,x_{N_{k+1}}^{(k)},t_k\right)\sum_{j=1}^{N_K}\sum_{k=1}^{k}x_i\frac{t_k-t_{k-1}}{2}\omega_j^{(k)}g_j^{(k)} \tag{11}$$

$$\sum_{j=1}^{N_{k+1}}x_i^{(k)}D_{ij}^{(k)}\frac{t_k-t_{k-1}}{2}f\left(x_i^{(k)},U_i^{(k)},\tau_i^{(k)};t_{k-1},t_k\right)=0 \tag{12}$$

$$C(x_i^{(k)},U_i^{(k)},\tau_i^{(k)};\tau_{k-1},\tau_k)\le 0 \tag{13}$$

$$\varphi(x_0^{(1)},t_0,x_{N_{k+1}}^{(k)},t_k)=0 \tag{14}$$

$$\varphi(x_0^{(1)},t_0,x_{N_{k+1}}^{(k)},t_k)=0 \tag{15}$$

Where, $x_i^{(k)},U_i^{(k)}$ represent the state variable and the control variable at the ith distribution point in the Kth subinterval; N_K is the number of distribution points in the Kth subinterval; $\phi(\cdot)$, g$(\cdot)$ represent the final value index and the integral index respectively; $\omega_j^{(k)}$ represents the LGR orthogonal weight at the jth distribution point of the Kth subinterval; $D_{ij}^{(k)}$ represents the Kth subinterval $N_k\,(N_k+1)$ dimension Radau pseudospectral differential matrix. The calculation formulas of $\omega_j^{(k)},D_{ij}^{(k)}$ are as follows:

$$D_{ij}^{(k)}=\begin{cases} \dfrac{\dot{h}(\tau_k)}{(\tau_k-\tau_i)\dot{h}(\tau_i)},k\ne i \\ \dfrac{\ddot{h}(\tau_i)}{2\ddot{h}(\tau_i)},k=i \end{cases} \tag{16}$$

$$\begin{cases} \omega_1^{(k)}=\dfrac{2}{N_K} \\ \omega_j^{(k)}=\dfrac{1}{\left(1-\tau_j^{(k)}\right)\left[P_{N_{k-1}}(\tau_j^{(k)})\right]^2} \end{cases} \tag{17}$$

$$\mathrm{h}(\tau_i)=(1+\tau_i)\left[P_{N_K}(\tau_i)-P_{N_{K-1}}(\tau_i)\right] \tag{18}$$

3.3 h-p Adaptive Strategy Design

3.3.1 Error Evaluation Criteria

By setting the maximum allowable deviation, the state variable in the transformed time subinterval and the maximum residual error of the control variable at the distribution point are compared as the error evaluation criterion of the h-p adaptive strategy. When the accuracy requirements are met, iterative calculation is stopped, and when the accuracy requirements are not met, methods should be adopted to improve the accuracy until the requirements are met. The h-p adaptive strategy provides a double-layer strategy to improve the accuracy requirements, namely thinning the mesh and increasing the order of inter-polation polynomial.

3.3.2 The Decision of Double-layer Strategy

The core of the double-layer strategy is to deal with variables in different time subintervals differently to meet the dual requirements of accuracy and solving speed.

Refined mesh needs to use low-order interpolation basis functions to discretize each element, and in the optimization process, the element length h is subdivided by gradually encrypting mesh generation, but with the increase of dimension, the time cost is greatly increased, which is not conducive to the solution of the problem. Increasing the order of interpolation polynomial requires using high-order interpolation basis function to discretize each element, keeping the element unchanged in the optimization process, and increasing the order p of the basis function to improve the accuracy. However, when the smoothness of the function is poor, increasing the order of polynomial cannot obtain very high accuracy and the convergence speed is extremely slow. In order to solve the above problems, curvature can be taken as the judging condition and two methods can be reasonably selected. The curvature can be expressed by the following formula:

$$k^{(k)}\left(\tau_s^{(k)}\right)=\frac{\left|\dot{x}^{(k)}(\tau_s^{(k)})\right|}{\left|\left[1+\dot{x}^{(k)}(\tau_s^{(k)})^2\right]^{\frac{3}{2}}\right|} \tag{19}$$

Where, $k^{(k)}$ represents the kth curvature in a sub-interval, and $x(\tau_s^{(k)})$ represents the curve of τ_s at the sth collocation point.

Set the maximum curvature threshold. If the curvature of the curve exceeds the threshold, the smoothness of the curve is not enough and the grid needs to be added. On the contrary, increasing the polynomial order improves the accuracy.

3.3.3 Change of Variable Calculation Formula

The number of refined grids can be expressed by the following formula:

$$n_k = ceil[Bl\,\mathrm{g}\left(e_{\max}^{(k)}/\varepsilon\right)] \tag{20}$$

Where, B is a constant of change, $ceil(\cdot)$ indicating rounding up.

The order of interpolation polynomial can be expressed by the following formula:

$$N_k = N_{k0} + ceil\left[\lg\left(\frac{e_{max}^{(k)}}{\varepsilon}\right)\right] + A \tag{21}$$

Where, N_{k0} is the previous polynomial order and A is a variable constant.

3.3.4 h-p Adaptive Pseudospectral Algorithm Steps and Flow

Based on the above description, the calculation steps of h-p adaptive pseudo-spectral method are as follows:

1)Dividing the grid, transforming the problem into NLP problem with the divided independent variable grid, and solving NLP problem;

2)Calculate the precision of subinterval to see if it meets the maximum permissible error, meets and calculates the next grid, otherwise, proceed to the next step;

3)Judge whether the curvature exceeds the curvature threshold, exceed it, refine the mesh, and on the contrary, increase the polynomial order.

4)Judging whether the relative error between the state quantity and the path constraint meets the requirements, meets the requirements, and the optimization ends; otherwise, returning to step 1.

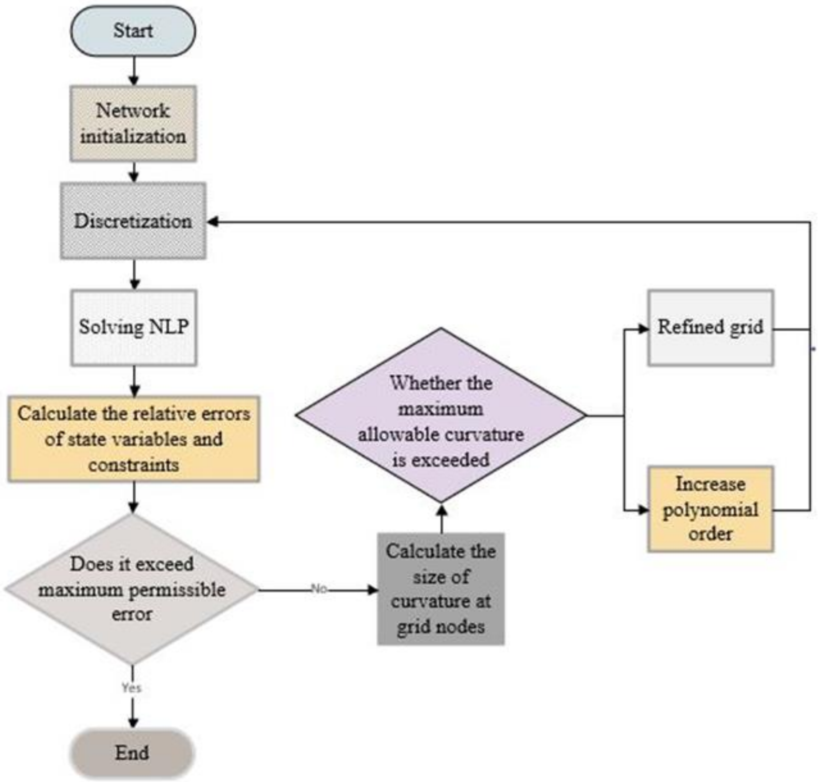

Figure 2. h-p RBM flowchart

4. Example Simulation

4.1 Example Description

The aerodynamic rudder surface of the UAV adopts the "double flat tail-V" layout, and the whole structure is symmetrical. The flight process can be described as a solid rocket motor boosting in zero-length launching mode at a specific launch angle. When the aircraft reaches a mission window of 50m in altitude and 240m/s in speed, the solid rocket motor shuts down and separates, and the turbojet engine works and enters the ascending stage. In order to satisfy the specific altitude and speed requirements of UAV at the end of boost phase, the initial trajectory angle guess of UAV in ascending phase should satisfy the constraints of 15-30°. Combining with the actual situation, this paper puts forward the following four selection strategies of initial guess, and focuses on the study of the attitude and fuel consumption of UAV based on different strategies. In addition, the reference values of the optimal initial trajectory angle guess of UAV under this example are given.

Strategy 1: Initial track angle guess is 15°;

Strategy 2: Initial track angle guess is 20°;

Strategy 3: Initial track angle guess is 25°;

Strategy 4: Initial track angle guess is 30°.

The initial mass of the aircraft is $330.35kg$, the aerodynamic reference area is $1.54686m^2$, the given initial altitude H_0= $50m$, the initial velocity V_0= $120m/s$, the terminal altitude H_f= $8km$, the terminal velocity V_f= 246 m/s, and the terminal trajectory angle θ_f= 0°.The boundary limits of state variables, control variables and path constraints during ascending phase are expressed as follows:

Table 1. Constraints.

θ_{min} /(°)	θ_{max} /(°)	n_{max} /(°)	t_{min} /(s)	t_{max} /(s)	α_{min} /(°)	α_{max} /(°)	V_{min} /(m/s)	V_{max} /(m/s)
-10	30	8	400	1000	-4	4	120	246

The software environment in the simulation process is: Windows7 32-bit operating system and MATLAB R2012a; hardware environment: $IntelCore\ i5\ -2450$processor, 4G memory, precision requirement [1e-3 2e-3].

4.2 Simulation Results

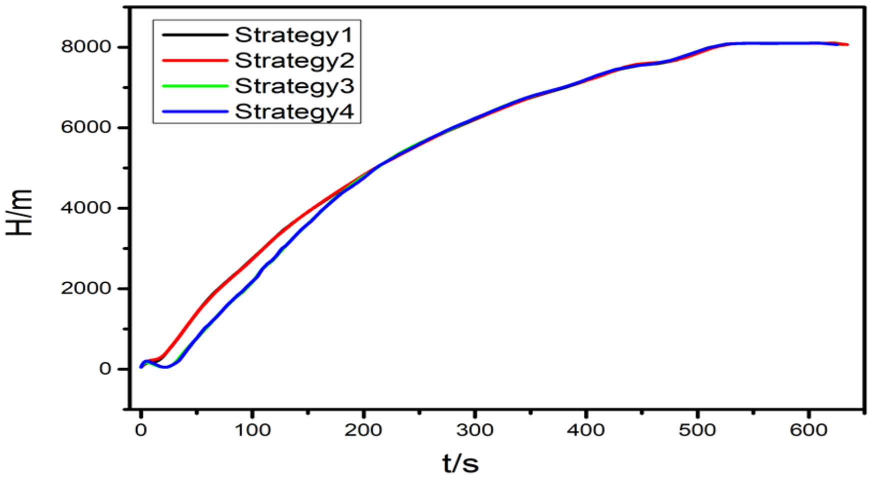

Figure 3. Height changes under different strategies.

As can be seen from the figure, the ascending trajectory of the aircraft under the four initial guessing strategies is not much different. However, under strategy 3 and strategy 4, the aircraft will rise to 200 meters in the first five seconds, then slowly descend to the lowest altitude in the next 15 seconds, and maintain the lowest altitude of 50 meters flying flat for 5 seconds, and then all the way up to the specified altitude for cruise. It can be concluded that the initial conjectures of strategy 3 and strategy 4 are more stringent.

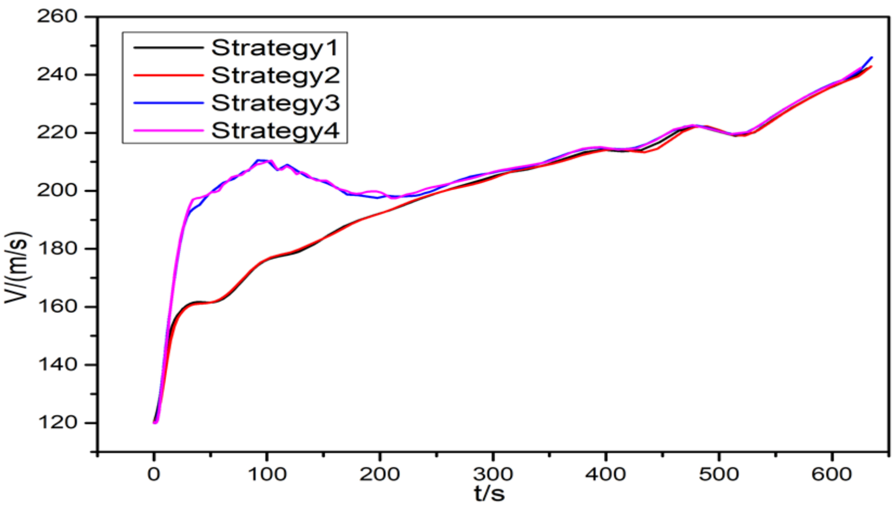

Figure 4. Speed changes under different strategies.

As can be seen from the figure, the velocity changes of UAV under four initial guessing strategies are different. Generally speaking, the speed changes of the initial guessing strategy 1 and the initial guessing strategy 2 are similar. During the whole time period, the speed increases continuously. Combining with the analysis of Fig. 3, the height slope decreases at about 480s. It can be concluded that the engine power is insufficient at this time.

In the first 100 seconds, the speed of the initial guessing strategy 3 and the initial guessing strategy 4 will increase sharply to 200 m/s, then the speed will decrease to 10 m/s, and the speed will increase sharply to 160 m/s in the first 15 seconds.

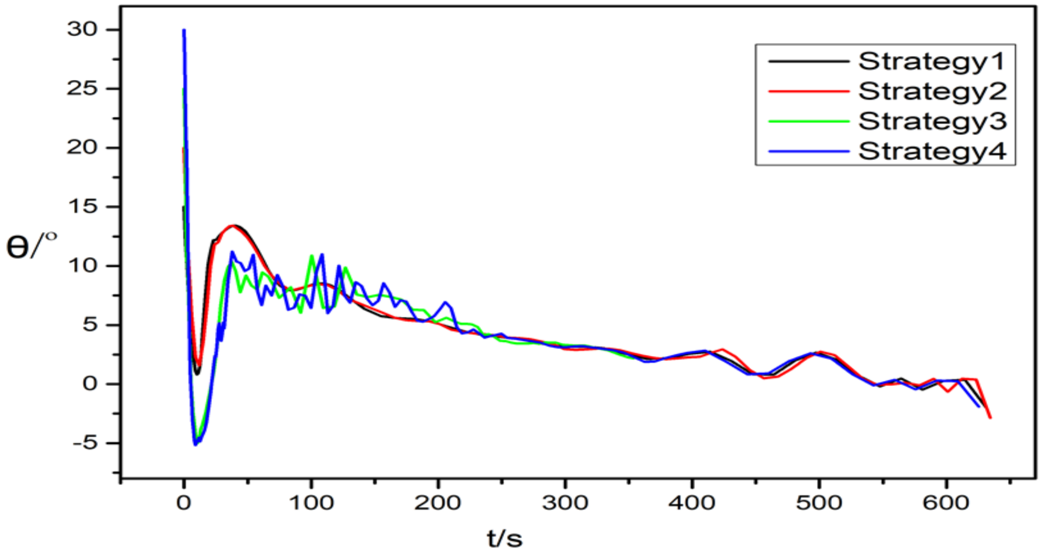

Figure 5. Track angle under different strategies.

As can be seen from the figure, the change rule of track angle of strategy 1 and strategy 2 is relatively similar, and the change rule of track angle of strategy 3 and strategy 4 is relatively similar. Strategy 1 and Strategy 2 have good smoothness in the course of changing the track angle, while the latter two strategies will have a course of oscillation between 50s-200s.

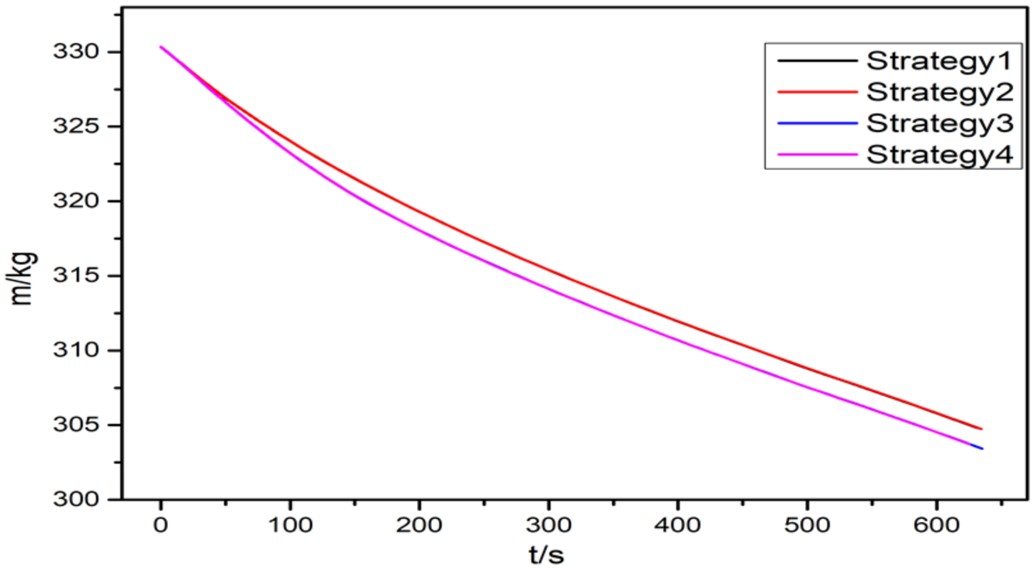

Figure 6. Quality under different strategies.

As can be seen from the figure, strategy 1 and strategy 2 are similar in terms of fuel changes, while strategy 3 and strategy 4 are similar. With unmanned aerial vehicles under strategy 1 and strategy 2, fuel consumption is about 1.5 kg less than strategy 3 and strategy 4.

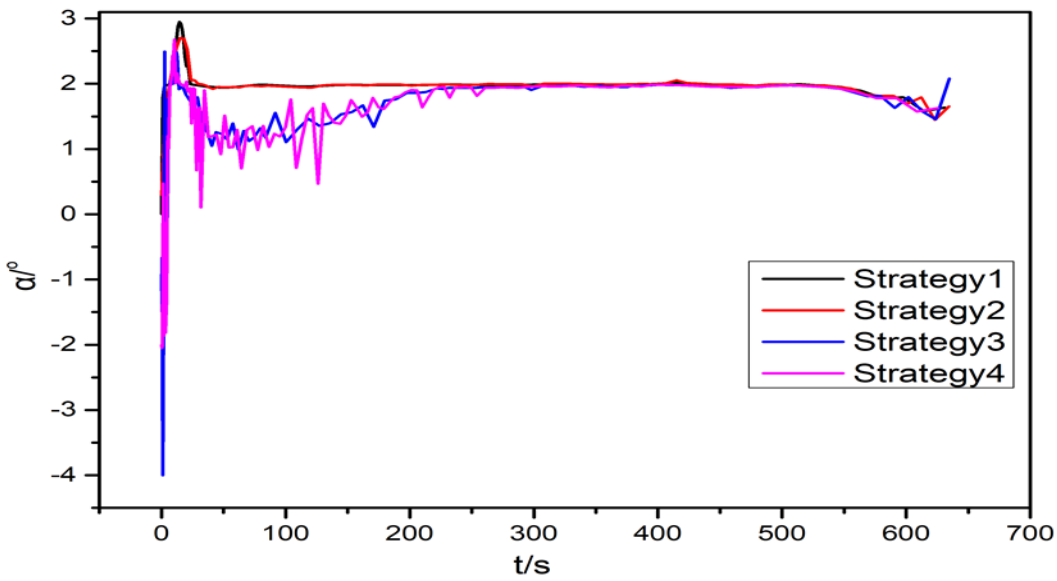

Figure 7. Angle of attack under different strategies.

As can be seen from the figure, the rules of the attack angle about strategy 1 and strategy 2 are relatively similar, and the rules of angle of attack of strategy 3 and strategy 4 are relatively similar. During the whole process of change, the angle of attack of strategy 1 and strategy 2 changed smoothly, and the angle of attack was stabilized at 2°in a very short period of time. For strategy 3 and strategy 4, the angle of attack will fluctuate more severely within the first 200s and converge to 2°after 200s and remain unchanged.

4.3 Summary

Based on the above simulation results, it is not difficult to see that the difference between strategy 1, strategy 2 and strategy 3, strategy 4, is mainly between 50s and 200s. During this period, the initial guessing strategy 3 and 4 will produce more severe shocks, which shows that the existing constraints on the initial track angle guessing strategy 3 and 4 are more stringent. For the UAV under the example, it is reasonable to select strategy 1 and strategy 2 as the initial guess of ascending phase.

5. Conclusion

Through the above simulation results, it can be seen that different initial trajectory angle guessing strategies have a great impact on the trajectory optimization in the ascending phase. Whether it is the flight attitude of the aircraft or the ultimate optimization target, under different initial guesses, different results will be produced, and it can be concluded that there must be an optimal initial trajectory angle, so that the flight attitude and index of the aircraft can be optimized. For the UAV under the example, when the initial trajectory angle is selected at 20 °, the attitude and fuel of the aircraft will be optimized. The index is better. This paper reveals the influence of guessing the initial state on the trajectory optimization of hp-RPM, which has a certain reference significance for practical engineering.

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