# Conjecture and Proof of Summation of Series Equal to One

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**Abstract.** Convergence of series is the rudiments of study series, but for the divergent series, people tend not to pay close attention, however the proof of this paper what is based on the revelation of the divergent series. Doing a process of combination unconditional convergence with a proper pattern exists some interesting finding. When we do a subtraction with harmonic series expressed as geometric progression and primitive series equal to one. It exists an obvious regularity, so the following paper will extract the regularity of this series by using the complete induction to and by using reduction ad absurdum to demonstrate this conclusion.

**Keywords:** reduction ad absurdum, Convergence of series, harmonic series, geometric progression, primitive series.

## 1. Introduction

The theory of series is the branch of analytics, combine with integral calculus which called the other branch of analytics as the rudiments and tool appear in others branches. Series along with integral calculus study analytics from two sides that are discreteness and continuer that taking limits as the basic tool and combining to study the objection of analytics-the relationship of variable-function. Series is a vital tool for study of functions and plays an important role in both theoretical and practical application, that is because on one hand series can express large numbers elementary function, such as the solution of a differential equation; on the other hand, function can represent series, therefore using the series to study function, such as using power function as a tool to study nonelementary function, as well as doing an approximate calculation, etc.

We were proving the divergence of harmonic series, when we found a certain form function's summation equal to one. There, that might do a simplification in the future calculation.

Our guess is as follows:

We try a variety of methods to prove the divergence of harmonic series, however one method that is classify each of number of terms in series, then do a summation. Dismantling function to infinitely many geometric progressions, then sum the finitely many geometric progressions, which means transform harmonic series to a summation of geometric progression. But we have some brand new found in this progress, research the summation of geometric progression as the conjecture follow:

$$M, x, n \in N^{+}$$
$$M = \mathrm{x}^{n} (n \geq 2, x \neq 1)$$
$$\sum_{M} \frac{1}{M-1} = 1$$

M, x, n are all positive integer; M can express n-degree power of a certain positive integer; n is any number which is greater than 1; all the form, such as $\frac{1}{M-1}$ equal to 1. The rigorous proof will follow.

## 2. The Proof Process is as Follows

Harmonic series $S_0=\sum_{n=1}^{\infty}\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+\cdots$

Dismantle the harmonic series to geometric progression, which the purpose is making $S_1+S_2+S_3+\cdots$ each of the term cover all the term in harmonic series and will not repeat.

When common ratio $q=\frac{1}{2}$, $S_1=1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots$

When common ratio $q=\frac{1}{3}$, $S_2=\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots$

When common ratio $q=\frac{1}{4}$, each of number of terms in geometric progression repeat with the $S_1$, discard.

When common ratio $q=\frac{1}{5}$, $S_3=\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\cdots$

In accordance with above term, the following summations are just unilateral:

$$q=\frac{1}{6},q=\frac{1}{7},q=\frac{1}{10},q=\frac{1}{11},q=\frac{1}{12},q=\frac{1}{13},q=\frac{1}{14},q=\frac{1}{15},q=\frac{1}{17},q=\frac{1}{17}$$

According to the geometric sequence sum formula $S_n=a_1\bullet\frac{1-q^n}{1-q}$

Result $S_1=\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\left(n\longrightarrow\infty\right)=2$

By the same token $S_2=\frac{1}{2}$, $S_3=\frac{1}{4}$, $S_4=\frac{1}{5}$, $S_5=\frac{1}{6}$, $S_6=\frac{1}{9}$, $S_7=\frac{1}{10}$, $S_8=\frac{1}{11}$,

Compared $S_0$ with $S_1$, $S_2$, $S_3$, $S_4\cdots$,we can found, make $S_k=S_1+S_2+S_3+S_4\cdots$

$S_q=S_k-S_0=\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\cdots$

## 3. Doing an Explanation of Some Process May Possible Lead to Doubt and Prove

1 The Riemann theorem states that if an infinite convergent real number is conditional convergence, then its terms can be arranged in a permutation so that the new series convergences to an arbitrary real number or diverges.

In the text, the proof is absolutely a permutation. And the divergence of harmonic series will not prove again. $S_q$ is the positive term series, so it is an absolute convergence series.

2 prove: the summation of geometric progression covers all the term of harmonic series, and will not repeat.

$$\because q_n \notin S_{n-1}$$

Prove the term will not repeat.

$$a=n^{2k} \cap a=m^{2q-1}(a,n,m,k,q \in N^+)$$

$m=X^{v}$ *and* $n=X^{u}$ (this formula is discussed at the election of common ratio q, so discard)

$$a=n^{2k}$$

$$a=m^{2q-1}$$

$2q-1 \neq 2k$ (odd number is not equal to even number)

When $n \div m=p$ ($p \in N^{+}$) and $2q-1 \succ 2k$, $k=\sqrt[(2q-1)-2k]{\frac{1}{p}}$ ($k \notin N^{+}$), is a negative proposition.

By the same token, when $2q-1 \prec 2k$ is a negative proposition.

When $n \div m=p$ ($p \notin N^{+}$), and $2q-1 \succ 2k$, is obvious $k \notin N^{+}$, which is a negative proposition.

By the same token, when $2q-1 \prec 2k$, is a negative proposition.

## 4. Conclusion

Through the above logical reasoning, comprehensive analysis and reasonable induction of taking power series as a basic tool to prove the proposition in this paper, which could be a brand-new discovery of series theory. The proposition gives a clearly logic thought to deal with 1/n, deriving the summation of each term of series, deleting the repeating item, which conclude an expression of the proposition- summation of series equal to one. This proposition might provide a convenience in the future series study, as well as a brand-new possibility.

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