

# The impact of transport costs on network price variation

Petr Bobrik

*Institute of Problems of Transport of  
N.S. Solomenko of the RAS  
Moscow, Russia  
bobrikpp@mail.ru*

Irina Sukhorukova

*Plekhanov Russian University of  
Economics  
Moscow, Russia  
sukhorukovaira@yandex.ru*

Bobrik Galina

*Plekhanov Russian University of  
Economics  
Moscow, Russia  
lbobrikpp@mail.ru*

**Abstract** - It is often possible to find an erroneous opinion that if the threshold values to start arbitration between network nodes are limited, then the general price spread across the network is also limited. However, in the case of an increase in the number of nodes, transportation costs on delivery also increase. The article shows that price deviations can grow to infinity as the size of the network grows, even with a limited change in the supply-demand ratio. This circumstance must be considered when planning the transport policy of the regions.

**Keywords**—*network pricing; competitive bidding; arbitration operations; graphs; transportation networks; spatial development strategy*

## I. INTRODUCTION

The article continues research in the field of exchange pricing [1] and develops further mathematical constructions in [2].

Let some financial instrument be independently traded in a certain number of places (markets)  $\{x_i\}$  total  $n + 1$ . As a result of trading, each item  $x_i$  will have its own price  $p_i$ . If trades are carried out with the use of exchange mechanisms, then the classical problem of network exchange pricing arises [3], [4]. We will assume that in each local market price is locally determined by the exchange law

$$\Delta p_i = k_i (Z_{i,k} - Z_{i,bid})$$

Here,  $Z_{i,k}$ ,  $Z_{i,bid}$  - the volume of applications for the purchase and sale at each point in time,  $k_i$  - the elasticity coefficients. Generally speaking, the elasticities may vary for each market.

Although in reality the price dynamics is much more complicated [5], [6], [7], but in the first approximation we will assume that the price changes in the direction of supply/demand imbalance.

Since each market is traded independently, the set of prices can be different even in a stationary state. This situation may be due to various reasons. In the case of product markets, this may be transport costs [8]. For financial instruments, differences between regulation and taxation in different countries are important [9]. The need to allocate capital for collateral on all exchanges leads to

capital requirements [10]. Transaction costs hamper transactions between participants from the same market, but with different trading strategies.

In case of a strong price divergence, prerequisites arise for conducting profitable arbitrage operations. It is the presence of arbitrage that turns a set of individual markets into a network, which leads to a number of qualitatively new properties of exchange pricing. In particular, as a result of arbitrage operations, each price tends not to deviate from the average values over the ensemble. From this point of view, it is possible to draw an analogy between the dynamics of the entire set of prices  $\{p_i\}$  and the swarm movement problems [11].

There is a widespread erroneous opinion that if the threshold values for the inclusion of arbitration are small, then the general price spread across the network is also limited. However, in the case of an increase in the number of nodes, such an assertion became false. The article shows that price deviations can grow to infinity as the size of the network grows, even with a limited change in the supply-demand ratio.

## II. MATERIALS AND METHODS (MODEL)

### A. Spatial Development Strategy.

The strategy for the spatial development of Russia [12] approved at the 2019 Sochi Investment Forum in one of its early versions suggested the allocation of economic growth centers only in a limited number of agglomerations. Although in the final version of the strategy the emphasis has shifted to the selection of promising regions, but the former logic is also preserved, and it does not contradict the general provisions of the strategy. Agglomerations are selected in accordance with the general theory of globalization, which, as a special case, includes global urbanization [13], [14].

In these conditions, there is a need for highly effective transport communications between the selected points of growth. One of the solutions to this problem can be the creation of a high-speed network between agglomerations, which is directly mentioned in the strategy. However, the specific mechanisms for the implementation of this task are not yet available, as well as many minor questions about how exactly the final high-speed network should look like have not been worked out. Due to the high cost of high-speed highways, there is an urgent need to build such a

network that, on the one hand, would meet the basic requirements of the demand for transport links, and on the other hand, would be as cheap as possible in construction and operation.

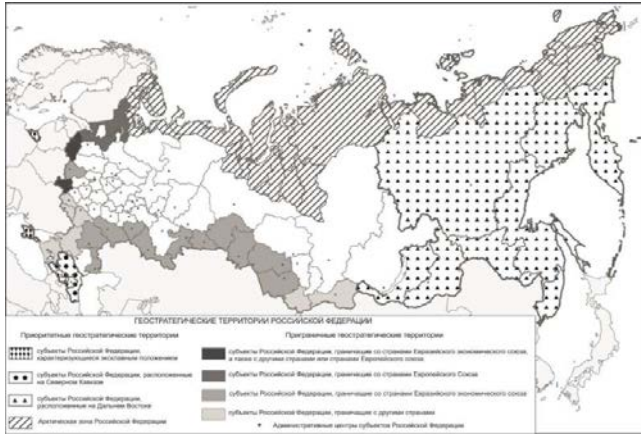


Fig. 1. Layout of geostrategic territories of the Russian Federation [12]

We formalize this problem using graph theory [15], [16]. Denote each agglomeration or region by the network node. Edges will be drawn only between those nodes whose regions have a common border. That is, the graph  $G$  of this future network is naturally defined. That allows us to formalize the task of studying transport accessibility and pricing mechanisms in the context of regions.

Let a graph

$$G(X, E)$$

describes the transport network, where  $X$  is the set of its vertices (nodes), and  $E$  the set of its edges. It is assumed that the graph is connected, that is, at least one path exists from each vertex to each vertex. A graph is weighted. For each its edge  $e_j$ , a non-negative cost

$$l_j = l(e_j)$$

is defined. Its physical meaning lies in the cost or time of transportation along this edge. For each vertex  $x_i$ , a set of its adjacent vertices is defined.

Suppose that for some commodity  $C$  there are several vertices

$$\{x_{c1}, x_{c2}, x_{c3}, \dots, x_{ck}\}$$

from which it is delivered. This can be either the locations of its production or the places of its import from external regions. We construct from each vertex-source  $x_{ci}$  a tree  $D$  of shortest routes  $\{\gamma_{ci}\}$  to all other vertices of the graph. That is, for each vertex  $x_i$ , a vertex

$$x_i = D(x_i)$$

is defined, from which the shortest path to it comes from  $x_{ci}$ . For the shortest path between the vertices

$$\gamma(v_{ci}, v_o),$$

its length is defined, which is equal to the cost of its delivery of commodity to the point  $x_o$ . Adding to this value

commodity release price from the node  $x_{ci}$ , we obtain the total cost of the commodity at the point  $x_o$

$$p(v_o) = p(v_{ci}) + l(\gamma(v_{ci}, v_o)).$$

When moving along any of the branches of the tree  $D$ , there is a monotonous increase in the value of the goods due to transport costs. In general, each node may include deliveries of goods from different suppliers. The presence of exchange pricing mechanisms allows you to cut off non-competitive prices, and thereby determine the area of dominance of each of the suppliers. As a result, some distribution of prices among nodes is established over the entire network.

We note without proof that each branch of the shortest route tree is a linear path without cycles, at each node of which commodity price increases by the amount

$$l_j = l(e_j)$$

of transport costs. Further in the article only the subgraph of the graph  $G(X, E)$  will be considered as one of the branches of the shortest route tree.

### B. Arbitration Mechanisms

In the event of a strong divergence of prices in neighboring nodes of the network, arbitration operations may start. A sufficient condition for their launch is a price spread above the cost of transporting goods  $l(v_i, v_j)$  between network nodes. In this case, the conduct of arbitration will make a profit. At the same time, in practice situations are sometimes observed when market participants neglect such opportunities for a long time, which allows imbalances to be present for long periods of time. [7].

A much more interesting and frequently encountered situation is a smaller price range than is due to transport costs. This may be due to several reasons.

It should be understood that there is no single price  $l(v_i, v_j)$  of transportation, which is carried out by different operators and in terms of orders at different prices. Frequently, shipments are carried out along the way when performing other shipments. In this case, their cost can not be correctly determined, but in any case, its assessment will be significantly less than if a separate independent transport of only this consignment of goods with the cost.

In addition, a large proportion of the cost of transportation is occupied by auxiliary operations: loading and unloading, sorting, and paperwork. negotiations and other logistic procedures. Their cost can sometimes exceed directly the cost of transportation. If they are carried out within the framework of the planned distribution of goods in large batches according to optimal pre-calculated plans, then the number of operations is less and the cost, respectively, is also lower.

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A common case is the delivery of goods to neighboring nodes of the network by different routes on different days and even by different means of transport. For example, this happens when two different branches of the shortest route lead to two neighboring nodes. If their lengths are close to each other, then the final cost of goods in neighboring nodes will also be close. The last example shows that a weak price spread in the general case is not an exception, but rather a typical situation. And that transport costs are the determining factors for prices only in the case of weak network development. For example, in the absence of alternative routes for the connection of two points Formulation of the problem.

To prove an unlimited price increase as the number of network nodes increases, it suffices to construct an appropriate example.

Let there are  $n$  customers of financial instrument and  $n + 1$  markets where this instrument is supplied. We first consider the stationary problem and set that the volumes of purchases and sales per unit of time are constant and equal to one item in each market, except the last. In the case of commodity markets, it is convenient to submit a supply by individual producers with fixed volumes of output. To change the volume of output, its readjustment is necessary, which requires a willful decision and time. Those, at a minimum, there is a lag in a bid change. Therefore, for short periods of time, the volume of supply may be a constant value. The last  $x_{n+1}$  market will represent in this example the reserve capacity of the supply (in the last market we put zero turnover). We will assume that it has the highest reaction rate to changing market conditions and it is the first among all network nodes to start increasing the supply volume.

Although each investor can enter any market, due to different transaction costs, incorporation into various trading communities and personal preferences, which ultimately also have their own monetary value, for each investor there are different preferences where to make exchange transactions. We will evaluate them in the form of some additional costs to the price of exchange transactions. In order to uniquely identify the system, we will set the cost matrix  $C_{i,j}$  in accordance with the distance formula

$$C_{i,j} = |i - j|,$$

where  $i$  is the number of the customer, and  $j$  is the number of the market where the financial instrument is being traded [14]. For the first investor, the purchases have zero added value in the first market, a unit in the second market, etc. The additional cost grows in a linear fashion depending on the range of the market. Each investor seeks to select such markets so that the total cost of his purchases would be minimal. Thus, a game problem arises. You can also draw an analogy with the classical assignment problem.

It is easy to see that in the stationary case there is a trivial solution. The first investor will purchase the entire volume of supply in the first market, the second in the second, etc. The latter  $n + 1$  market remains unclaimed. Thus, the demand coincides with the supply. Therefore,

prices will be stable for any current distribution  $\{p_{0,i}\}$ . And the sum of additional costs is zero, which, in the sense of the task, means the creation of the most efficient market. Note that in this example, for each customer there is an opportunity to enter a foreign market with a unit additional cost, i.e. the minimum cross-border costs for each participant in this example are limited throughout the network and do not exceed one.

### III. RESULTS AND DISCUSSION

#### A. Disturbed case.

Let us increase the intensity of purchases of the first investor to  $1 + \varepsilon$ , where  $\varepsilon$  is an arbitrarily small amount. In this case, he cannot fully satisfy all its demand in the first market. He needs to purchase this small supplement in other markets. The task is to determine the prices arising from this small perturbation.

Although a small variation of the initial conditions does not change the linear nature of the problem, and it is still relatively easy to solve by linear programming methods, in this case, a simpler solution can be found.

Since in the first market there was an excess of demand over supply, then, in accordance with the exchange law, price growth will begin on it. It will continue until the first investor becomes interested in making transactions in the second market. For this, the change in the stock price must exceed the additional costs of the first participant entering the second market.

$$p_1 = p_{0,1} + C_{12} = p_{0,1} + 1.$$

After the release of the first investor to the second market, the total supply for the first two markets is 2 units, and the demand  $2 + \varepsilon$ . Therefore, prices  $p_1$  and  $p_2$  will also start to grow until they reach

$$p_1 = p_{0,1} + C_{13} = p_{0,1} + 2,$$

$$p_2 = p_{0,2} + C_{23} = p_{0,2} + 1.$$

This will lead to the fact that the first two investors will start entering the third market too. It should be noted that within the framework of this model nothing is assumed about how the volumes transferred to the third market will be distributed among investors. Only their common infinitesimal volume  $\varepsilon$  is known. Repeating these arguments by induction, we obtain a running wave of price increases, which gradually covers the entire network and forms local trends in each of the markets [3]. When the wave reaches the last  $n + 1$  market, the excess demand will spill over it. In accordance with the assumptions of the model, after a while there will appear a corresponding supply, which will balance the demand and thereby stop the growing trend.

Within this model, nothing is said about when the additional volume will appear in time. As well as about what transients (corrections, waves, etc.) will flow. Therefore, generally speaking, one cannot even argue that the system will necessarily reach equilibrium. Moreover, observations from practice show that the case of chaotic oscillations around a certain price band is quite possible.

Note that in the current formulation, this example is a type of the classical assignment problem, which can be

solved in various ways using linear programming methods. The problem is that, although the volumes of purchases and sales in our example are known, there is no such information on real exchange trading. Therefore, future price dynamics is unknown. This does not allow the use of various mathematical optimization methods in solving practical problems.

One of the missions of the exchange is just the collection of initial orders and thus the determination of the current market situation. Those, on the stock exchange, this problem is solved with the help of informational approaches, rather than mathematical approaches. This includes solving the problem of finding the optimal price with current profiles of supply and demand that is so important for practice.

### B. Price Sensitivity.

Although the issues of convergence of prices to a certain limiting state in case of demand perturbation remain debatable, nevertheless, there is an equilibrium solution in prices expressed by the formulas

$$p_i = p_{0,i} + C_{i,n+1} = p_{0,i} + n - i, \quad i = 1, \dots, n.$$

From the above we easily get the following

#### Theorem.

With an arbitrarily small perturbation  $\varepsilon$  of the supply/demand ratio, the price can change indefinitely with the increase in the number of network nodes  $n + 1$ .

This example also allows us to formulate a stronger proposition about the asymptotics of price growth.

#### Lemma.

The price increase may follow a linear law on the number of network nodes.

For this example, it is important to assume that there is a fixed supply and that there are no significant reserves of supply capacity, which allows a wave of disturbances to run along with the network and form price trends in certain markets.

The question arises, how this model example can be applied to real trading. Two points can be made.

In practice, the condition of the fixed supply is not always satisfied. Although it will take some time due to the lag effect, but with sufficiently large times there will certainly be ways to increase supply and thereby compensate for the lack of demand locally, without requesting the entire network. But even in this case, there will be long periods of trends with strong price changes [1].

A more important limitation is the condition of the lack of reserve supply capacity. At least in the commodity markets such a situation is rare. But even in this case, from a qualitative point of view, many of the conclusions of the article remain true if we replace the infinitely small perturbation of demand with a limited, but finite. If this imbalance of demand exceeds local reserves of supply, then it is possible to obtain long periods of price growth in one direction due to the network nature of price distribution. At the same time, the price change will significantly exceed the value of the imbalance of demand.

## IV. CONCLUSION

Mathematical graph of the future network between the points of advanced economic growth in the strategy of spatial development of the Russian Federation is formalized.

An example of a mathematical network pricing model was constructed, in which an arbitrarily small perturbation of the supply/demand ratio leads to an indefinite increase in prices.

The price increase may have linear asymptotics from the number of local network markets.

The wave-like nature of the propagation of disturbance of supply-demand imbalance through the network is shown, which determines price trends at each point of the network.

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