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Dynamic Model of Controlling the Behavior of an Economic Agent Using the Mechanism of Self-Regulation of Resource Flows

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***Abstract*—The problem of development of the analytical system of simulation modeling (ASSM) of functioning processes of the economic system investigated at the macro level is considered. The structure of the complex of dynamic models of economic agents, which are considered at the macro level and presented in the form of firms, households (population), banks and the state, is presented. The concept of dynamic non-equilibrium mode of functioning of the macroeconomic system (MES) is introduced. The system principles for constructing dynamic models of behavior of macroeconomic agents (MEA) are stated. The dynamic model of the controlled behavior of the MEA which is analyzed from the standpoint of various approaches that demands performance of a number of transformations of the scheme of dynamic model is presented. The three variants of interpretation of the scheme of the dynamic model of the MEA behavior (from the standpoint of economic theory, control theory and systems theory) allowed to reveal the regularities of the MEA behavior in non-equilibrium modes. It is shown that non-equilibrium modes arising in the behavior of one agent extend to other agents. The proposed algorithm provides the restoration of the dynamic equilibrium mode of the whole MES. The application of controlled dynamic models of the MEA and the developed ASSM is advisable as a research system for the purpose of analyzing different management scenarios of the functioning process of the MES as a whole.**

Keywords*—*dynamic model, economic agents, reserves of resources, self-regulation mechanism, system approach, management

I. INTRODUCTION

In modern conditions, the problems of studying the dynamics of reproductive processes at the macroeconomic level are particularly acute, since the systemic crisis of the Russian economy, accompanied by violations of the main reproductive proportions, although it was softened, but not completely overcome, and then worsened under the influence of the global financial crisis [1,2]. Within this direction the researches on creation of complexes of economic and mathematical models and program tools for the solution of various problems of the macroeconomic analysis [3-15] are conducted.

For the analysis of the proportions of the reproduction process and the mechanisms of distribution of financial flows in dynamics, macro-structural models are developed using various tools: interindustry balance; integrated social matrices - *SAM*-matrices (*Social Account Matrix*); system dynamics, computable general equilibrium models - *CGE*-models (*Computable General Equilibrium*), models of inter-temporal equilibrium, *SFC*-models (*Stock-Flow Consistent*), macroeconomic and interindustry models developed by the international community *INFORUM* [3-6,10-15].

Despite the large number of works in the studied direction, it should be recognized that due to the complexity of the problems solved, some features of the dynamics of reproductive processes are not fully reflected in the developed model complexes. In particular, the issues of analysis of the dynamics of reproduction processes, which are considered taking into account the closure of the macroeconomic cycle of financial flows and the impact of stocks of macroeconomic agents, remain insufficiently studied.

To support the procedures of analysis and formation of scenarios of macroeconomic system management (MES), an analytical system of simulation modeling (ASSM) of MES functioning processes is developed on the basis of the previously developed complex of dynamic models of macroeconomic circulation of flows forming the reproduction process [16-18]. It should be noted that the macroeconomic system is understood as a system of social production carried out on a national scale, carried out by the interacting sectors of the economy (agents) and regulated by public administration.

The complex of dynamic models of macroeconomic circulation includes four dynamic models of macroeconomic agents' behavior: firms, households (population), banks and the state [18]. The interrelated functioning of macroeconomic agents covers all stages of the reproduction process and forms the macroeconomic cycle [18]. This article solves the problem of developing an algorithm to control the behavior of a single macroeconomic agent on the basis of information about the reserves of previously accumulated resources.

II. SYSTEM PRINCIPLES OF CONSTRUCTING DYNAMIC MODELS OF MEA BEHAVIOR

The system principles of construction of dynamic models of behavior of macroeconomic agents are formulated.

First, a macroeconomic agent (MEA) is a person in the economy who can make economic decisions, act as a producer or buyer of economic goods (goods, services). Every macroeconomic agent: has reserves of financial resources; in accordance with its role in the reproduction process, performs one or more functional processes, which is accompanied by the formation of expenses (outflows) of financial resources; receives income (inflows) through one or more channels from other MEA; forms a balance of income and expenditure rates; forms reserves, which, depending on the sign of the balance, either increase or decrease. It should be noted that according to the logic of the macroeconomic cycle, the construction of a closed reproductive cycle is ensured by observing the rule: the expenses of one MEA necessarily become the income of another MEA.

Secondly, the situation where balance of flow is zero, and the quantity of reserves is not changed, corresponds to a *dynamically equilibrium mode* of functioning of the MEA. The formation of conditions for the balance of income and expenditure for each MEA allows to derive the main known macroeconomic identities [17]. Only if the balance of expenditures and incomes is observed for all MEAs, the operation of the whole MEA as a whole in dynamically equilibrium modes is possible at the same time.

Third, *dynamically non-equilibrium* modes of operation of the MEAs arise from the violation of the equality of expenditure and income for at least one MEA. The rebalancing of MES is carried out using a self-regulation mechanism that is implemented when building the model of the managed behavior of MEA.

III. DYNAMIC BEHAVIOR MODEL OF MEA AND ITS ANALYSIS FROM THE STANDPOINT OF ECONOMIC THEORY

The model of controlled behavior of MEA is developed in the class of dynamic nonlinear models with logical elements. The model is implemented using the libraries of the MATLAB Simulink application, as well as using modules, the operation logic of which is programmed in the MATLAB language [18].

The generalized scheme of the functional model of the controlled behavior of MEA is presented in the form of parallel channels of income generation and several, also parallel, channels of expenses formation for the implementation of functional processes. The goal of managing each functional cost-building process is to ensure the planned rate of resource consumption. The formation of the plan is carried out in three steps. The first step involves taking into account autonomous, independent of information on the behavior of other sectors, expenditure plans (for example, autonomous consumption by the population). In the second step, this information is taken into account, for example, in the form of dependence of population consumption on disposable income received from firms as wages. Obtained in this step plans should be balanced by the flows to ensure the equilibrium mode. The third step is the adjustment of plans based on information about reserves using an algorithm based on the mechanism of self-regulation of financial flows.

Let us consider the features of the algorithm operation on the example of a simplified model of the MEA, which performs one functional process (has one outflow) and receives income on one channel (has one inflow). This is necessary to ensure clarity of the following transformations of the functional scheme of the model.

From the standpoint of the economic content, the functional scheme of the model of the controlled behavior of the i-th MEA (Fig. 1) should be represented as a parallel connection of channels for the formation of the inflow (income) with rate $In_i(t)$ and outflow (expenditures) with rate $Out_i(t)$, as well as series-connected adder to calculate the balance of flows $Fl_i(t)$ and integrator to calculate the current amount $St_i(t)$ of accumulated reserves of financial resources [19]. The integrator in this way of representation of the scheme is the final element. The block of implementation of the functional process is represented by a continuous dynamic model built in the class of controlled systems based on the principle of combined control.

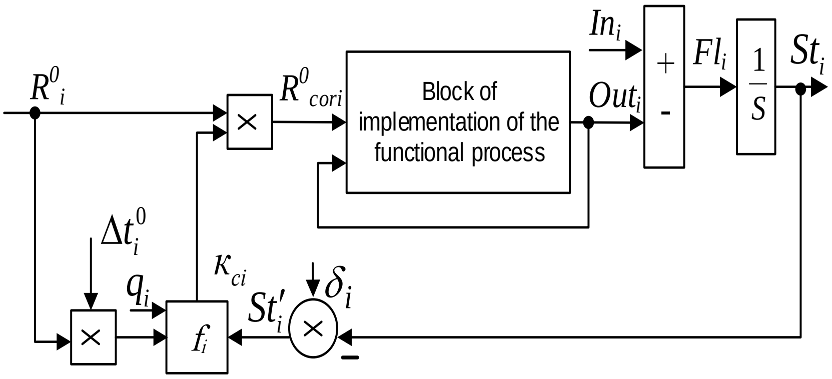

Fig. 1. Functional scheme of the model of controlled behavior of i-th MEA (from the standpoint of economic theory)

The peculiarity of the proposed model is to build a channel of resource consumption, which contains a logical block f_i , implementing an algorithm for adjusting the planned rate of consumption based on information about the current available volume of reserves $St_i'(t)$. The coefficient k_{ci} is calculated by the formula:

$$k_{ci}=\begin{cases}1, R_i^0\Delta t_i^0 \le St_i' \le R_i^0\Delta t_i^0 q_i\\ \dfrac{St_i'}{R_i^0\Delta t_i^0}, St_i' \le R_i^0\Delta t_i^0\\ \dfrac{St_i'}{R_i^0\Delta t_i^0 q_i}, St_i' > R_i^0\Delta t_i^0 q_i\end{cases} \quad (1)$$

where $St_i' = St_i(t) - \delta_i(t)$, $\delta_i(t)$ – emergency reserve, Δt_i^0 – the time interval during which the functional process must be provided with reserves of resources, q_i – a coefficient that determines how many times the current amount of reserves must exceed the required in order to make a decision to increase the rate of consumption of resources (Fig. 2).

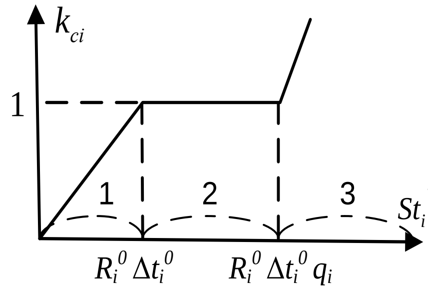

Fig. 2. Graph of nonlinear dependence $k_{ci} = f_i(St_i')$

Note that the algorithm of calculation of the coefficient k_{ci} is implemented in software in the *MATLAB* language.

The calculation of the correction coefficient k_{ci} is performed on the basis of analysis of information about the volume of current reserves. At the same time, an analysis of the sufficiency of current reserves for the consumption of resources with a planned rate over time is carried out Δt_i^0 ($\Delta t_i^0 \ge 1$). It should be noted that the reasons for changes in the volume of reserves are either changes in the rate of income from other MEAs, or in the adjustment of plans due to changes in the behavior of other agents. Any change in the volume of reserves indicates the onset of a dynamically non-equilibrium MEA functioning mode.

In case of insufficient reserves (situation for zone 1, $k_{ci} < 1$) the rate is reduced with such a coefficient, multiplying by which gives a possible rate, which is provided by the current reserves for the planned time interval. The decrease in the rate of resource consumption associated with a decrease in reserves is forced, to track the time of occurrence of such situations for a person (system analyst) is not possible, therefore, for the control algorithm "for reducing" the automatic mode is applied. Control of the condition of sufficiency of the reserve of resources is carried out at each step of modeling during the whole period of modeling.

For favorable situations of reserves growth (situation for zone 3, $k_{ci} > 1$) when reserves exceed a certain threshold, which is q ($q \ge 1$) times more than the required volume, the recalculation of plans in the direction of increase is performed. The introduction of this condition for decision-making to increase the stepped-up pace is explained by the fact that the rate of resource consumption should not increase as soon as there is a slight excess of reserves, but only when a certain limit is reached.

Such a decision to increase the planned rate of consumption of resources must be based on the analysis of a large number of data on the status of MEAs in general, therefore, the algorithm adjustments "to increase" is implemented in the automated mode. In general, two modes of operation of the control algorithm are proposed in ASSM: the mode of adjusting the rate of resource consumption as "to decrease and "to increase"(*regU*$_i$ flag=2) and the mode of adjusting the rate of resource consumption only "to decrease" (*regU*$_i$ flag = 1).

For situations of zone 2 ($k_{ci} = 1$) the MEA supports the planned consumption of resources $R_i^0(t)$ despite the fluctuations in volume of reserves within the acceptable range defined by formula (1). The size of the dead zone (resistance of the agent to external disturbances) is determined by both the reserves planning horizon Δt_i^0 and the parameter q. Since the values of these parameters determine the behavior of the MEA in dynamically non-equilibrium modes, we refer them to the control and call the control parameters "damping" reserves. The values of these parameters determine not only the volume of reserves at which any changes in the rate of consumption begin, but also, as a consequence, the time of the beginning of these changes. For example, at small values of control parameters, that is, at small volumes of "damping" reserves, in situations of decrease in reserves the MEA reacts very late to decrease in reserves, and in situations of growth of reserves (positive balance) – very early.

IV. ANALYSIS OF DYNAMIC BEHAVIORS OF THE MEA FROM THE STANDPOINT OF CONTROL THEORY

In order to analyze the dynamics of the MEA behavior in these groups of situations, let us perform scheme transformations. The considered type of the model scheme corresponds to its economic content, formulated earlier in accordance with economic theory. Experiments carried out for different forms of change in the time rate of income $In_i(t)$, showed that the proposed algorithm for adjusting the planned rate of consumption of resources provides an approximation of the rate of consumption of resources $Out_i(t)$ to the form of changes in the rate of income $In_i(t)$. The functional scheme of the model presented from the standpoint of control theory (Fig. 3) explains this effect.

The scheme is obtained by moving the adder calculating the balance $Fl_i(t)$ (and now the error) and the integrator calculating the stock volume $St_i'(t)$ to the beginning of the scheme. Now it becomes obvious that there is a second control circuit for feedback from the driving force in the form of the rate of $In_i(t)$ income (inflow). Consequently, the MEA managed behavior model is dual-circuit and is designed to control the rate of resource consumption $Out_i(t)$. In situations where $k_{ci} = 1$ the control purpose will be recorded: $Out_i(t) \to R_i^0(t)$, in other situations $Out_i(t) \to In_i(t)$.

It is as if the switch of control purpose occurs: for the dead zone, this is the development of the initial plan; for inclined sections of the nonlinear characteristic, this is the approximation of the rate of consumption to the current rate of income.

In other words, the dynamic non-equilibrium modes ensure *self-regulation* of the rate of flow of expenditures and revenues of the macroeconomic agent.

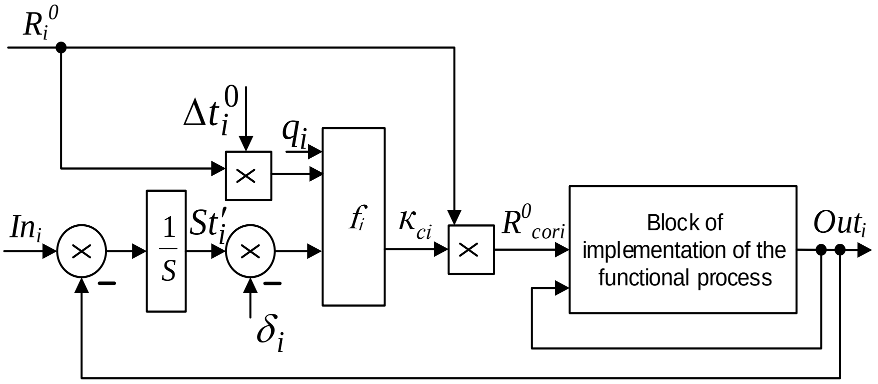

Fig. 3. Functional scheme of the model of controlled behavior of *i*-th MEA (from the standpoint of control theory)

V. ANALYSIS OF DYNAMIC BEHAVIOR MODEL OF THE MEA FROM THE STANDPOINT OF THEORY OF SYSTEMS

The effect of self-regulation can be explained from the third side - from the standpoint of the theory of systems, if we involve the system principle of non-equilibrium state in the analysis [20-22]. According to the latter, the system is in a dynamically non-equilibrium state if the equality between two oppositely directed forces of the system is violated:

$$\frac{dX}{dt} = k\big(F(+) - F(-)\big).$$

For the MEA , the rate of income generation is considered as a force $F(+)$ contributing to the movement of the system. In the general case, by modeling the operation of all MES as a whole, its graph has a free form, in the simplest case is a straight line (Fig. 4). The rate of expenditure is considered $F(-)$ as a counteracting force. The type of flow-reserve characteristic for the rate of resource consumption repeats the nonlinear dependence (1). The reserves are the regulating X systemic factor. Then the equation of motion of the system will take the form:

$$\frac{dSt_i}{dt} = k\big(In_i(St) - Out_i(St)\big).$$

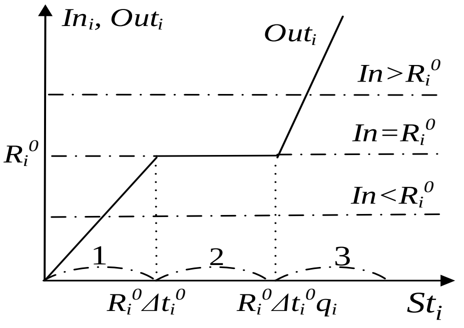

Fig. 4. Graphical view of the dependence of the force contributing to (income rate $In_i(t)$) and counteracting (expenditure rate $Out_i(t)$) the movement of the MEA (from the standpoint of the theory of systems)

The transformations of this equation, performed under the conditions of ideal (non-inertial) implementation of the functional process with a transfer coefficient equal to 1, as well as with zero emergency reserve $\delta_i(t)$, allow to obtain a differential equation of the inertial link, the input of which is the rate of income $In_i(t)$, and the output is the volume of reserves $St_i(t)$. The transfer coefficient k_p and time constant τ are determined by the control parameters Δt_i^0 and q. For group 1 situations $k_p = \Delta t_i^0$; $\tau = \Delta t_i^0$; and for group 3 – $k_p = \Delta t_i^0 q_i$; $\tau = \Delta t_i^0 q$. Let us introduce a coefficient ρ:

$$\rho = \frac{1}{k_p} = \frac{1}{\tau}$$

(equal to the tangent of the slope angle of the flow curve) and call it the self-regulation coefficient, since its increase contributes to a faster achievement of a new equilibrium state, and vice versa.

The behavior of the agent is the more reactive, that is, the faster the equilibrium of flow rates will occur and the less the new equilibrium state will differ from the previous one, the greater the self- regulation coefficient ρ. However, according to the logic of reserves planning, the remaining reserves will be smaller due to the smaller transfer coefficient, that is, the smaller values of the control parameters Δt_i^0 and q. In other words, a higher reactivity behavior is inherent to the agent if the it's "damping" reserves are less, and Vice versa. At the same time, the large reactivity of the MEA allows it to react by expenditure changes later - with a reserves decrease and earlier, if reserves grow. If "damping" reserves of agent is scheduled more (larger values of control parameters Δt_i^0 and q), and MEA will react earlier, but with greater persistence with a lack of reserves; and later and more slowly when there are the reserves excess requiring increasing in the rate.

Note that by changing the combination of the values of the control parameters Δt_i^0 and q, it is possible to describe the different dynamic characteristics of the MEA response to changes in reserves. For example, if the MEA plans small tactical reserves (small values Δt_i^0) and large strategic reserves (large *q values)*, then the agent is lagging (retarded): first, it reacts to the downward trend of reserves very late, although it reduces the rate very quickly; and, secondly, it reacts to the upward trend of reserves also very late, waiting for the accumulation of large volumes of reserves, and very

TABLE I. TYPE OF THE MANAGED BEHAVIOR OF MEA ON NON-EQUILIBRIUM MODES

Values Δt_i^0 и q_i / Characteristics MEA behavior	Δt_i^0 - small		Δt_i^0 - large		Situations
	q_i - small	q_i - large	q_i - small	q_i - large	
Time t_d to start reduction in the rate	late	**late**	**early**	early	decrease in reserves (group 1)
The amount of remaining reserves at the time of reduction in the rate	**small**	small	large	**large**	
Dynamic characteristics of the reaction at reduction in the rate	fast	fast	slow	slow	
Time t_{inc} of the start of rate increase	early	**late**	**early**	late	increase of reserves (group 2)
Amount of accumulated reserves at the time of increasing the rate	**small**	large	small	**large**	
Dynamic characteristics of the reaction at the increase in the rate	fast	average	average	slow	
Type of the managed MEA behavior	*Fast MEA* with *small* reserves	*Lagging* MEA	*Leading* MEA	*Slow* MEA with *large* reserves	

slowly increases the pace. Different variants of the dynamic characteristics of the controlled behavior of the MEA depending on the control parameters Δt_i^0 and q are presented in Table 1.

Note that higher initial values of reserves, of course, in all situations lead to a later reaction of the MEA.

Thus, the obtained three variants of interpretation of the model of controlled behavior of the MEA allow to form a three-fold view of the proposed model, which embodied the laws of both economic theory and management theory, as well as the theory of systems. The obtained *triad of theories* that form the mathematical basis for describing the behavior of the MEA, is a reflection of the *triad of approaches* that form the basis of the concept of research and modeling of MES. The concept is based on the integration of approaches: system, dynamic (embodied in the theory of management), as well as cognitive and scenario, reflected in the economic theory [17,19-22].

VI. ANALYSIS OF DYNAMIC BEHAVIOR MODEL OF THE MEA FROM THE STANDPOINT OF THEORY OF SYSTEMS

Studies of the MEA behavior model for group 1 situations with a negative flow balance are carried out.

The algorithm operation mode only "on the decrease" is applied $regU_i$=1, the projected income rate $In^0 = R_i^0 = 10$, the reserves volume $St_i^0 = 59$, the safety planning period $\Delta t_i^0 = 3$. Figure 5 shows: the basic experiment 1 with the perturbation, applied at time t=23 in the form of lower rate of income $\Delta In = -5$; experiment 2, characterized by the initial value of the reserve $St_i^0 = 95$; and experiment 3, which increased the planning horizon of reserves $\Delta t_i^0 = 5$.

The first two experiments do not differ in dynamic characteristics (equal time constants and transfer coefficients, new steady-state values of reserves $St_i = 15$). Only the moment of the beginning of the slowdown for higher initial reserves in experiment 2 comes later. In the third experiment, due to a larger reserve of stocks, the agent becomes more inertial, reacts more slowly to a decrease in reserves, but earlier; therefore, the reserves remain larger when the equilibrium mode is restored $St_i = 25$ (due to a large transfer coefficient). The balance is restored $In_i(t) = Out_i(t)$ in all situations.

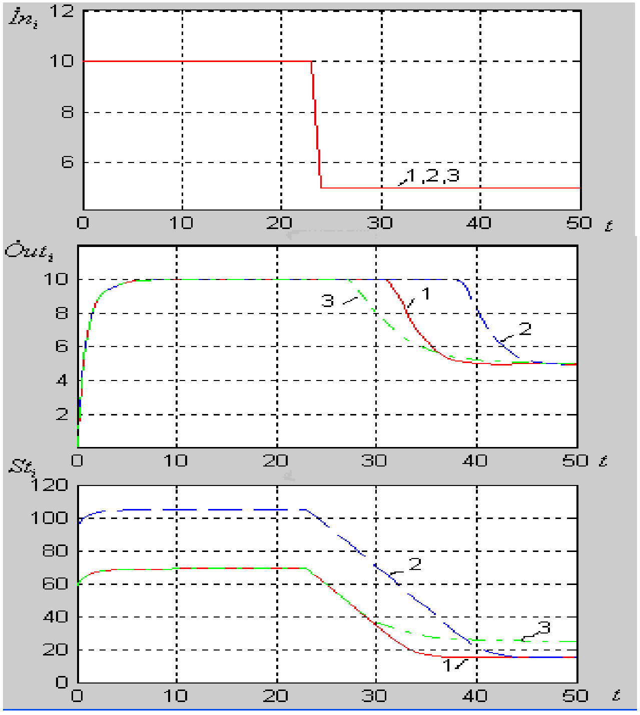

Fig. 5. Graphs of transient processes of the functioning of the MEA

VII. CONCLUSION

Thus, the proposed algorithm for controlling the behavior of macroeconomic agents provides self-regulation of income and expenditure rates on non-equilibrium modes of operation. The proposed variants of interpretations of the functional scheme of the MEA behavior model allowed: to explain the economic content of the model structure; to formulate management goals in various non-equilibrium situations; and to describe the dynamic features of the MEA transition process from one dynamically equilibrium state to another. The study of the dynamics of interrelated functioning of the MEA in the composition of the MES have shown that non-equilibrium modes that emerged in the behavior of a single agent are propagated to other agents. The proposed algorithm provides the restoration of the dynamic equilibrium mode of the whole MES. The use of managed dynamic models of the MEA and the developed ASSM is advisable as a research system for the analysis of different scenarios of management of the reproduction process of the macroeconomic system.

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