# Multi Criteria Group Decision Making Approach for Smart Phone Selection Using Intuitionistic Fuzzy TOPSIS

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#### Abstract

The objective of this study is to provide an effective multi criteria decision making (MCDM) approach with group decision making to evaluate different smart phone alternatives according to consumer preferences. The choice of the most appropriate phone is a very complex decision, involving several perspectives. In such complex situations intuitionistic fuzzy sets with TOPSIS (IF-TOPSIS) can be utilized to eliminate uncertainty and to better represent decision makers' preferences. The originality of the paper comes from its ability to handle high uncertainty in a smart phone selection and provide a real case study with IFS for the first time.

Keywords: Smart phone selection, Intuitionistic fuzzy sets, IF-TOPSIS, Group decision making.

## 1. Introduction

The mobile phone technology has been developed and launched in the late 1980ies, and since then it has demonstrated a continuous, rapid and widespread growth. With its young population and developing economy, Turkey has also experienced a similar trend since the introduction of this technology in 1994. As of 2014, the mobile communication industry in Turkey enjoys approximately 68 million registered mobile phones, 58% of which having access to the 3G technology, double of the European average<sup>1</sup>.

Presently, according to Information and Communication Technologies Authority (ICTA) there are around 72 million mobile subscribers in Turkey, corresponding to a ratio of 92.7% of active mobile phone lines for a consumer base. When the consumer base is limited by excluding the ages between 0-9, then this ratio even exceeds  $100\%^2$ .

Number of mobile phones in Turkey is expected to increase and the number of 3G subscribers has reached to 59.4 million<sup>2</sup>. Turkish consumers are also displaying high mobile phone usage rates with an average monthly airtime of 299 minutes, the highest in Europe<sup>1</sup>. Turkey is selected for the reason that its position as an emerging economy in Europe with a rapid smart phone market growth rate in terms of penetration rate as well as talking time per user. The spread of the technology has caused a decline of smart phone prices and a large selection of available models, giving better access for the masses.

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Following a wider consumer base, producers have started to better react to consumer expectation<sup>3</sup> and come up with various designs and features to address different needs in the market, with regards to social and financial segmentation of the market. Moreover, access to the internet using Wi-Fi or mobile network is certainly an important feature for smart phones. With many built-in applications a mobile phone can nowadays be defined as a "smart phone".

Recently, consumers are being offered with a range of smart phones in a specific price band and the selection of a smart phone becomes an important decision problem<sup>5</sup>. Although smart phones can be used more or less for a life time of ten years, most users prefer to use their phones for a shorter time and change them with newer models. The smart phone industry proposes new features to attract users, operates competitively with rivalries and even shortens the life time of their phone models<sup>4</sup>. In addition, presenting a series of new features such as high resolution cameras, physical durability and other trends complicates the smart phone selection by consumers. Therefore, this study aims to propose a decision procedure for smart phone selection on a Turkish company. The selection procedure is ever more important for the case company, as it purchases large amounts of smart phones for its white collar personnel.

Multi criteria decision making (MCDM) refers to finding the most appropriate option where many different decision criteria are to be taken into account simultaneously<sup>6-7</sup>. In these processes, Group Decision Making (GDM) involves multiple decision makers (DMs) who have different goals or ways of thinking and can assess the decision process distinctively different from others. Nevertheless, for each of the assigned DMs there is a common interest to interact with each other in order to reach collective decision<sup>8-11</sup>. Especially when uncertainty exists, achieving consensus for a decision in a group with different opinions becomes more important<sup>11</sup>. Generally, GDM problems are solved by utilizing classic approaches, such as the majority rule, minority rule or total agreement. 10 However, these techniques do not guarantee a solution that is accepted by all DMs. Therefore, Consensus Reaching Processes (CRPs) are becoming more and more necessary. 10-11

Briefly, this study aims to solve this MCDM problem by using intuitionistic fuzzy sets (IFS) with

GDM approach. In this pursuit, the proposed methodology makes use of intuitionistic fuzzy technique for order performance by similarity to ideal solution, shortly called IF-TOPSIS, which has been developed by Boran<sup>12</sup>. The underlying logic of TOPSIS<sup>13</sup>, a popular MCDM technique, is its consideration of the positive and negative ideal solutions for dealing with decision problems<sup>14,15</sup>. In MCDM, various criteria need to be evaluated in order to reach a single alternative. However, evaluations of DMs can be rather difficult to collect and process, as the feedbacks DMs express are usually not precise. For this reason, IFS<sup>16</sup> are utilized to eliminate uncertainty and to better represent DMs' preferences<sup>10</sup>. In recent literature, some studies have clearly indicated that IFS present a powerful method to cope with uncertainty in decision problems<sup>17-19</sup>. In contrast to the fuzzy set theory, in IFS the data information assigns a membership degree, a nonmembership degree and a hesitancy degree to each component. As stated in Xu & Liao's research<sup>19</sup>, triangular fuzzy numbers, trapezoidal fuzzy numbers and interval-valued fuzzy numbers can only be used to depict the fuzziness of agreement but cannot reflect the disagreement of the DMs. However in real life, human beings frequently disagree, which is a common way for expressing their ideas. Hence in GDM problems IFS cope with these situations and aggregate experts' opinions for a collective decision 19-21. Additionally, a TOPSIS model under intuitionistic environment is utilized to establish a flexible and robust way for DMs to better understand a decision problem in case of uncertainty and vagueness in DMs' perceptions<sup>20-21</sup>. Several publications have reported advantages of effective consideration of vague information 19-23.

The main contribution of the paper is the definition and development of smart phone selection problem with IF-TOPSIS for the first time. To authors' best knowledge, no study so far has utilized this technique for the smart phone selection problem. Besides, this article also contributes to literature by presenting a case study to help corporate customers to better take decisions by evaluating and accordingly selecting the most appropriate smart phone.

This study is structured such that the 2<sup>nd</sup> section presents a comprehensive literature review about the smart phone selection and techniques that is employed, and the 3<sup>rd</sup> section provides a detailed description of the

methods used. The 4<sup>th</sup> section gives a case study of smart phone selection by using application steps of the framework. Comparison and sensitivity analysis of the results are presented in 5<sup>th</sup> section. Finally, concluding remarks and guidance for future studies are presented in 6<sup>th</sup> section.

#### 2. Literature Survey

#### 2.1. Smart phone selection/evaluation

Over years, technology and style of mobile phones such as technical features, color, design, size etc. have changed significantly. At the beginning, mobile phones focused were largely limited with voice calling functions. Nowadays, cellular phones offer many more services such as camera, messaging (e.g. SMS, MMS, email), internet access, other connection options (e.g. infrared, Bluetooth), music, business and gaming applications. Cellular phones offering such functions in addition to voice calling are generally referred to as "smart phones"<sup>24</sup>.

In the current telecommunications market, mobile communication is becoming popular, thus gradually increasing consumers' desire to use smart phones. Many of the products on the shelves also provide hints about the social and financial status of users, their preferences and attitudes. This turns the selection of a smart phone by a consumer into an important decision problem in which the most appropriate handset is selected even though the user is interested in smart phones in a fixed range of price. As user expectations can vary for each consumer, the selection of a smart phone can be seen as a complex MCDM problem. This study proposes a solid mechanism to support users in deciding on the most suitable mobile phone in the marketplace<sup>5</sup>.

Smart phones are becoming an integral component of daily and business life, so that interest in this technology and in available product alternatives in the market is rising. The Worldwide Quarterly Mobile Phone Tracker published by International Data Corporation suggests that manufacturers have shipped more than 330 million units globally in the 1<sup>st</sup> quarter of 2015. Accordingly, the Android operating system dominated the market with more than <sup>3</sup>/<sub>4</sub> share in the same period. Samsung, an electronics manufacturer, leads this large market by offering a wide range of smart phones, offering cutting edge as well as low-cost smart

phones. In this picture, Turkey has been one of the forerunners in the adoption of mobile communications. Presently, more than ten global smart phone vendors are actively operating in Turkey, each one having a relatively large product line<sup>25</sup>. As of 2015, Apple, Nokia, Samsung, LG and HTC are the top five in the Turkish smart phone market. Overall, smart phone penetration continues to rise and the number of mobile broadband subscribers (computer and mobile handset) is around 33.9 million, indicating the intensity of mobile telecommunication in Turkey.

As smart phones become powerful mobile tools, Economides and Grousopoulou<sup>26</sup> presented students' considerations about the importance and costs of the features and services of their smart phones. In their study, the opinions of male and female students are investigated with a survey to gain insight about the importance and costs of these gadgets. Another research from Bayraktar, Tatoğlu, Turkyilmaz, Delen and Zaim<sup>25</sup> focused on customer satisfaction and loyalty (CS&L) for existing mobile phone brands in the Turkish market. Hu, Lu and Tzeng<sup>27</sup> concentrated on smart phone improvement for promoting product value to satisfy customer needs. Rahul and Majhi<sup>28</sup> investigated consumer satisfaction and loyalty in the Indian mobile phone market. Mobile phone use behavior and differences in connection with male and female consumers is evaluated in Haverila's study<sup>24</sup>, which investigated the relationship between feature preferences and customer satisfaction and repurchase intent among young male users. An interesting research topic is the mobile phone feature preferences. Işıklar and Büyüközkan<sup>5</sup>, Haverila<sup>29</sup> and Haverila<sup>30</sup> published articles on this topic. For instance, Haverila<sup>30</sup> discussed the progressive evolution of specific smart phone feature preferences in Finland among different high school and college students.

### 2.2. Smart phone evaluation criteria

Selecting criteria is one of the most important dimensions while constructing a decision model. Therefore, criteria are important components that enable alternatives to be compared from a specific point of view. Generally, users get satisfied with a particular product when its properties match with their preferences and expectations. In order to develop an effective

decision model, most important product selection criteria by consumers need to be identified.

In order to prepare this study and develop the proposed selection model, a series of recent publications dealing with the smart phone selection problem are reviewed. Moreover, the opinions of smart phone market experts are taken into account to identify selection criteria. Based on Işıklar and Büyüközkan's<sup>5</sup> Haverila's<sup>24</sup>, Mokhlis and Yaakop's<sup>31</sup>, Hu, Lu and Tzeng's<sup>27</sup>, Hsiao and Chen's<sup>4</sup> studies evaluation model criteria are determined and given in Table 1. Existing literature provides various models on smart phone selection research. For instance, Işıklar Büyüközkan<sup>5</sup> and Chen<sup>32</sup> concentrated on product and user-related criteria. Işıklar and Büyüközkan<sup>5</sup> introduced product-related criteria which consist of basic requirements, physical specifications and technical features. The user-related criteria was divided into functionality, brand choice and customer excitement. Another research by Chen, Hu, Kuo and Liang<sup>33</sup> identified the brand, price, hardware feature/functionality, basic built-in functions and extended built-in functions as being the most essential 5 selection criteria for purchasing a smart phone. Mokhlis and Yaakop31 summarized the selection criteria as innovative features, image, price, personal recommendation, durability and portable aspects, media influence, and post-sales service. More recently, Hu, Lu and Tzeng<sup>27</sup> defined three dimensions which are customer equity, product function and mobile convenience as influencing factors for consumer willingness to purchase a smart phone. Hsiao and Chen<sup>4</sup> came up with three demand dimensions and emphasized their differences. Accordingly, the smart phone handset, subscription to the 2G/3G network and mobile services are the basic dimensions of consumer demand.

Table 1. Summary of criteria for mobile/smart phone selection

Criteria	Descriptions	Studies in literature
Durability	The ability to endure	Işıklar and Büyüközkan <sup>5</sup> ,Mokhlis and Yaakop <sup>31</sup> , Hsiao and Chen <sup>4</sup> .
Battery life	The duration of a rechargeable battery of a product	Işıklar and Büyüközkan <sup>5</sup> , Economides and Grousopoulou <sup>26</sup> , Chen, Hu, Kuo and Liang <sup>33</sup> , Hsiao and Chen <sup>4</sup> .
Changeable parts	The components that can be changed easily	Mokhlis and Yaakop <sup>31</sup> , Hsiao and Chen <sup>4</sup> .
Dimensions	The physical characteristics of a product	Işıklar and Büyüközkan <sup>5</sup> , Chen, Hu, Kuo and Liang <sup>33</sup> , Haverila <sup>24</sup> , Hsiao and Chen <sup>4</sup> ,Ling, Hwang and Salvendy
Memory capacity	The storage capacity of data during the use of the smart phone	Chen, Hu, Kuo and Liang <sup>33</sup> , Haverila <sup>24</sup> , Hu, Lu and Tzeng <sup>27</sup> , Hsiao and Chen <sup>4</sup> .
Processor speed	The speed of programmable integrated circuit	Hu, Lu and Tzeng <sup>27</sup> , Hsiao and Chen <sup>4</sup> .
Internet connection speed (4G/5G)	Quality and performance of the internet connection	Hsiao and Chen <sup>4</sup> .
Camera specifications	The physical characteristics of camera such as resolution image quality etc.	Chen, Hu, Kuo and Liang <sup>33</sup> , Hsiao and Chen <sup>4</sup> .
Operation system easiness	User interface and interaction functions of a smart phone should be simple, plain and intuitive	Işıklar and Büyüközkan <sup>5</sup> , Haverila <sup>24</sup> , Hsiao and Chen <sup>4</sup> .
Variety of applications	Provides application services to consumers	Haverila <sup>24</sup> ,Hsiao and Chen <sup>4</sup> .
Brand choice	The relation to how the customer insists on the brand	Işıklar and Büyüközkan <sup>5</sup> , Chen, Hu, Kuo and Liang <sup>33</sup> , Haverila <sup>24</sup> , Hu, Lu and Tzeng <sup>27</sup> , Hsiao and Chen <sup>4</sup> .
Prestige	The reputation of a product	Mokhlis and Yaakop <sup>31</sup> , Haverila <sup>24</sup> , Bayraktar, Tatoğlu, Turkyilmaz, Delen and Zaim <sup>24</sup> .
Fashionable style/ Aesthetics	Conforming to the current styles or trends of a product	Işıklar and Büyüközkan <sup>5</sup> ,Chen, Hu, Kuo and Liang <sup>33</sup> , Haverila <sup>24</sup> ,Mokhlis and Yaakop <sup>31</sup> , Hsiao and Chen <sup>4</sup> .
Personal information and media security	The resistance degree or protection from harm	Işıklar and Büyüközkan <sup>5</sup> , Haverila <sup>24</sup> .
Price	It refers to reasonable cost/price and often serves as a quality indicator	Işıklar and Büyüközkan <sup>5</sup> , Chen, Hu, Kuo and Liang <sup>33</sup> , Haverila <sup>24</sup> , Mokhlis and Yaakop <sup>31</sup> , Hsiao and Chen <sup>4</sup> .
Warranty / Service Availability	It represents guarantee which provide assurance to consumers dealing with problems.	Işıklar and Büyüközkan <sup>5</sup> , Haverila <sup>24</sup> , Mokhlis and Yaakop <sup>31</sup> .

The structure of the proposed model make use of the following criteria: Consumers highly regard physical conditions in selecting their phones, including Durability, Battery life, Changeable parts, Dimensions. Physical conditions refer to the design standards of a product and constitute an essential component. These conditions can differ from product to product. Haverila<sup>24</sup> underlined the importance of various feature preferences. This preference consideration becomes particularly important as manufacturers have many feature alternatives to add to a phone model among a large set of potential features. A large number of technical features, however, do not necessarily affect consumers towards a positive purchase decision. Embedding too many features might even have a negative impact on consumers if these features can be perceived to be unnecessary or complicated. Therefore, basic conditions appear to be grouped together.

Technical conditions refer to the technological assistance of products and include Memory capacity, Processor speed, Internet connection speed (4G/5G) and Camera specifications. Beside hardware, consumers also value the software on a mobile phone, i.e. Functionality conditions. Operation system easiness and Variety of applications are among the most interesting features<sup>35</sup>.

Brand/Market conditions involves the subjective and invisible evaluation of the brand based on the brand awareness, the brand ethics and the brand attitude of the customer. Brand/Market conditions include Brand choice, Prestige, Fashionable style/Aesthetics, Personal information and media security, Price Warranty/Service Availability. In mobile sector, consumers are facing higher security and privacy risks because of the data transaction in a wireless environment.

#### 2.3. IF-TOPSIS with GDM environment

MCDM is one of the popular methods to deal with complicated problems that exhibit high uncertainty, clashing objectives, various interests and multiple perspectives<sup>32-34</sup>. Besides, MCDM methods are effective in decision making, weighting and selecting the most appropriate alternatives<sup>17-18</sup>. A number of researchers have used different MCDM techniques in the fields of information, mobile communications, music business and gaming applications<sup>35,36</sup>. For instance; Işıklar and

Büyüközkan<sup>5</sup> developed a MCDM technique where they evaluated different mobile phone options with respect to users' preferences by applying AHP and TOPSIS. Chen, Hu, Kuo and Liang<sup>33</sup> proposed recommendation systems for online mobile phone stores by using an AHP-based mechanism. Hu, Lu and Tzeng<sup>27</sup> proposed a hybrid MCDM model for promoting a smart phone's product value by using DEMATEL, ANP and VIKOR. Apart from these MCDM techniques, this study utilized Intuitionistic Fuzzy Sets (IFS) introduced by Atanassov<sup>16, 37</sup>. IFS can be seen as the extension of fuzzy sets which was originally proposed by Zadeh<sup>38</sup>. IFS can be characterized by the components of a membership function, a non-membership function and a margin for hesitation. It is flexible for dealing with uncertainty, whereas fuzzy sets are characterized by only their membership function<sup>16</sup>. When the individual evaluations of DMs in a GDM under uncertainty are concerned, it must be taken into account that not all DMs have the same level of knowledge, background and experience. They can differ in terms of skills, personality or area of research<sup>39,40</sup>. Therefore, uncertainty is natural for DMs providing their preferences, characteristics of affirmation, negation, and hesitation to some extent<sup>41,42</sup>. IFS can in such cases be a helpful tool with its flexibility and robustness<sup>17,18</sup>. IFS can also take the degree of hesitation into account and can deal with any error or lack of knowledge in defining the membership function. Beside these advantages, IFS can also cope with uncertain and vague objects. As such, it provides researchers and DMs with a powerful tool for expressing data under different fuzzy environments<sup>17,19</sup>.

Integration with GDM can be exemplified in Xu and Liao's research<sup>19</sup>. It is stated that triangular fuzzy numbers, trapezoidal fuzzy numbers and interval-valued fuzzy numbers can only be used to depict the fuzziness of agreement but cannot reflect the disagreement among DMs. However, in real life recognition of human beings' disagreement is a common expression for effectively describing and communicating opinions. Hence IFS copes with such situations and aggregates experts' opinions for a collective decision in GDM problems<sup>43</sup>.

A key approach in GDM problems is to include many alternatives and DMs from different or same disciplines to find the most suitable solution among a set of available alternatives and attempt to reach a collective decision<sup>44,45</sup>. Aggregation of expert opinions plays a central role in reaching a collective decision with an evaluation process<sup>46,47</sup>. For this purpose, the technique called intuitionistic fuzzy weighted averaging (IFWA) operator is proposed by Xu<sup>40</sup>. IFWA can be used for merging the opinions of each individual DM together, where the aggregated result is used for assigning a value to the importance of selection criteria and available alternatives 12,43. Besides, Xu and Yager 48 introduced additional intuitionistic fuzzy geometric aggregation operators, such as intuitionistic fuzzy weighted geometric averaging operator (IFWGA), intuitionistic fuzzy ordered weighted geometric averaging operator (IFOWGA) and intuitionistic fuzzy hybrid geometric averaging operator (IFHGA). Xu<sup>40</sup> presented some other intuitionistic fuzzy aggregation operators, such as intuitionistic fuzzy ordered weighted averaging (IFOWA) operator and the intuitionistic fuzzy hybrid averaging (IFHA) operator. He, Chen, Zhou, Liu and Tao<sup>49</sup> developed another operator, intuitionistic fuzzy geometric interaction averaging (IFGIA) operator.

Following these valuable developments in literature and other important contributions, researchers recently started to investigate IFS in MCDM. As one of best known MCDM methods, TOPSIS technique was initially proposed by Chen and Hwang<sup>50</sup>. The developed

TOPSIS method proposed by Hwang and Yoon<sup>13</sup> incorporates a simple computation process, systematic procedure, and a solid logic that considers the rationale of people's choices. In this paper, TOPSIS is utilized as a ranking technique based on its rational logic and understandability<sup>14</sup>. Although TOPSIS is very popular to solve MCDM problems, this approach also has some weaknesses. A better approach may be to use IFS rather than fuzzy sets, where criteria ratings and weights are found with intuitionistic numbers. Recently, many researchers extended IF-TOPSIS for MCDM. These methodologies have been applied satisfactorily to different research areas for evaluation purposes and these are summarized in Table 2.

As seen in Table 2, IF-TOPSIS is integrated with different techniques in various fields of application. On the other hand, IF-TOPSIS is utilized with interval valued approach which is different from Boran's <sup>12-14</sup> proposed approach. To authors' best knowledge; there exists no publication in which IF-TOPSIS is used for the smart phone selection problem. Therefore, this paper contributes to the literature by addressing this research gap and demonstrating the applicability of the proposed method with a case study.

Table 2. Several studies make use of IF-TOPSIS

Year	Authors	Intuitionistic type	Application area
2009	Boran, Genç, Kurt and Akay <sup>14</sup>	TOPSIS, GDM	Illustrative example (Supplier selection)
2009	Boran <sup>12</sup>	TOPSIS	Case study (Personnel selection)
2010	$Ye^{51}$	Interval-valued IFS, TOPSIS, GDM	Illustrative example (Partner selection)
2011	Boran <sup>52</sup>	TOPSIS	Illustrative example (Facility location selection)
2011	Tan <sup>53</sup>	Interval-valued intuitionistic fuzzy sets Choquet integral, <b>TOPSIS</b>	Illustrative example (Investment selection)
2011	Su, Chen, Xia and Wang <sup>54</sup>	TOPSIS, consensus	Illustrative example (3PL logistic provider selection)
2012	Boran, Boran and Menlik <sup>43</sup>	TOPSIS	Case study (Renewable energy resource selection)
2013	Intepe, Bozdağ and Koç <sup>55</sup>	Interval-valued IFS, TOPSIS, GDM	Case study (3D TV technology selection)
2013	Vahdani, Mousavi, Moghaddam and Hashemi <sup>56</sup>	ELECTRE, TOPSIS,GDM	Illustrative example (Flexible manufacturing systems selection)
2014	Kucukvar, Gumus, Egilmez, and Tatari <sup>57</sup>	TOPSIS	Illustrative example (Asphalt pavement selection)
2014	Joshi and Kumar <sup>23</sup>	TOPSIS, entropy	Case study (Portfolio selection)
2014	Maldonado-Macías, Alvarado, García, and Balderrama <sup>22</sup>	TOPSIS, AHP	Illustrative example (Milling machine selection)
2014	Yue <sup>20</sup>	TOPSIS	Illustrative example (Chinese universities' satisfaction evaluation)
2015	Chen <sup>58</sup>	Interval-valued IFS, TOPSIS, GDM	Illustrative example (Medical treatment method selection)
2015	Zhang and Xu <sup>59</sup>	Interval-valued IFS, TOPSIS	Illustrative example (Supplier selection)

### 3. The Applied Methodology

This section briefly summarizes the methodology to be utilized in the paper for solving mobile phone selection problem. First, basic knowledge about IFS is presented and then the computational steps of IF-TOPSIS with GDM are explained.

#### 3.1. Preliminaries

The methodology explained in this section first presents the basic definitions and notations of IFS, most of which are taken from Atanassov's study.<sup>37</sup> In a finite set of X, IFS S can be stated as:

$$S = \{\langle x, \mu_S(x), \nu_S(x) \rangle x \in X\}$$

Here,  $\mu_S(x)$ ,  $v_{(S)}(x)$ :X $\rightarrow$  [0,1] is the membership function and the non membership function respectively, so that,

$$0 \le \mu_S(x) + \nu_S(x) \le 1 \tag{1}$$

In IFS, there is another parameter  $\pi(x)$ , called the "hesitation degree" that checks if x belongs to S,

$$\pi_{S}=1-\mu_{S}(x)-v_{S}(x)$$
 (2)

Here, for every  $x \in X$ :

$$0 \le \pi_S(x) \le 1 \tag{3}$$

As  $\pi_S(x)$  becomes smaller, the certainty of the knowledge about x becomes higher. As  $\pi_S(x)$  gets higher, then the knowledge about x becomes less certain. In this case, when  $\mu_S(x) = 1 - \nu_S(x)$  for each and every element of the universe, the concept of ordinary fuzzy set is recovered. Defining M and N as two IFSs that belong to the set of X, then the multiplication operator can be defined as the following.<sup>39</sup>

$$M \otimes N = \{ \mu_M(x), \mu_N(x), \nu_M(x), \nu_N(x) - \nu_M(x), \nu_N(x) \}$$
 (4)

#### 3.2. IF-TOPSIS

The general view of GDM based approach of IF-TOPSIS is given in "Fig. 1". The methodology of IF-

TOPSIS is adapted from Boran's studies 12-14. The summary view of this framework starts with identifying evaluation criteria and alternatives using experts' opinions and a detailed literature review is required to search and collect information. Next, a committee of experts is necessary to provide group decision. According to the group qualifications, different weights are determined for each DM. Then a comparison scale to weight criteria set and rate alternatives are selected. In the next phase an aggregated intuitionistic fuzzy decision matrix is constructed based on DMs assessments. Then criteria weights based on DMs assessments is obtained. The framework concludes with focusing on the selection process. Here, TOPSIS is adapted to the system for ranking available alternatives of smart phones in a decreasing order using their relative closeness coefficient. The steps of the IF-TOPSIS are explained briefly as follows:

Define  $A = \{A_1, A_2, ..., A_m\}$  as a set of alternatives and  $X = \{X_1, X_2, ..., X_n\}$  as a set of criteria.

**Step 1:** Identify the evaluation criteria and alternatives for the smart phone selection problem. The objective is to find out the most appropriate smart phone alternatives among the others.

**Step 2:** Find the weights of DMs' evaluations. Here, the decision committee consists of three DMs. The importance degrees of each of the DM evaluations are processed as linguistic terms expressed in IFS. Assume that  $D_k = [\mu_k, \nu_k, \pi_k]$  is an intuitionistic fuzzy number with the rating of  $k^{th}$  DM. Accordingly, the weight of the  $k^{th}$  DM can be obtained as:

$$\lambda_k = \frac{\left(\mu_k + \pi_k \left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)}{\sum_{k=1}^l \left(\mu_k + \pi_k \left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)} \quad \text{and} \quad \sum_{k=1}^l \lambda_k = 1$$
 (5)

**Step 3:** Select a comparison scale to weight criteria set and rate alternatives. Linguistic label sets with their respective IFS are given in Table 3 and 4 as follows:

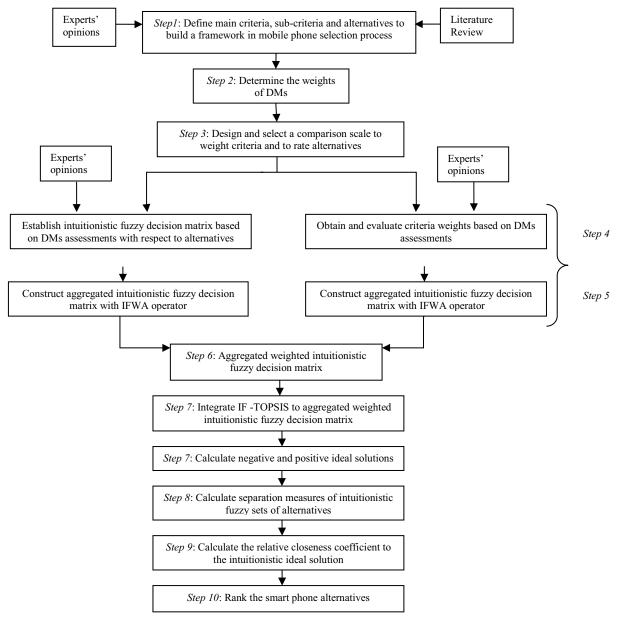


Fig. 1. Schematic diagram of IF-TOPSIS

Table 3. Linguistic terms for rating criteria

Linguistic term	IFS
Very Important (VI)	[0.9; 0]
Important (I)	[0.8; 0.1]
Moderately Important (MI)	[0.7;0.2]
Medium (M)	[0.5;0.5]
Unimportant (U)	[0.3;0.5]
Very Unimportant (VU)	[0.2;0.7]

**Step 4:** Form an aggregated intuitionistic fuzzy decision matrix that is based on DMs' assessments. Assume that  $R^{(k)} = (r_{ij}^k)_{mxn}$  is an intuitionistic fuzzy decision matrix for each DM. Let  $\lambda = (\lambda_1, \lambda_2, ..., \lambda_l)$  be the weight of each DM and  $\sum_{k=1}^{l} \lambda_k = 1$ ,  $\lambda_k \epsilon [0,1]$ . Operator IFWA will then be used to aggregate DMs' evaluations in order to rate the importance values of decision criteria and available alternatives in the GDM process.

Table 4. Linguistic terms for rating alternatives

Linguistic terms	IFS
Extremely Good (EG)	[1,0,0]
Very Good (VG)	[0.75, 0.1, 0.15]
Good (G)	[0.6,0.25,0.15]
Moderately Good (MG)	[0.5, 0.4, 0.1]
Medium (M)	[0.5, 0.5, 0]
Moderately Bad (MB)	[0.4, 0.5, 0.1]
Bad	[0.25, 0.6, 0.15]
Very Bad (VB)	[0.1, 0.75, 0.15]
Very very Bad (VVB)	[0,0.9,0.1]

$$R = (r_{ij})_{mxn} \text{ and } r_{ij} = (\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j))$$

$$i=(1,2,...,m; j=1,2,...,n)$$

$$r_{ij} = IFWA_{\lambda}(r_{ij}^{(1)}, r_{ij}^{(2)}, ..., r_{ij}^{(l)})$$

$$= \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \lambda_3 r_{ij}^{(3)} \oplus ... \oplus \lambda_l r_{ij}^{(l)}$$

$$= [1 - \prod_{k=1}^{l} (1 - \mu_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l} (\nu_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l} (1 - \mu_{ij}^{(k)})^{\lambda_k}, - \prod_{k=1}^{l} (\nu_{ij}^{(k)})^{\lambda_k}]$$
(6)

Then intuitionistic fuzzy decision matrix can be defined as:

$$R = \begin{bmatrix} \mu_{A_1}(x_1), \nu_{A_1}(x_1), \pi_{A_1}(x_1) & \mu_{A_1}(x_2), \nu_{A_1}(x_2), \pi_{A_1}(x_2) & \dots & \mu_{A_1}(x_n), \nu_{A_1}(x_n), \pi_{A_1}(x_n) \\ \mu_{A_2}(x_1), \nu_{A_2}(x_1), \pi_{A_2}(x_1) & \mu_{A_2}(x_2), \nu_{A_2}(x_2), \pi_{A_2}(x_2) & \dots & \mu_{A_2}(x_n), \nu_{A_2}(x_n), \pi_{A_2}(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_m}(x_1), \nu_{A_m}(x_1), \pi_{A_m}(x_1) & \mu_{A_m}(x_2), \nu_{A_m}(x_2), \pi_{A_m}(x_2) & \dots & \mu_{A_m}(x_n), \nu_{A_m}(x_n), \pi_{A_m}(x_n) \end{bmatrix}$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \dots & r_{1m} \\ r_{21} & r_{22} & r_{23} & \dots & r_{2m} \\ r_{31} & r_{32} & r_{33} & \dots & r_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & r_{n3} & \dots & r_{nm} \end{bmatrix}$$

**Step 5:** Construct and evaluate weights of criteria according to DMs' viewpoints. The importance degrees of each criteria can be shown with "W". Then, in order to evaluate importance degrees, all individual opinions have to be fused. Here,  $W_j^{(k)} = \left[\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)}\right]$  is defined as IFS that is assigned to criteria  $X_j$  by the  $k^{th}$  DM. Their criteria weights are calculated as follows:

$$W_{j} = IFW A_{\lambda} (W_{j}^{(1)}, W_{j}^{(2)}, ..., W_{j}^{(l)})$$

$$= \lambda_{1} W_{j}^{(1)} \oplus \lambda_{2} W_{j}^{(2)} \oplus \lambda_{3} W_{j}^{(3)} \oplus ... \oplus \lambda_{l} W_{j}^{(l)}$$

$$= \left[1 - \prod_{k=1}^{l} (1 - \mu_{j}^{(k)})^{\lambda_{k}}, \prod_{k=1}^{l} (v_{j}^{(k)})^{\lambda_{k}}, \prod_{k=1}^{l} (1 - \mu_{j}^{(k)})^{\lambda_{k}}, - \prod_{k=1}^{l} (v_{j}^{(k)})^{\lambda_{k}}\right]$$

$$W = \left[W_{1}, W_{2}, W_{3}, ..., W_{j}\right]$$

$$W_{j} = \left[\mu_{j}, v_{j}, \pi_{j}\right] (j=1, 2, ..., n).$$

$$(7)$$

**Step 6:** Establish the aggregated weighted intuitionistic fuzzy decision matrix based on the previously constructed criteria weights (W) and the aggregated intuitionistic fuzzy decision matrix, as shown below:

$$R \otimes W = \{\langle x, \mu_{A_i}(x), \mu_w(x), v_{A_i}(x) + v_w(x) - v_{A_i}(x), v_w(x) \rangle | x \in X \}$$
(8)  
Next,  

$$\pi_{A_i}w(x) = 1 - v_{A_i}(x) - v_w(x) - \mu_{A_i}(x), \mu_w(x) + v_{A_i}(x), v_w(x)$$
(9)

Finally, the aggregated weighted intuitionistic fuzzy decision matrix is found as:

$$R' = \begin{bmatrix} r'_{11} & r'_{12} & r'_{13} & \dots & r'_{1m} \\ r'_{21} & r'_{22} & r'_{23} & \dots & r'_{2m} \\ r'_{31} & r'_{32} & r'_{33} & \dots & r'_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r'_{n1} & r'_{n2} & r'_{n3} & \dots & r'_{nm} \end{bmatrix}$$

 $r'_{ij} = (\mu'_{ij}, v'_{ij}, \pi'_{ij}) = (\mu_{A_iW}(x_j), v_{A_iW}(x_j), \pi_{A_iW}(x_j))$  is an element of the aggregated weighted intuitionistic fuzzy decision matrix, as found above.

**Step 7:** Integrate IF-TOPSIS to aggregated weighted intuitionistic fuzzy decision matrix. Following that, calculate the distances from positive and negative ideal points. Assume that  $J_1$  represents the benefit criteria and  $J_2$  represents the cost criteria. Here,  $A^+$  is defined as the intuitionistic fuzzy positive-ideal solution and  $A^-$  is defined as the intuitionistic fuzzy negative-ideal solution, as shown below:

$$A^{+} = \left( (\mu_{A^{+}W}(x_{j}), v_{A^{+}W}(x_{j}) \right) \text{ and}$$

$$A^{-} = \left( (\mu_{A^{-}W}(x_{j}), v_{A^{-}W}(x_{j}) \right) \text{ where,}$$
(10)

$$\mu_{A^+W}(x_j) = \left( \left( \max_i \mu_{A_i,W}(x_j) \mid j \in J_1 \right), \left( \min_i \mu_{A_i,W}(x_j) \mid j \in J_2 \right) \right)$$
(11)

$$v_{A+W}(x_j) = \left( \left( \min_i v_{A_i,W}(x_j) \mid j \in J_1 \right), \left( \max_i v_{A_i,W}(x_j) \mid j \in J_2 \right) \right)$$

$$\mu_{A-W}(x_j) =$$
(12)

$$\left(\left(\min_{i} \mu_{A_{i},W}(x_{j}) \mid j \in J_{1}\right), \left(\max_{i} \mu_{A_{i},W}(x_{j}) \mid j \in J_{2}\right)\right) \tag{13}$$

$$v_{A-W}(x_j) = \left( \left( \max_i v_{A_i.W}(x_j) \mid j \in J_1 \right), \left( \min_i v_{A_i.W}(x_j) \mid j \in J_2 \right) \right)$$

$$(14)$$

Step 8: Calculate the separation measures of the intuitionistic fuzzy sets of the available alternatives. The distance of each alternative from the positive and negative ideal points are computed as follows:

$$S^{+} = \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left[ \left( \mu_{A_{i}W}(x_{j}) - \mu_{A^{+}W}(x_{j}) \right)^{2} + \left( v_{A_{i}W}(x_{j}) - v_{A^{+}W}(x_{j}) \right)^{2} + \left( \pi_{A_{i}W}(x_{j}) - \pi_{A^{+}W}(x_{j}) \right)^{2} \right]}$$

$$S^{-} = \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left[ \left( \mu_{A_{i}W}(x_{j}) - \mu_{A^{-}W}(x_{j}) \right)^{2} + \left( v_{A_{i}W}(x_{j}) - v_{A^{-}W}(x_{j}) \right)^{2} + \left( \pi_{A_{i}W}(x_{j}) - \pi_{A^{+}W}(x_{j}) \right)^{2} \right]}$$

$$= \sqrt{\frac{1}{2n}} \sum_{j=1}^{n} \left[ \left( \mu_{A_{i:W}}(x_{j}) - \mu_{A^{-W}}(x_{j}) \right)^{2} + \left( v_{A_{i:W}}(x_{j}) - v_{A^{-W}}(x_{j}) \right)^{2} + \left( \pi_{A_{i:W}}(x_{j}) - \pi_{A^{+W}}(x_{j}) \right)^{2} \right]$$
(16)

Step 9: Find the relative closeness coefficient (CC<sub>i</sub>) for the intuitionistic ideal solution. For an alternative A<sub>i</sub>, its CC<sub>i</sub> with respect to A<sup>+</sup> can be found as:

$$CC_{i} = \frac{S_{i-}}{S_{i-} + S_{i+}} \tag{17}$$

Step 10: Rank the alternatives in decreasing order of  $CC_i$  values. At this step, the alternative which has the maximum  $CC_i$  is selected.

## **Case Study**

The methodology is applied to a case of smart phone selection and evaluation in a Turkish company. Turkish retail market with modern retail formats and large chains has played an important role in the Turkish economy. One of the most notable components can be defined as electronics and telecommunication. There are more than 68 million phone users in Turkey. The high number of mobile phone usage rates and devices in Turkey, as well as recent publications have raised our interest to establish a decision model for choosing the most suitable smart phone. In this perspective, the evaluation methodology is used in a Turkish Company X (company name is not given due to privacy concerns) which aim to purchase smart phones for their white collar employees. The process is critical because of a relatively high number (approximately 60) of smart phones to be purchased. A decision committee consisting of three experts is formed with members DM1, DM2 and DM3 with the aim to determine the most appropriate smart phone among three possible alternatives. In this evaluation process, DM1 is the procurement manager who is well informed about the mobile telecom market and is knowledgeable about available smart phone alternatives. DM2 is the human resources manager of the company, who has deep information about the profile of the Company's employees and their smart phone preferences. DM3 is the quality manager who focuses on the quality of the products.

Step 1: In this step, smart phone selection criteria presented in the 2<sup>nd</sup> section are used and decision making process is done by the aid of experts. There are three alternatives; Iphone 6 16 GB (A1), Samsung Galaxy S6 32GB (A2) and HTC One M9 (A3). These models are selected because of their comparable prices in the Turkish market. The criteria are summarized in Fig. 2.

Step 2: Establish DMs' weights. In this case study, the decision making procedure is carried out with the support of three DMs and the weights of DMs are determined by using Table 3 and Equation (5). The importance of linguistic variables of the three DMs is as the following:  $\lambda_1$  presents as "medium importance"  $\lambda_2$ presents as "very important",  $\lambda_3$  presents as "very important". By applying Equation (5) the DMs weights are found as 0.2266, 0.3867 and 0.3867 respectively.

Step 3: The nine- and six-label linguistic evaluation scale has already been discussed in the previous section. In order to do comparisons, DMs analyze alternatives and criteria according to their interest, expertise and their intuition.

Step 4: The evaluations given by each DM for each of the three alternatives are shown in Table 5. Rating the alternatives is carried out based on Table 3. Next, according to these assessments R is constructed, as shown in Table 6. As an example, by using Equation 6 and 7;  $\mu_{11}$ = 1-((1-0.5)0.2266 \*(1-0.5) 0.3867 \*(1-0.5) 0.3867) = 0.500.

Next  $v_{11} = (0.5)^{0.2266} * (0.5)^{0.3867} * (0.5)^{0.3867} =$ 0.5006 and  $\pi_{11} = 1 - 0.500 - 0.500 = 0.000$ . The other values are calculated similarly as stated above. For A2 and A3, the remaining R values are calculated in same way.

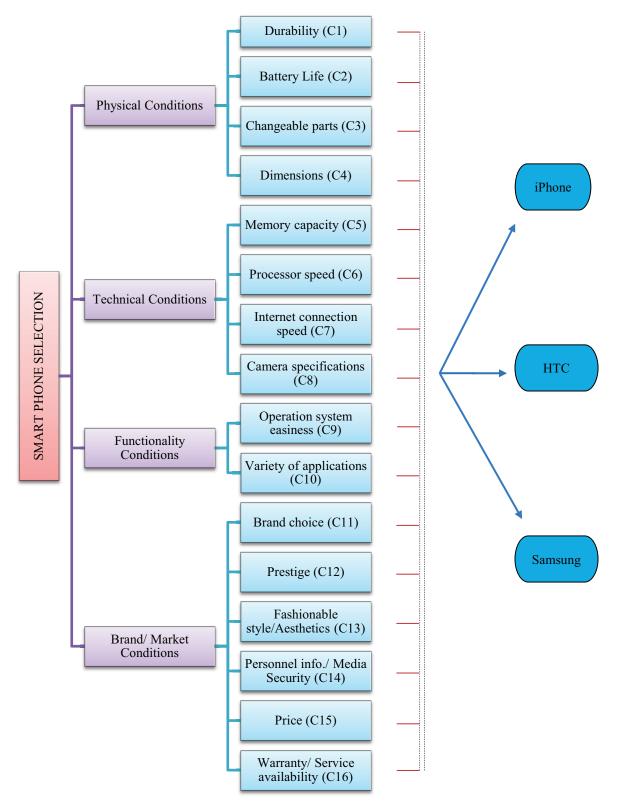


Fig. 2. The evaluation criteria for smart phone selection

Table 5. Linguistic evaluation data of alternatives with respect to criteria

Alternatives	Criteria	DM1	DM2	DM3
	C1	M	M	M
	C2	M	M	В
	C3	M	M	M
	C4	VG	G	G
A1	C5	MD	MD	MD
	C6	G	G	G
	C7	G	G	G
	C8	MD	MD	MD
	C9	G	G	VG
	C10	MD	MD	M
	C11	VG	VG	VG
	C12	VG	VG	VG
	C13	VG	VG	VG
	C14	M	M	В
	C15	VG	VG	VG
	C16	VG	VG	VG
	C1	G	G	MD
	C2	G	G	G
	C3	M	M	M
	C4	MD	MD	MD
	C5	G	G	G
	C6	G	VG	G
	C7	G	G	G
A2	C8	G	M	G
	C9	G	G	G
	C10	M	MD	MD
	C11	VG	VG	G
	C12	VG	G	G
	C13	M	M	M
	C14	G	G	G
	C15	G	G	G
	C16	G	G	G
	C1	G	G	G
	C2	G	MD	MD
	C3	M	M	M
	C4	M	M	M
	C5	VG	VG	VG
	C6	G	VG	G
	C7	G	G	G
A3	C8	VG	VG	VG
	C9	G	G	G
	C10	M	MD	MD
	C11	MD	G	G
	C12	MD	G	MD
	C13	M	M	M
	C14	G	G	G
	C15	MD	M	M
	C16	VG	G	G

**Step 5:** Construct and evaluate weights of criteria according to DMs' viewpoints. Table 2 is utilized for corresponding linguistic terms for evaluation. Using DMs' assessments, Table 7 is constructed and then these assessments are aggregated with the same approach. By using Equation 7, the weights of criteria are calculated and can be seen in Table 8.

Table 6. Aggregated intuitionistic fuzzy decision matrix (R)

Alternatives	Criteria	DM1	DM2	DM3
	C1	0.500	0.500	0.000
	C2	0.415	0.537	0.048
	C3	0.500	0.500	0.000
	C4	0.640	0.203	0.156
	C5	0.500	0.400	0.100
	C6	0.600	0.250	0.150
	C7	0.600	0.250	0.150
A1	C8	0.500	0.400	0.100
	C9	0.666	0.175	0.158
	C10	0.500	0.436	0.064
	C11	0.750	0.100	0.150
	C12	0.750	0.100	0.150
	C13	0.750	0.100	0.150
	C14	0.415	0.537	0.048
	C15	0.750	0.100	0.150
	C16	0.750	0.100	0.150

Table 7. Linguistic evaluation data of criteria

Criteria	DM1	DM2	DM3
C1	MI	MI	MI
C2	M	MI	I
C3	M	MI	MI
C4	VI	I	I
C5	M	MI	I
C6	M	M	I
C7	M	M	I
C8	M	MI	I
C9	U	M	MI
C10	U	M	MI
C11	I	VI	I
C12	I	I	I
C13	I	MI	MI
C14	MI	I	I
C15	MI	VI	I
C16	MI	I	I

Table 8. Calculated weight (W) of each criteria

μ	v	П
0.700	0.200	0.100
0.712	0.188	0.100
0.663	0.246	0.091
0.829	0.000	0.171
0.712	0.188	0.100
0.649	0.268	0.082
0.649	0.268	0.082
0.712	0.188	0.100
0.557	0.351	0.092
0.557	0.351	0.092
0.847	0.000	0.153
0.800	0.100	0.100
0.726	0.171	0.103
0.781	0.117	0.102
0.832	0.000	0.168
0.781	0.117	0.102

**Step 6:** The aggregated weighted intuitionistic fuzzy decision matrix is formed with the Equation 8, and the result is provided in Table 9.

Table 9. Intuitionistic fuzzy decision matrix of A1

Alternatives	Criteria	μ	v	$\Pi$
	C1	0.350	0.600	0.050
	C2	0.296	0.624	0.081
	C3	0.332	0.623	0.045
	C4	0.531	0.203	0.266
	C5	0.356	0.513	0.131
	C6	0.390	0.451	0.159
	C7	0.390	0.451	0.159
A1	C8	0.356	0.513	0.131
	C9	0.371	0.465	0.164
	C10	0.279	0.634	0.088
	C11	0.635	0.100	0.265
	C12	0.600	0.190	0.210
	C13	0.545	0.254	0.201
	C14	0.324	0.591	0.085
	C15	0.624	0.100	0.276
	C16	0.586	0.205	0.209

For A2 and A3, the remaining  $\mu$ , v and  $\prod$  values are calculated in same way.

**Step 7:** Obtain the intuitionistic fuzzy positive and negative ideal solutions with the Equations 10-14. The results are given in Table 10. For A2 and A3, the remaining r values are calculated in same way.

Table 10. Intuitionistic fuzzy positive and negative ideal solutions of A1

r1	0.420	0.400	0.180
r2	0.427	0.391	0.182
r3	0.332	0.623	0.045
r4	0.531	0.203	0.266
r5	0.534	0.269	0.197
r6	0.433	0.397	0.171
r7	0.390	0.451	0.159
r8	0.534	0.269	0.197
r9	0.371	0.465	0.164
r10	0.279	0.624	0.097
r11	0.635	0.100	0.265
r12	0.600	0.190	0.210
r13	0.545	0.254	0.201
r14	0.468	0.338	0.194
r15	0.624	0.100	0.276
r16	0.586	0.205	0.209

**Step 8:** The separation measures of intuitionistic fuzzy sets are calculated by using Equation 15 and 16. As an example, the positive separation measure of A1 is calculated as follows:

$$S^{+} = \frac{1}{2}(|0.350 - 0.420| + |0.600 - 0.400| + |0.500 - 0.180| ... + |0.209 - 0.209|) = 1.237.$$

The remaining S<sup>-</sup> values are calculated in same way.

**Step 9:** Calculate the CC<sub>i</sub> to the intuitionistic ideal solution. Table 11 shows the final ranking and CC<sub>i</sub>.

Table 11. Separation measures and CC<sub>i</sub> of each alternative

Alternatives	$S^+$	S-	$CC_i$
A1	1.237	1.573	0.560
A2	1.340	1.470	0.523
A3	1.621	1.190	0.423

**Step 10:** According to the  $CC_i$  values, the product A1 (Iphone 6 16 GB) is identified as the best ranking phone with a score of 0.560.

## 5. Comparison / Sensitivity Analysis

#### 5.1. Comparison with Chen's fuzzy TOPSIS

A comparative analysis is carried out to investigate the consistency of the rank and weight of the alternatives selection. The work that makes use of IF-TOPSIS is tested and compared with the results of Chen's<sup>60</sup> fuzzy TOPSIS method. Table 12 presents the ranking of the alternatives according to their performance indices. According to the overall result of Fuzzy TOPSIS, A2 is the best alternative followed by A1 and A3 (A2>A1>A3). The ranking of the alternatives changes very small comparing to the results of utilized IF-TOPSIS method (A1>A2>A3).

Table 12. Performance indices of each alternative by using fuzzy TOPSIS

Alternatives	Performance index
A1	0.0506
A2	0.0522
A3	0.0499

From this research, the ranking orders are slightly inconsistent with Fuzzy TOPSIS due to use of different preference scales of the DMs. However, the utilized preference scale with hesitation degree is based on the IFS notation. As a summary, it can be seen that the differences in the values may come from the intuitionistic evaluations of the utilized method, providing more flexible and informative definitions of fuzzy sets.

#### 5.2. Sensitivity analysis

A sensitivity analysis is performed in order to determine whether the final solution is robust to changes of the weights of a specific expert. Considering that the priorities are remarkably dependent on subjective judgments of the DMs, the stability of the final ranking under different weights should be checked out. With regard to this purpose, it is better to execute a sensitivity analysis based on a set of cases that reflect different views on the relative importance of the determinants. By altering the weights and providing some insights into the results are the main idea of the analysis. Initially, DM1 has "medium", DM2 and DM3 have "very important" linguistic importance weights. This is the current situation and named as CASE 1. In the sensitivity analysis (see Table 13) the importance weights are changed to "very important" for DM1, "medium" for DM2 and "important" for DM3 - named as CASE 2. In another case, named as CASE3, weights are changed to "important" for DM1, "very important" for DM2 and "medium" for DM3. The results of cases are presented in Tables 14 and 15.

Table 13. Different importance weights of experts

Experts (DM)	Importance weights CASE 1	Importance weights CASE 2	Importance weights CASE 3
DM1 (Marketing and sales manager)	0.227	0.406	0.344
DM2 (Human resources manager)	0.387	0.238	0.414
DM3 (Quality manager)	0.387	0.356	0.242

Table 14. Separation measures and CC<sub>i</sub> of each alternative (CASE 2)

Alternatives	$S^+$	$S^{-}$	$CC_i$
A1	1.203	1.631	0.575
A2	1.310	1.525	0.538
A3	1.628	1.207	0.426

Table 15. Separation measures and CC<sub>i</sub> of each alternative (CASE 3)

Alternatives	S <sup>+</sup>	<i>S</i> -	CCi
A1	1.207	1.584	0.568
A2	1.305	1.486	0.532
A3	1.597	1.193	0.428

It is seen that as no significant changes occur in the most important alternative, the results are not sensitive to the importance weights of DMs. In other words, the sensitivity analysis shows that the changes in the DMs' weights do not cause any change in the ranking of the considered smart phone alternatives. This means that our decision is robust against possible changes in DMs' weights.

#### 6. Conclusion

The main objective of this paper is to identify the most suitable smart phone alternative by taking various decision criteria and consumer preferences into account. Evaluation of smart phone options includes subjective and qualitative judgments and requires different complex factors. For this reason, the evaluation problem needs MCDM methods to correctly select the most appropriate smart phone alternative. In GDM problems, DMs' opinions may differ substantially. Therefore, to come to a meaningful and reliable solution, it is preferable to consider group decision in decision process. In this study, IF-TOPSIS is used with a GDM approach which allowed us to mathematically represent the uncertainty and vagueness and reflect the DMs' perception in the decision process. Besides, the model is illustrated with a case study to exemplify the decision framework.

This study presents a novel technique that uses an MCDM based GDM approach with IF-TOPSIS. To authors' best knowledge, this study has originality as it is the first application of IF-TOPSIS in literature with a case study of the smart phone selection problem. However, literature needs more studies conducted in other industrial fields using IF-TOPSIS.

In this paper, preferences of clients (i.e. employees of Company X) are gathered and integrated into the model with the opinions of the three managers of the same company. The aim of the next paper is to come up with another structure where every employee provides his or her personal assessment into a computer-supported, automated system that is able to collect a large number of assessments from employees and to easily integrate them into the decision procedure.

Future studies are encouraged to consider more alternatives by taking the inherent complexity of the problem into account. This decision framework is planned to be extended to other disciplines as next steps. Other extended studies can include "the hesitant fuzzy linguistic terms" with TOPSIS method<sup>61-64</sup> as well.

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#### References

- 1. Turk-Telekom 2015 Data, available at: http://www.ttinvestorrelations.com. (accessed: 20.06.2015).
- Information and Communication Technologies Authority (ICTA), electronic communications market in Turkey market data (2015 q1): available at:http://eng.btk.gov.tr. (accessed: 20.06.2015).
- 3. P. Mahatanankoon, H. J. Wen, and B. Lim, Consumer-based m-commerce: exploring consumer perception of mobile applications, *Computer Standards & Interfaces* **27**(4) (2005) 347-357.
- 4. M.-H. Hsiao, and L.-C. Chen, Smart phone demand: An empirical study on the relationships between phone handset, Internet access and mobile services, *Telematics and Informatics* **32**(1) (2015) 158-168.
- G. Işıklar, and G. Büyüközkan, Using a multi-criteria decision making approach to evaluate mobile phone alternatives, *Computer Standards & Interfaces* 29(2) (2007) 265–274.
- G. Çifçi, and G. Büyüközkan, A Fuzzy MCDM Approach to Evaluate Green Suppliers, *International Journal of Computational Intelligence Systems* 4(5) (2011) 894-909.
- A. Özgen, G. Tuzkaya, U. R. Tuzkaya, and D. Özgen, A Multi-Criteria Decision Making Approach for Machine Tool Selection Problem in a Fuzzy Environment, International Journal of Computational Intelligence Systems 4(4) (2012) 431-445.
- G. Wei, X. Zhao, and R. Lin, Some Induced Aggregating Operators with Fuzzy Number Intuitionistic Fuzzy Information and their Applications to Group Decision Making, *International Journal of Computational Intelligence Systems* 3(1) (2010) 84-95.
- 9. G. Büyüközkan, O. Feyzioglu, and G. Çifçi, Fuzzy Multi-Criteria Evaluation of Knowledge Management Tools, International Journal of Computational Intelligence Systems 4(2) (2012) 184-195.
- I. Palomares, R. M. Rodríguez, and L. Martínez López, An attitude-driven web consensus support system for heterogeneous group decision making, *Expert Systems* with Applications 40(1) (2013)139-149.
- I. Palomares, F J. Estrella, L. Martínez, and F. Herrera, Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study, *Information Fusion* 20 (2014) 252-271.
- 12. F. E. Boran, An application of intuitionistic fuzzy set on personnel selection, Thesis for the degree of M.S.c. in

- industrial engineering, Gazi University Institute of Science and Technology, 2009.
- C.L. Hwang, and K.P. Yoon, Multiple attribute decisionmaking: Methods and application, Springer, New York, 1981.
- F. E. Boran, S. Genç, M. Kurt, and D. Akay, A multicriteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, *Expert Systems* with Applications 36 (8) (2009)11363-11368.
- S. Perçin, and C. Kahraman, An Integrated Fuzzy Multi-Criteria Decision-Making Approach for Six Sigma Project Selection, *International Journal of Computational Intelligence Systems* 10 (2010) 610-621.
- 16. K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **20** (1986) 87-96.
- L. Abdullah, and L. Najib, A new preference scale of intuitionistic fuzzy analytic hierarchy process in multicriteria decision making problems, *Journal of Intelligent* & Fuzzy Systems 26(2) (2014) 1039-1049.
- L. Abdullah, and L. Najib, Sustainable energy planning decision using the intuitionistic fuzzy analytic hierarchy process: choosing energy technology in Malaysia, *International Journal of Sustainable Energy*, DOI: 10.1080/14786451.2014. 907292, 2014.
- 19. Z. Xu, and H. Liao, Intuitionistic Fuzzy Analytic Hierarchy Process, *IEEE Transactions on Fuzzy Systems* **22**(4) (2014) 749-761.
- Z. Yue, TOPSIS-based group decision-making methodology in intuitionistic fuzzy setting, *Information Sciences* 277(1) (2014) 141-153.
- 21. K. Govindan, R. Khodaverdi, and A. Vafadarnikjoo, Intuitionistic fuzzy based DEMATEL method for developing green practices and performances in a green supply chain, *Expert Systems with Applications* **42**(20) (2015) 7207–7220.
- A. Maldonado-Macías, A. Alvarado, J.L. García, and C.O. Balderrama, Intuitionistic fuzzy TOPSIS for ergonomic compatibility evaluation of advanced manufacturing technology, *International Journal of Advanced Manufacturing Technology* 70(9) (2014) 2283-2292
- 23. D. Joshi, and S. Kumar, Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making, *Egyptian Informatics Journal* **15** (2) (2014) 97-104.
- M. Haverila, Mobile phone feature preferences, customer satisfaction and repurchase intent among male users, Australasian Marketing Journal 19(4) (2011) 238-246.
- E. Bayraktar, E. Tatoglu, A. Turkyilmaz, D. Delen, and S. Zaim, Measuring the efficiency of customer satisfaction and loyalty for mobile phone brands with DEA, Expert Systems with Applications 39(1) (2012) 99-106
- A.A. Economides, and A. Grousopoulou, Students' thoughts about the importance and costs of their mobile devices' features and services, *Telematics and Informatics* 26(1) (2009) 57-84.
- 27. S.-K. Hu, M.-T. Lu, and G.-H. Tzeng, Exploring smart phone improvements based on a hybrid MCDM model,

- Expert Systems with Applications **41**(9) (2014) 4401-4413.
- T. Rahul, and R. Majhi, An adaptive nonlinear approach for estimation of consumer satisfaction and loyalty in mobile phone sector of India, *Journal of Retailing and Consumer Services* 21(4) (2014) 570-580.
- 29. M. Haverila, What do we want specifically from the cell phone? An age related study, *Telematics and Informatics* **29**(1) (2012) 110-122.
- M. Haverila, Cell phone usage and broad feature preferences: A study among Finnish undergraduate students, *Telematics and Informatics* 30(2) (2013) 177-188
- S. Mokhlis, and A. Y. Yaakop, Consumer Choice Criteria in Mobile Phone Selection: An Investigation of Malaysian University Students, *International Review of Social Sciences and Humanities*, 2(2) (2012) 203-212.
- C.H. Chen, L.P. Khoo, and W. Yan, Evaluation of multicultural factors from elicited customer requirements for new product development, *Research in Engineering Design* 14(3) (2003) 119-130.
- D.-N. Chen, P. J.-H. Hu, Y.-R. Kuo, and T.-P. Liang, A Web-based personalized recommendation system for mobile phone selection: Design, implementation, and evaluation, *Expert Systems with Applications* 37(12) (2010) 8201-8210.
- C. Ling ,W. Hwang, and G. Salvendy, Diversified users' satisfaction with advanced mobile phone features, Universal Access in the Information Society 5(2) (2006) 239-249.
- J.M. Noguera, M J. Barranco, R.J. Segura, and L. Martínez López, A Mobile 3D-GIS Hybrid Recommender System for Tourism, *Information Sciences* 215 (2012) 37-52.
- Y. Yun, and S.-H. Cha, Guitar application programming for smart phone, Communications in Computer and Information Science 300–305. Springer Berlin, Heidelberg, 2012.
- 37. K. T. Atanassov, *Intuitionistic fuzzy sets*, (Heidelberg: Springer, 1999).
- 38. L.A Zadeh, Fuzzy sets. *Journal of Information and Control* **8** (1965) 338-353.
- M. Xia, and Z. Xu, Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment. *Information Fusion* 13(1) (2012) 31-47.
- Z. S. Xu, Intuitionistic preference relations and their application in group decision making, *Information Sciences* 177(11) (2007) 2363-2379.
- L. Martinez, D. Ruan, and F. Herrera, Computing with Words in Decision support Systems: An overview on Models and Applications, *International Journal of Computational Intelligence Systems* 3(4) Special Issue (2010) 382-395.
- L. Martinez, M. Espinilla, and L. G. Perez, A Linguistic Multigranular Sensory Evaluation Model for Olive Oil, International Journal of Computational Intelligence Systems 1(2) (2008) 148-158.
- 43. F. E. Boran, K. Boran, and T. Menlik, The evaluation of renewable energy technologies for electricity generation

- in Turkey using intuitionistic fuzzy TOPSIS, *Energy* sources 7(1) (2012) 81-90.
- 44. R. de Andres, M. Espinilla, and L. Martinez, An Extended Hierarchical Linguistic Model for Managing Integral Evaluation, *International Journal of Computational Intelligence Systems* 3(4) Special Issue (2008) 486-500.
- 45. R M. Rodríguez, and L. Martínez, A Consensus Model for Group Decision Making with Hesitant Fuzzy Linguistic Information, International Conference on Intelligent Systems and Knowledge Engineering 2015, DOI 10.1109/ISKE.2015.31.
- F. J. Quesadaa, I. Palomares, and L. Martínez, Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators, *Applied* Soft Computing 35 (2015) 873–887.
- J. Liu, R M. Rodríguez, and L. Martínez, Preface New trends of information fusion in decision making, *Information Fusion* 29 (2016) 87–88.
- 48. Z. S. Xu, and R.R. Yager Some geometric aggregation operators based on intuitionistic fuzzy sets, *International Journal of General Systems* **35**(4) (2006) 417-433.
- Y. He, H. Chen, L. Zhou, J. Liu, and Z. Tao, Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making, *Information Sciences* 259 (2014)142-159.
- S. J. Chen, and C. L. Hwang, Fuzzy multiple attribute decision-making methods and application, In Lecture notes in economics and mathematical systems, (New York: Springer, 1992).
- 51. F. Ye, An extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection, *Expert Systems with Applications* **37**(10) (2010) 7050-7055.
- 52. F. E. Boran, An integrated intuitionistic fuzzy multi criteria decision making method for facility location selection, *Mathematical and Computational Applications* **16**(2) (2011) 487-496.
- C. Tan, A multi-criteria interval-valued intuitionistic fuzzy group decision making with Choquet integralbased TOPSIS, Expert Systems with Applications 38(4) (2011) 3023-3033.
- 54. Z.-X. Su, M.-Y. Chen, G.-P. Xia, and L. Wang, An interactive method for dynamic intuitionistic fuzzy multi-attribute group decision making, *Expert Systems with Applications* 38(12) (2011) 15286-15295.
- 55. G. Intepe, E. Bozdag, and T. Koc, The selection of technology forecasting method using a multi-criteria interval-valued intuitionistic fuzzy group decision making approach, *Computers & Industrial Engineering* **65**(2) (2013) 277-285.
- 56. B.Vahdani, S. Meysam Mousavi, R. Tavakkoli-Moghaddam, and H. Hashemi, A new design of the elimination and choice translating reality method for multi-criteria group decision-making in an intuitionistic fuzzy environment, *Applied Mathematical Modelling* 37(4) (2013) 1781-1799.
- 57. M. Kucukvar, S. Gumus, G. Egilmez, and O. Tatari, Ranking the sustainability performance of pavements: An

- intuitionistic fuzzy decision making method, *Automation in Construction* **40**(15) (2014) 33–43.
- T.Y. Chen, The inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making, *Applied Soft Computing* 26 (2015) 57-73.
- 59. X. Zhang, and Z. Xu, Soft computing based on maximizing consensus and fuzzy TOPSIS approach to interval-valued intuitionistic fuzzy group decision making, *Applied Soft Computing* **26** (2015) 42-56.
- 60. C.T. Chen, Extensions of the TOPSIS for group decision making under fuzzy environment, *Fuzzy Sets and Systems*, **114**(1) (2000) 1-9.
- R M. Rodríguez, L. Martínez, and F. Herrera, Hesitant Fuzzy Linguistic Term Sets for Decision Making, *IEEE Transactions on Fuzzy Systems* 20(1) (2012) 109-119.

- 62. H. Liu, and R M. Rodríguez, A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multi criteria decision making, *Information Sciences* **258**(10) (2014) 220-238.
- 63. F. J. Estrella, R. M. Rodríguez, and L. Martínez, A Hesitant Linguistic Fuzzy TOPSIS Approach Integrated into FLINTSTONES, Proceedings of the 2015 Conference of the International Fuzzy Systems Association and The European Society for Fuzzy Logic and Technology.
- 64. R.M. Rodríguez, B. Bedregal, H. Bustince, Y.C. Dong, B. Farhadinia, C. Kahraman, L. Martínez, V. Torra, Y.J. Xu, Z.S. Xu, and F. Herrera, A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making, *Information Fusion* 29 (2016) 89–97.