Direction-of-arrival estimation for noncircular signals*

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Abstract - Direction-of-arrival (DOA) estimation for near-completely noncircular signals is considered in this paper, and two DOA estimators based on *Widely Linear Minimum Variance Distortionless Response* (WL-MVDR) beamforming are proposed, called the spatial conjugate power spectrum (SCPS) method and subspace matching (MBNB) method, respectively. The performance of the proposed DOA estimators is illustrated via numerical examples and compared with other estimators.

Index Terms - array signal processing; direction-of-arrival estimation; noncircular signal

1. Introduction

The temporal features of signals, e.g., cyclostationarity, non-Gaussianity, and constant modulus properties are widely exploited to improve the performance of array signal processing [1-3]. The second-order (SO) noncircularity, as one of these available characteristics, is often utilized to achieve aperture extension and blind identification [4-15]. Signals with SO noncircularity are extensively used in communication and radar systems, e.g., binary phase shift keying (BPSK) and amplitude modulation (AM) signals. A particular kind of SO noncircular signals, namely rectilinear signals (i.e., signals with unity SO noncircular rate), have attracted interests in terms of direction-of-arrival (DOA) estimation. In [4], the standard MUSIC algorithm is extended to a nearly doubled aperture and a one-dimensional searching scheme, namely NC-MUSIC, is proposed. A closed-form NC-MUSIC via polynomial rooting techniques for uniform linear arrays (ULAs) is given in [5]. An extended 2*q*-MUSIC algorithm, which is based on even-order statistics, is formulated in [6]. DOA estimation algorithms for noncircular signals using certain types of arrays, e.g., acoustic vector sensor and conjugate symmetric arrays, are also studied in [7-8] and [9], respectively. Some reviews and performance analysis of the use of noncircularity in array signal processing can be found in [10-13].

All the high-performance DOA estimation algorithms mentioned above are based on the stringent assumption on the coherence between the signal $s(t)$ and its conjugate form $s^*(t)$, i.e., the assumption of unity noncircular rate. In practice, this hypothesis is often unsatisfied due to reasons such as frequency offsets or nonnull carrier residues [14]. Besides, in satellite communication systems, the unbalanced quadrature phase shift keying (UQPSK) signals are often used, of which the two orthogonal components have unequal powers [16]. In both aforementioned scenarios, the noncircular signals possess noncircular rates less than unity and are called partially noncircular signals. However, DOA estimation for partially noncircular signals is rarely considered in the literature.

Recently, Chevalier and Blin proposed a widely linear minimum variance distortionless response (WL-MVDR) beamformer for noncircular (both rectilinear and partially noncircular) signals [14]. Motivated by this fact, we herein propose a WL-MVDR based direction-finding algorithm for partially noncircular signals. To achieve further noise suppression, the spatial conjugate power spectrum (SCPS), instead of power spectrum, is utilized. Sequentially, a direct SCPS approach and a mixed blind/nonblind beamforming (MBNB) approach are proposed.

The rest of the paper is organized as follows. In Section 1, we introduce the array model and the WL-MVDR beamformer. The two proposed DOA estimation algorithms are formulated in Section 2 and 3. Then the performance is illustrated using simulations in Section 4. And we give our conclusion in Section 5.

2. Noncircular Signal and WL-MVDR Beamformer

Consider a zero-mean stationary complex signal $s(t)$, and

$$E\{|s(t)|^2\} = \sigma^2 > 0 \tag{1}$$

$$|E\{s^2(t)\}| = \hbar\sigma^2 \geq 0 \tag{2}$$

where $\hbar \in [0,1]$ is referred to as the noncircularity rate of the signal. $s(t)$ is said to be fully/completely noncircular if $\hbar = 1$ (for this case, $s(t)$ is perfectly correlated with $s^*(t)$), partially noncircular if $0 < \hbar < 1$, and circular if $\hbar = 0$. In this paper, we consider signals that are almost completely noncircular, i.e., $\hbar \to 1$.

Assume an N-element array illuminated by M statistically independent near-perfectly noncircular far-field and narrowband signals, $\{s_m(t)\}_{m=1}^{M}$, the output vector of the array can be written as:

$$\boldsymbol{x}(t) = \sum_{m=1}^{M} \boldsymbol{a}_m s_m(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{3}$$

where $\boldsymbol{a}_m$ is the steering vector of $s_m(t)$, and

$$\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_M] \tag{4}$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_M(t)]^{\mathrm{T}} \tag{5}$$

and $\boldsymbol{n}(t)$ is the zero-mean, white and circular noise vector.

The covariance and conjugate covariance matrices of the

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array output are respectively given by

$$\boldsymbol{R}_{xx} = E\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\} = \boldsymbol{A}\boldsymbol{R}_{ss}\boldsymbol{A}^{\mathrm{H}} + \sigma_{\mathrm{n}}^2\boldsymbol{I}_N \tag{6}$$

$$\boldsymbol{R}_{xx^*} = E\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{T}}(t)\} = \boldsymbol{A}\boldsymbol{R}_{ss^*}\boldsymbol{A}^{\mathrm{T}} \tag{7}$$

where σ_{n}^2 is the variance of noise, and $\boldsymbol{I}_N$ denotes an $N \times N$ identity matrix, and

$$\boldsymbol{R}_{ss} = E\{\boldsymbol{s}(t)\boldsymbol{s}^{\mathrm{H}}(t)\}, \quad \boldsymbol{R}_{ss^*} = E\{\boldsymbol{s}(t)\boldsymbol{s}^{\mathrm{T}}(t)\} \tag{8}$$

The output of the Widely Linear Minimum Variance Distortionless Response (WL-MVDR) beamformer [14] is:

$$y(t) = \boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{x}(t) + \boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{x}^*(t) = \boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{x}}(t) \tag{9}$$

where

$$\boldsymbol{w} = [\boldsymbol{w}_1^{\mathrm{T}}, \boldsymbol{w}_2^{\mathrm{T}}]^{\mathrm{T}} \tag{10}$$

$$\tilde{\boldsymbol{x}}(t) = [\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{H}}(t)]^{\mathrm{T}} \tag{11}$$

The design criterion for WL-MVDR beamformer weight vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$, is as follows [14]:

$$\min_{\boldsymbol{w}_1, \boldsymbol{w}_2} E\{|\, y(t)\,|^2\} \quad \text{s.t.} \quad \boldsymbol{w}_1^{\mathrm{H}}\boldsymbol{a} = 1,\ \boldsymbol{w}_2^{\mathrm{H}}\boldsymbol{a}^* = 0 \tag{12}$$

where $\boldsymbol{a}$ is the steering vector of the signal-of-interest. Using the Lagrange multiplier technique, the solution to (12) can be easily obtained, as

$$\boldsymbol{w}_{\text{WL-MVDR}} = \boldsymbol{R}_{\tilde{x}\tilde{x}}^{-1}\boldsymbol{B}[\boldsymbol{B}^{\mathrm{H}}\boldsymbol{R}_{\tilde{x}\tilde{x}}^{-1}\boldsymbol{B}]^{-1}\boldsymbol{f} \tag{13}$$

where

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{a}^* \end{bmatrix} \tag{14}$$

$$\boldsymbol{R}_{\tilde{x}\tilde{x}} = E\{\tilde{\boldsymbol{x}}(t)\tilde{\boldsymbol{x}}^{\mathrm{H}}(t)\} \tag{15}$$

$$\boldsymbol{f} = [1, 0]^{\mathrm{T}} \tag{16}$$

where $\boldsymbol{0}_N$ denotes an $N \times 1$ zero vector.

Thus, the WL-MVDR beamformer weight vector extracting the signal from a certain direction θ is given by

$$\boldsymbol{w}_{\text{WL-MVDR}}(\theta) = \boldsymbol{R}_{\tilde{x}\tilde{x}}^{-1}\boldsymbol{B}(\theta)[\boldsymbol{B}^{\mathrm{H}}(\theta)\boldsymbol{R}_{\tilde{x}\tilde{x}}^{-1}\boldsymbol{B}(\theta)]^{-1}\boldsymbol{f} \tag{17}$$

where

$$\boldsymbol{B}(\theta) = \begin{bmatrix} \boldsymbol{a}(\theta) & \boldsymbol{0}_N \\ \boldsymbol{0}_N & \boldsymbol{a}^*(\theta) \end{bmatrix} \tag{18}$$

3. DOA Estimation via Spatial Conjugate Power Spec trum

In [17], Borgiotti and Kaplan propose an *Adapted Angular Response* (AAR) approach utilizing the spatial power spectrum for DOA estimation of point-source signals. In [18], AAR is shown to better than the standard Capon method [19]. The AAR spatial spectrum is given by:

$$P_{\text{AAR}}(\theta) = \frac{\boldsymbol{w}_{\text{CAPON}}^{\mathrm{H}}(\theta)\boldsymbol{R}_{xx}\boldsymbol{w}_{\text{CAPON}}(\theta)}{\boldsymbol{w}_{\text{CAPON}}^{\mathrm{H}}(\theta)\boldsymbol{w}_{\text{CAPON}}(\theta)} = \frac{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_{xx}^{-2}\boldsymbol{a}(\theta)} \tag{19}$$

where

$$\boldsymbol{w}_{\text{CAPON}}(\theta) = \frac{\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)} \tag{20}$$

Motivated by AAR, we define the following *Spatial Conjugate Power Spectrum* (SCPS) expression:

$$P_{\text{SCPS}}(\theta) = \frac{|\,\boldsymbol{w}_{\text{WL-MVDR}}^{\mathrm{H}}(\theta)\boldsymbol{R}_{\tilde{x}\tilde{x}^*}\boldsymbol{w}_{\text{WL-MVDR}}^*(\theta)\,|}{\boldsymbol{w}_{\text{WL-MVDR}}^{\mathrm{H}}(\theta)\boldsymbol{w}_{\text{WL-MVDR}}(\theta)} \tag{21}$$

where

$$\boldsymbol{R}_{\tilde{x}\tilde{x}^*} = E\{\tilde{\boldsymbol{x}}(t)\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\} \tag{22}$$

4. Simulation Results

In this section, we provide some numerical examples to illustrate the performance of the two proposed estimators: SCPS and MBNB, compared with the standard Capon and AAR algorithms. We assume a uniform linear array composed of 6 sensors spaced half wavelength apart, illuminated by two equal-power narrowband far-field UQPSK signals, from 40° and 60°, with noncircular rates equal to 0.98 and 0.99, respectively. The background noise is assumed to be circular additive white Gaussian. All the results are averaged via 1000 Monte-Carlo simulation runs. Fig. 1 and Fig. 2 illustrate the RMSE versus signal-to-noise ratio (SNR) for 200 snapshots and the RMSE versus the number of snapshots for SNR = 10 dB, respectively.

Then we investigate the performance in the presence of element position errors (0.5%). Fig. 3 and Fig. 4 depict the RMSE versus SNR for 200 snapshots and the RMSE versus the number of snapshots for SNR = 10 dB, respectively.

At last, we investigate the performance in the presence of channel mismatch (1% in magnitude and 1° in phase). Fig. 5 and Fig. 6 show the RMSE versus SNR for 200 snapshots and the RMSE versus the number of snapshots for SNR = 10 dB, respectively.

It can be seen from Figs. 1-6 that SCPS and MBNB have a better performance in terms of RMSE compared with Capon and AAR, and the improvements is more pronounced in the presence of low SNR values and model errors.

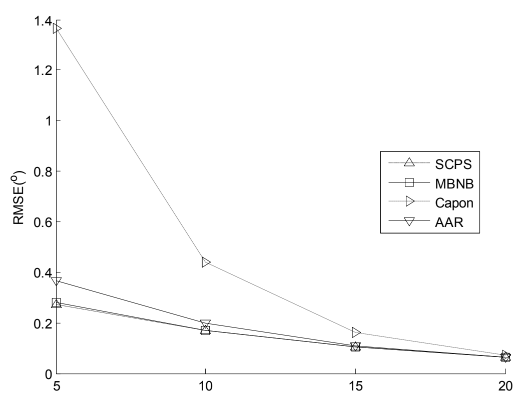

Fig. 1 RMSE versus SNR

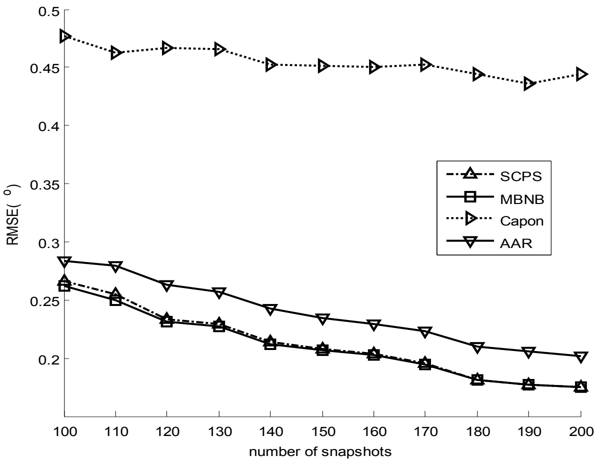

Fig. 2 RMSE versus number of snapshots

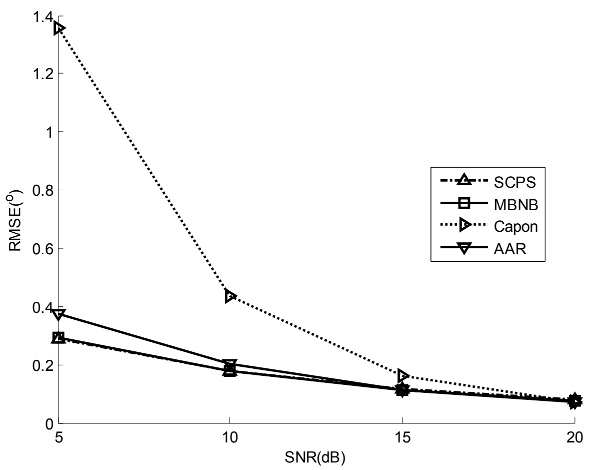

Fig. 3 RMSE versus SNR in the presence of element position errors

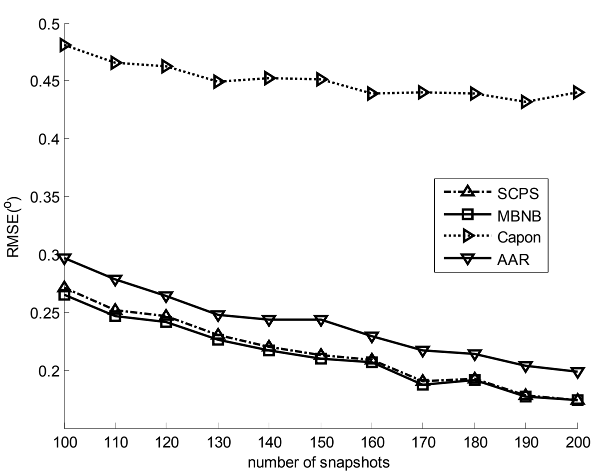

Fig. 4 RMSE versus number of snapshots in the presence of element position errors

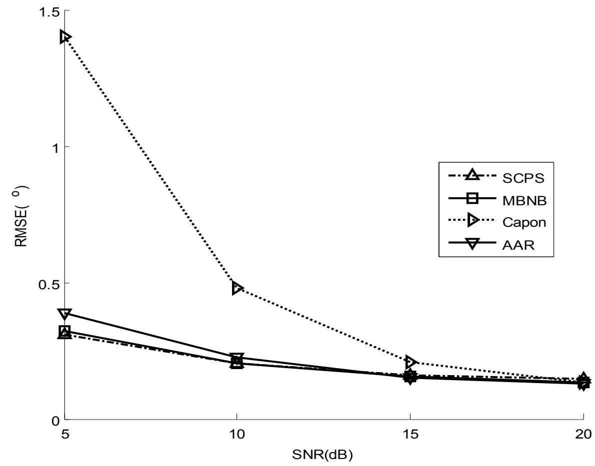

Fig. 5 RMSE versus SNR in the presence of channel mismatch

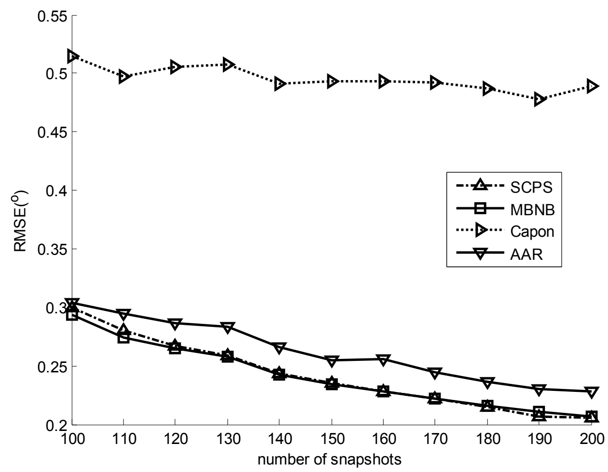

Fig. 6 RMSE versus number of snapshots in the presence of channel mismatch

5. Conclusion

We have proposed two widely linear MVDR-based DOA estimators for partially noncircular signals. Numerical simulations have demonstrated the performance improvements compared with other algorithms. To conclude the paper, we remark that WL-MVDR beamformer used in SCPS and MBNB can be replaced with any other nonblind beamformers, e.g., WL-MVDR$_2$ beamformer [20] (while more *a priori* knowledge needed) and the standard Capon beamformer.

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