Joint Estimation of Doppler Frequency and Angle in Bistatic MIMO Radar System*

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Abstract - Addressed with bistatic multiple-input and multiple-output (MIMO) radar, a joint estimator of both Doppler frequency and angle is proposed. The angle of the target are estimated using beamforming technique after the Doppler frequency is estimated via MUSIC or ESPRIT approach, which are automatically paired without extra computation burden. The method has better angle resolution than conventional subspace decomposition algorithms. The simulation results demonstrate its effectiveness and robustness.

Index Terms - Array signal processing, MIMO radar, bistatic, beamforming, Doppler frequency estimation, angle estimation

1. Introduction

A multiple-input multiple-output (MIMO) radar uses multiple antennas to simultaneously transmit several waveforms, which can be either correlated or not, and it also uses multiple antennas to receive the reflected signals. It has been shown that by exploiting the waveform diversity and spatial diversity, MIMO radar outperforms conventional phased-array radar in terms of parameter estimation, target detection and radar imaging, etc. [1-3].

In bistatic MIMO radar, which has the configuration with separated transmitting array and receiving array, the reflected signal contains the information about the angle of the target with respect to transmitting array normal and the receive array normal. It must estimate the transmit angle and receive angle simultaneously when locating the target. In much of current literature, the subspace decomposition algorithm such as MUSIC or ESPRIT is used for estimating parameters in the bistatic MIMO radar. In [4], the ESPRIT algorithm is firstly used for target direction estimation in a bistatic MIMO radar with utilization of the invariance property of the transmitting array and the receiving array, which decomposes the two-dimensional angle estimation problem into two 1-D angle estimation problems, and it requires the additional parameter pairing algorithm. In [5], the interrelation between the two one-dimensional ESPRIT is utilized to obtain automatically paired transmit angle and receive angle estimation without debasing the performance of angle estimation in a bistatic MIMO radar. In [6], the Doppler frequency of reflected signal is considered. The method estimates the Doppler frequency and angle via ESPRIT by means of the rotational factor produced by time delay sampling. However, the performance of estimation of Doppler frequency degrades drastically when SNR is low. What's more, the signal model for estimating the angle in the above literature is established when the array geometry is regular.

In recent years, there has been an increased investigation on utilizing the signal's temporal domain characters and the array's spatial domain characters simultaneously to improve the performance of parameter estimation and beamforming. The signal's temporal domain character mainly includes constant modulus property, cyclostationarity, noncircularity, Doppler frequency and so on [7-10]. In this paper, we propose a method for angle estimation via beamforming for Doppler frequency of reflected signal. The method has good angle resolution and the pairing is automatically obtained.

The remainder of this paper is organized as follows. In Section II, we propose the beamforming approach to estimate angle and describe the associated data model. In Section III, we estimate the Doppler frequency via subspace decomposition algorithm. We provide several simulation results in Section IV. Finally, Section V contains our conclusion.

2. Angle Estimation

Consider a MIMO narrowband radar system with M arbitrarily located transmitting antennas and N arbitrarily located receiving antennas as shown in Fig. 1. The system transmits K coherent pulses. Assume that there are P uncorrelated targets located at the same range bin, which are in the far field of the array. Let φ_p and θ_p be the transmit angle and receive angle of the pth target, respectively, and f_{dp} denotes the Doppler frequency of the pth target reflected signal. The output of the entire matched filters at the receive can be expressed as

$$\boldsymbol{X}(k)=\boldsymbol{C}(\varphi,\theta)\boldsymbol{s}(k)+\boldsymbol{N}(k),\ k=1,\cdots,K \tag{1}$$

where $\boldsymbol{C}(\varphi,\theta)=[\boldsymbol{c}(\varphi_1,\theta_1),\cdots,\boldsymbol{c}(\varphi_P,\theta_P)]\in\mathbb{C}^{NM\times P}$ denotes the array manifold matrix, and $\boldsymbol{c}(\varphi_p,\theta_p)=\boldsymbol{b}(\varphi_p)\otimes\boldsymbol{a}(\theta_p)$ is steering vector of the pth target, $\boldsymbol{b}(\varphi_p)$ is the steering vector of transmitting array for the pth target, and $\boldsymbol{a}(\theta_p)$ is the corresponding steering vector of receiving array, $\otimes$ denotes the Kronecker product; $\boldsymbol{s}(k)=[s_1(k),\cdots,s_P(k)]^{\mathrm{T}}$, $[\cdot]^{\mathrm{T}}$ denotes the transpose, $s_p(k)=\alpha_p e^{j(k-1)\psi(f_{dp})}$ with α_p denotes the complex amplitude of the reflected signal, which is related to the radar cross section (RCS) and path loss, $\psi(f_{dp})=2\pi f_{dp}/f_r$ with f_r denotes pulse repetition frequency; $\boldsymbol{N}(k)$ is an $NM\times 1$ noise vector assumed to be independent, zero-mean complex Gaussian distribution with covariance matrix $\sigma_n^2\boldsymbol{I}$, where $\boldsymbol{I}$ de-

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notes the identity matrix.

It is assumed that the target motion is uniform which leads to the Doppler frequency is constant and the complex amplitude is almost stationary. Further assume that the Doppler frequencies of different target reflected signals are not equal. Let $\boldsymbol{w} \in \mathbb{C}^{NM\times 1}$ is the weight vector of the beamformer, so the output of beamformer can be expressed as follows

$$y(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(k) \tag{2}$$

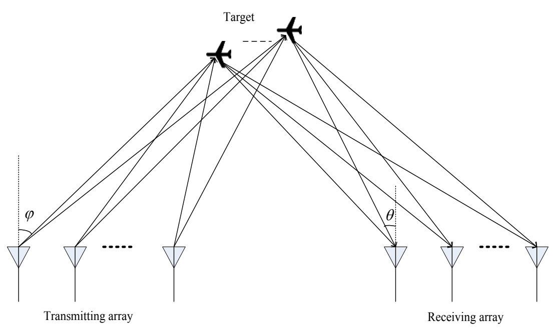

Fig. 1. Bistatic MIMO radar system

Construct the weight vector which is orthogonal to the steering vector $\boldsymbol{c}(\varphi_j,\theta_j),\ j \neq p$, and the inner product of $\boldsymbol{c}(\varphi_p,\theta_p)$ and the weight vector is equal to $1/\alpha_p$, so $y(k)$ approximates to $\exp(j(k-1)\psi(f_{dp}))$. As a result, the weight vector can be obtained by the following optimization problem

$$\min_{\boldsymbol{w},f_d} F(\boldsymbol{w},f_d) = \min_{\boldsymbol{w},f_d} \frac{1}{K}\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(k) - \exp\left(j(k-1)\psi(f_d)\right)\right|^2 \tag{3}$$

Rewrite the cost function as,

$$\begin{aligned}\mathrm{F}(\boldsymbol{w},f_d) = {}& \boldsymbol{w}^{\mathrm{H}}\left(\frac{1}{K}\sum_{k=0}^{K-1}\boldsymbol{X}(k)\boldsymbol{X}^{\mathrm{H}}(k)\right)\boldsymbol{w} - \\ & \boldsymbol{w}^{\mathrm{H}}\left(\frac{1}{K}\sum_{k=0}^{K-1}\boldsymbol{X}(k)\exp\left(-j(k-1)\psi(f_d)\right)\right) + 1 - \\ & \left(\frac{1}{K}\sum_{k=0}^{K-1}\exp\left(j(k-1)\psi(f_d)\right)\boldsymbol{X}^{\mathrm{H}}(k)\right)\boldsymbol{w}\end{aligned} \tag{4}$$

And

$$\hat{\boldsymbol{R}}_X = \frac{1}{K}\sum_{k=0}^{K-1}\boldsymbol{X}(k)\boldsymbol{X}^{\mathrm{H}}(k) \tag{5}$$

$$\hat{\boldsymbol{u}}(f_d) = \frac{1}{K}\sum_{k=0}^{K-1}\boldsymbol{X}(k)\exp\left(-j(k-1)\psi(f_d)\right) \tag{6}$$

thus the cost function yields

$$\mathrm{F}(\boldsymbol{w},f_d) = \boldsymbol{w}^{\mathrm{H}}\hat{\boldsymbol{R}}_X\boldsymbol{w} - \boldsymbol{w}^{\mathrm{H}}\hat{\boldsymbol{u}}(f_d) + 1 - \hat{\boldsymbol{u}}^{\mathrm{H}}(f_d)\boldsymbol{w} \tag{7}$$

take the gradient of the cost function with respect to $\boldsymbol{w}$

$$\nabla\mathrm{F}(\boldsymbol{w},f_d) = 2\hat{\boldsymbol{R}}_X\boldsymbol{w} - 2\hat{\boldsymbol{u}}(f_d) \tag{8}$$

when the covariance matrix $\hat{\boldsymbol{R}}_X$ is nonsingular, for a mixed Doppler frequency f_{dp}, the weight vector can be expressed as

$$\boldsymbol{w} = \hat{\boldsymbol{R}}_X^{-1}\hat{\boldsymbol{u}}(f_{dp}) \tag{9}$$

When obtaining the weight vector, the Doppler frequency of reflected signal rather than the array geometry or amplitude and phase response must be estimated firstly, so it is a blind beamformer. In order to avoid the performance degradation caused by the small eigenvalues of $\hat{\boldsymbol{R}}_X$, the diagonal loading technique can be used, and the diagonal loading value should be selected to [11]

$$\varepsilon = \frac{\mathrm{trace}\left(\hat{\boldsymbol{R}}_X\right)}{NM} \tag{10}$$

so the weight vector can be rewritten as

$$\boldsymbol{w} = \left(\hat{\boldsymbol{R}}_X + \varepsilon\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{u}}(f_d) \tag{11}$$

The output of the beamformer can be expressed as

$$\hat{s}_p(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(k) \tag{12}$$

the transmit angle and the receive angle of pth target can be estimated as

$$\left(\varphi_p,\theta_p\right) = \arg\max_{(\varphi,\theta)} \frac{1}{\det\left(\mathrm{corrcoef}\left(\boldsymbol{c}(\varphi,\theta)\hat{s}_p(k),\boldsymbol{X}(k)\right)\right)} \tag{13}$$

where $\det(\cdot)$ denotes the matrix determinant and $\mathrm{corrcoef}(\boldsymbol{x},\boldsymbol{y})$ denotes the correlation coefficient between two matrices.

3. Doppler Frequency Estimation

According to (1), the output of K coherent pulses can be expressed as

$$\boldsymbol{X} = \boldsymbol{C}(\varphi,\theta)\boldsymbol{S} + \boldsymbol{N} \tag{14}$$

the signal matrix can be expressed as

$$\begin{aligned}\boldsymbol{S} &= \begin{bmatrix} \alpha_1 & \alpha_1 e^{j\psi(f_{d1})} & \cdots & \alpha_1 e^{j(K-1)\psi(f_{d1})} \\ \alpha_2 & \alpha_2 e^{j\psi(f_{d2})} & \cdots & \alpha_1 e^{j(K-1)\psi(f_{d2})} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_P & \alpha_P e^{j\psi(f_{dP})} & \cdots & \alpha_P e^{j(K-1)\psi(f_{dP})} \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_P \end{bmatrix}}_{\boldsymbol{D}} \begin{bmatrix} 1 & e^{j\psi(f_{d1})} & \cdots & e^{j(K-1)\psi(f_{d1})} \\ 1 & e^{j\psi(f_{d2})} & \cdots & e^{j(K-1)\psi(f_{d2})} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\psi(f_{dP})} & \cdots & e^{j(K-1)\psi(f_{dP})} \end{bmatrix} \\ &= \boldsymbol{D}\boldsymbol{F}^{\mathrm{T}}(f_d)\end{aligned} \tag{15}$$

where $\boldsymbol{D}=\operatorname{diag}(\boldsymbol{\alpha})$, $\boldsymbol{\alpha}=[\alpha_1,\alpha_2,\cdots,\alpha_P]^{\mathrm{T}}$, and $\operatorname{diag}(\cdot)$ denotes a diagonal matrix whose the pth diagonal entry is α_p. By inserting (15) into (14), the equation can be rewritten as

$$\boldsymbol{X}=\boldsymbol{C}(\varphi,\theta)\boldsymbol{D}\boldsymbol{F}^{\mathrm{T}}(f_d)+\boldsymbol{N} \tag{16}$$

the transpose of the matrix $\boldsymbol{X}$ is

$$\boldsymbol{X}_t=\boldsymbol{X}^{\mathrm{T}}=\boldsymbol{F}(f_d)\boldsymbol{D}\boldsymbol{C}^{\mathrm{T}}(\varphi,\theta)+\boldsymbol{N}_t \tag{17}$$

where $\boldsymbol{N}_t=\boldsymbol{N}^{\mathrm{T}}$. The covariance matrix of $\boldsymbol{X}_t$ is

$$\hat{\boldsymbol{R}}_{X_t}=\boldsymbol{X}_t\boldsymbol{X}_t^{\mathrm{H}}/NM=\boldsymbol{F}(f_d)\hat{\boldsymbol{R}}_s\boldsymbol{F}^{\mathrm{H}}(f_d)+\hat{\boldsymbol{R}}_{N_t} \tag{18}$$

where $\hat{\boldsymbol{R}}_s=[\boldsymbol{D}\boldsymbol{C}^{\mathrm{T}}(\varphi,\theta)\boldsymbol{C}^{*}(\varphi,\theta)\boldsymbol{D}^{*}]/NM$, and $(\cdot)^{*}$ denotes the complex conjugate. When $\operatorname{rank}(\boldsymbol{D})=\operatorname{rank}(\boldsymbol{D}^{*})=P$ and the manifold matrix $\boldsymbol{C}(\varphi,\theta)$ is a full-rank matrix, so

$$\operatorname{rank}(\hat{\boldsymbol{R}}_s)=P \tag{19}$$

The eigenvalue decomposition of $\hat{\boldsymbol{R}}_{X_t}$ is

$$\hat{\boldsymbol{R}}_{X_t}=\boldsymbol{E}_s\boldsymbol{\Sigma}_s\boldsymbol{E}_s^{\mathrm{H}}+\boldsymbol{E}_n\boldsymbol{\Sigma}_n\boldsymbol{E}_n^{\mathrm{H}} \tag{20}$$

where $\boldsymbol{E}_s$ is the matrix of eigenvectors in the signal subspace, $\boldsymbol{\Sigma}_s=\operatorname{diag}(\lambda_1,\lambda_2,\cdots,\lambda_P)$ is a diagonal matrix containing the P largest eigenvalues associated to the columns of $\boldsymbol{E}_s$; $\boldsymbol{E}_n$ is the noise eigenvectors matrix, $\boldsymbol{\Sigma}_n=\operatorname{diag}(\lambda_{P+1},\lambda_{P+2},\cdots,\lambda_K)$ is a diagonal matrix with diagonal entries equal to the eigenvalues corresponding to the small eigenvalues. Using the MUSIC approach, we have

$$\left\|\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{f}(f_{dp})\right\|^2=\boldsymbol{f}^{\mathrm{H}}(f_{dp})\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{f}(f_{dp})=0,\quad p=1,\cdots,P \tag{21}$$

The Doppler frequency can also be estimated via ESPRIT algorithm. Using the Doppler frequencies estimated, the angles are estimated via (11).

4. Simulation Results

In this section, we present the simulation results in order to illustrate the performance of the proposed method.

Consider a bistatic MIMO radar system where a uniform linear array with $M=N=10$ antennas and half-wavelength spacing between adjacent antennas is used both for transmitting and receiving array. The pulse repetition frequency is 200 Hz, and other parameters are given in the examples.

First, we compare the estimation of Doppler frequency performance of the proposed method to the algorithm developed in [6] in term of RMSE. Assume that there exist $P=2$ uncorrelated targets located at the range bin, the Doppler frequency are 30 Hz and 60 Hz, respectively. The RMSE of Doppler frequency with SNR and number of pulses are shown in Fig. 2 and Fig. 3, respectively. In Fig. 2 the number of pulses is taken $K=100$ and in Fig. 3 the SNR is fixed at 5 dB. The number of Monte Carlo trials is 1000 in both two simulations.

From Fig.2 and Fig. 3, we can see that the proposed method outperforms the method in [5] in estimation of Doppler frequency.

Next, we consider a challenging scenario where there are two targets at the same range bin; the targets are located at $(\varphi_1,\theta_1)=(10^\circ,30^\circ)$ and $(\varphi_2,\theta_2)=(12^\circ,32^\circ)$ while all the other parameters are the same as before. The RMSE of angle with SNR and number of pulses are shown in Fig. 4 and Fig. 5, respectively. In Fig. 4 the number of pulses is taken $K=50$ and in Fig. 5 the SNR is fixed at 5 dB. The number of Monte Carlo trials is 1000 in both two simulations.

From Fig. 4 and Fig. 5, we can see that the proposed method has better angle resolution than ESPRIT, especially when the number of pulses is fewer.

5. Conclusion

We propose a new approach for Doppler frequency and angle estimation in bistatic MIMO radar. This method uses the subspace decomposition algorithm to estimate the Doppler frequency and the beamforming for transmit angle and receive angle estimation. Simulation results show that the proposed method has good Doppler frequency accuracy and better angle resolution than conventional high resolution algorithm. Furthermore, this method allows an automatic pairing between the Doppler frequency and angle.

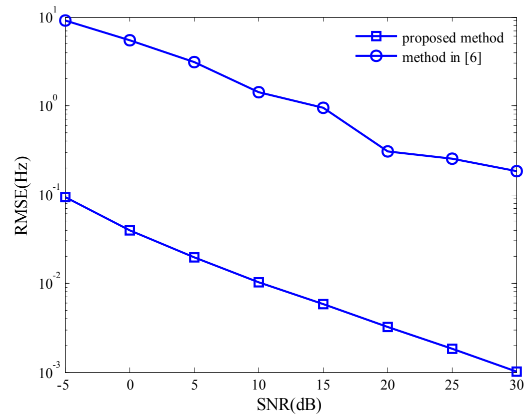

Fig. 2. RMSE of Doppler frequency estimation with SNR: $K=100$

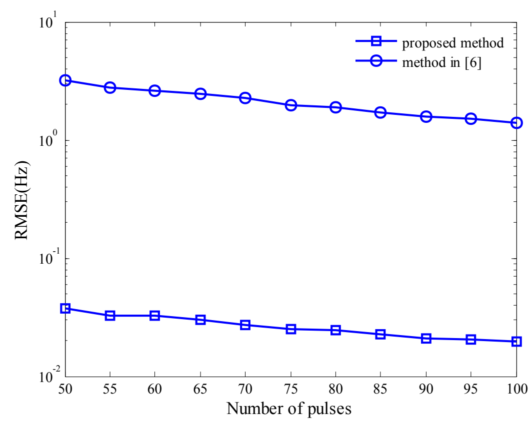

Fig. 3. RMSE of Doppler frequency estimation with number of pulses: SNR = 5 dB

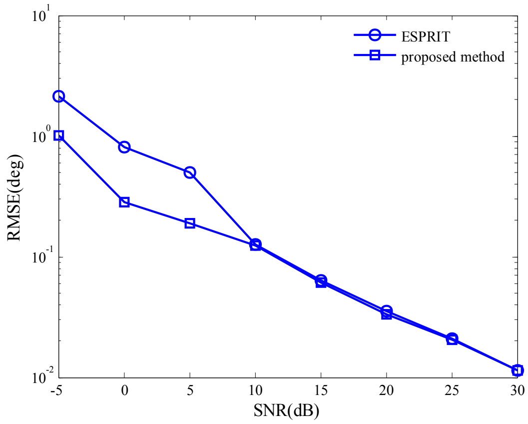

Fig. 4. RMSE of angle estimation with SNR: $K = 50$

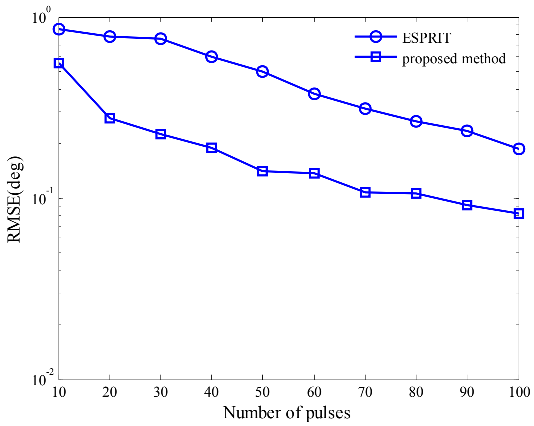

Fig. 5. RMSE of angle estimation with number of pulses: SNR = 5 dB

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